New Fuzzy Aggregations.
Part I: General Decision Making System and its Information Structure

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Abstract: - The Ordered Weighted Averaging (OWA) operator was introduced by R.R. Yager [58] to provide a method for aggregating inputs that lie between the max and min operators. In this article two variants of probabilistic extensions the OWA operator - POWA and FPOWA (introduced by J.M. Merigo [27, 28]) are considered as a basis of our generalizations in the environment of fuzzy uncertainty (parts II and III of this work), where different monotone measures (fuzzy measure) are used as uncertainty measures instead of the probability measure. For the identification of "classic" OWA and new operators (presented in parts II and III) of aggregations, the Information Structure is introduced where the incomplete available information in the general decision making system is presented as a condensation of uncertainty measure, imprecision variable and objective function of weights.

Key-Words: - mean aggregation operators, fuzzy aggregations, fuzzy measure, fuzzy numbers, fuzzy decision making.

1 Introduction

It is well recognized that intelligent decision support systems and technologies have been playing an important role in improving almost every aspect of human society. Intensive study over the past several years has resulted in significant progress in both the theory and applications of optimization and decision sciences.

Optimization and decision-making problems are traditionally handled by either the deterministic or the probabilistic approach. When working with complex systems in parallel with classical approaches of their modelling, the most important matter is to assume fuzziness ([3, 6, 14, 16-33, 36-44, 50-63] and others). All this is connected to the complexity of study of complex and vague processes and events in nature and society, which are caused by lack or shortage of objective information and when expert data are essential for construction of credible decisions. With the growth of complexity of information our ability to make credible decisions from possible alternatives with complex states of nature reduces to some level, below which some dual characteristics such as precision and certainty become mutually conflicting ([3, 11, 21-23, 37-39, 42, 50, 52, 55, 56] and others). When working on real, complex decision systems using an exact or some stochastic quantitative analysis is often less convenient, concluding that the use of fuzzy methods is necessary, because systems approach for development of information structure of investigated decision system [21, 37, 38] with combined fuzzy-stochastic uncertainty enables us to construct convenient intelligent decision support instruments. Obviously the source for obtaining combined objective + fuzzy + stochastic samplings is the populations of fuzzy-characteristics of experts knowledge ([23, 37, 39, 43, 52] and others). Our research is concerned with quantitative-information analysis of the complex uncertainty and its use for modelling of more precise decisions with minimal decision risks from the point of view of systems research. The main objects of our attention are 1) the analysis of Information Structures of expert's knowledge, its uncertainty measure and imprecision variable; 2) the construction of instruments of aggregation operators, which condense both characteristics of incomplete information - an uncertainty measure and an imprecision variable in the scalar ranking values of possible alternatives in the decision making system. The first problem is considered in this paper. The second problem will be presented in the Parts II and III of this work.

Making decisions under uncertainty is a pervasive task faced by many decision making persons (DMP), experts, investigators or others. The main difficulty is that a selection must be made between alternatives in which the choice of alternative doesn't necessarily lead to well determined payoffs (experts valuations, utilities and so on) to be received as a result of selecting an alternative. In this case DMP is faced with the problem of comparing multifaceted objects whose complexity often exceeds his/her ability to compare of uncertain alternatives. One approach to addressing this problem is to use valuation functions (or aggregation operators). These valuation functions convert the multifaceted uncertain outcome associated with an alternative into a single (scalar) value. This value provides a characterization of the DMP or expert perception of the worth the possible uncertain alternative being evaluated. The problems of Decision Making Under Uncertainty (DMUU) [52] were discussed and investigated by many well-known authors ([1-6, 9, 10, 14, 16-19, 24-61, 63] and others). In this work our focus is directed on the construction of new generalizations of the aggregation OWA operator in the fuzzy-probabilistic uncertainty environment.

In Section 2 some preliminary concepts are presented on the OWA operator; on the arithmetic of the triangular fuzzy numbers; on the some extensions of the OWA operator – POWA and FPOWA operators in the probabilistic uncertainty (developed by J.M. Merigo [27, 28]) and their information measures (see Section 3). In Section 4 a new conceptual Information Structure (IS) of a General Decision Making System (GDMS) with fuzzy-probabilistic uncertainty is defined. This IS classifies some aggregation operators and new generalizations of the OWA operator defined in the parts II and III of this work.

2 On the OWA Operator and Its some Fuzzy-Probabilistic Generalizations

In this type of problem the DMP has a collection $D=\{d_1,d_2,...,d_n\}$ of possible uncertain alternatives from which he must select one or some ranking of decisions by some expert's preference relation values. Associated with this problem is a variable of characteristics, activities, symptoms and so on, which acts on the decision procedure. This variable is normally called the state of nature, which affects the payoff, utilities, valuations and others to the DMP's preferences or subjective activities. This variable is assumed to take its values (states of nature) from some set $S=\{s_1,s_2,...,s_m\}$. As a result the DMP knows that if he selects d_i and the state of nature assumes the value s_j then his payoff (valuation, utility and so on) is a_{ij}. The objective of the decision is to select the "best" alternative, get the biggest payoff (valuation, utility and so on). But in DMUU [52] the selection procedure becomes more difficult. In this case each alternative can be seen as corresponding to a row vector of possible payoffs. To make a choice the DMP must compare these vectors, a problem which generally doesn't lead to a compelling solution. Assume d_i and d_k are two alternatives such that for all $j,\ j=1,2,....,m$ $a_{ij}\geq a_{kj}$ (Table 1). In this case there is no reason to select d_k. In this situation we shall say d_i dominates d_k $(d_i \succeq d_k)$. Furthermore if there exists one alternative that dominates all the alternatives then it will be optimal solution and as a result, we call this the *Pareto optimal*.

Faced with the general difficulty of comparing vector payoffs we must provide some means of comparing these vectors. Our focus in this work is on the construction of valuation function (aggregation operator) F that can take a collection of m values and convert it into a single value,

$F: R^m \Rightarrow R^1$.

Once we apply this function to each of the alternatives we select the alternative with the *largest scalar value*. The construction of F involves considerations of two aspects. The first being the satisfaction of some rational, objective properties naturally required of any function used to convert (aggregate) a vector of payoffs (valuations, utilities and so on) into an equivalent scalar value. The second aspect being the inclusion of characteristics particular to the DMP's subjective properties or preferences, dependences with respect to risks and other main external factors.

Table 1. Decision Matrix.

D \ S	s_1	s_2	…	s_k	…	s_m
d_1	a_{11}	a_{12}	…	a_{1k}	…	a_{1m}
d_2	a_{21}	a_{22}	…	a_{2k}	…	a_{2m}
…	…	…	…	…	…	…
d_i	a_{i1}	a_{i2}	…	a_{ik}	…	a_{im}
…	…	…	…	…	…	
d_n	a_{n1}	a_{n2}	…	a_{nk}	…	a_{nm}

First we shall consider the objective properties required of the valuation function (aggregation operator) F [52].

1) The first property is the satisfaction of Pareto optimality. To insure this we require that if $a_{ij} \geq a_{kj}$ for $j = 1, 2, ..., m$, then
$$F(a_{i1}, a_{i2}, ..., a_{im}) \geq F(a_{k1}, a_{k2}, ..., a_{km}). \qquad (1)$$
An aggregation operator satisfying this condition is said to be *monotonic*.
2) A second condition is that the value of an alternative should be bounded by its best payoffs (valuations, utilities) and worst possible one. $\forall i = 1, 2, ..., n$
$$\min_{j=1,m}\{a_{ij}\} \leq F(a_{i1}, a_{i2}, ..., a_{im}) \leq \max_{j=1,m}\{a_{ij}\}. \qquad (2)$$
This condition is said to be *bounded.*
3) Remark: if $a_{ij} \equiv a_i$ for all j, then from (2)
$\min_{j=1,m}\{a_{ij}\} = \max_{j=1,m}\{a_{ij}\}$ and $F(a_{i1}, a_{i2}, ..., a_{im}) = a_i$
This condition is said to be *idempotent*.
4) The final objective condition is that the indexing of the states of nature shouldn't affect the answer:
$$F(a_{i1}, a_{i2}, ..., a_{im}) = \\ = F(Permutation(a_{i1}, a_{i2}, ..., a_{im})), \qquad (3)$$
where $Permutation(\cdot)$ is some permutation of the set $\{a_{i1}, a_{i2}, ..., a_{im}\}$. An aggregation function satisfying this is said to be *symmetric* (or commutative).

Finally, we have required that our aggregation function satisfy four conditions: *monotonicity, boundedness, idempotency* and *symmetricity*. Such functions are called *mean* or *averaging operators* [52].

In determining which of the many possible aggregation operators to select as our valuation function we need some *guidance* from the DMP. The choice of a valuation function, from among the aggregation operators is essentially a "subjective" act reflecting the preferences of the DMP for one vector of payoffs over another. What is needed are tools and procedure to enable a DMP to reflect their subjective preferences into valuations. There are important problems in expert knowledge engineering for which we often use such intelligent technologies as neural networks, machine learning, fuzzy logic control systems, knowledge representations and others.

These problems may be solved by introducing information measures of aggregation operators ([1, 2, 4, 13, 14, 16, 17, 27-34, 36, 39, 41-43, 46-61, 63] and others). In this paper we will present new extensions of information measures of operators constructed bellow.

As an example we present some mean aggregation operators. Assume we have an m-tuple of values $\{a_1, a_2, ..., a_m\}$.

Then $F(a_1, a_2, ..., a_m) = \underset{i=1,m}{Min}\{a_i\}$ is one mean aggregation operator. The use of the operator Min corresponds to a pessimistic attitude, one in which the DMP assumes the worst thing will happen. Another example of a mean aggregation operator is $F(a_1, a_2, ..., a_m) = \underset{i=1,m}{Max}\{a_i\}$.

Here we have very optimistic valuations. Another example is the simple average: $Mean(a_1, a_2, ..., a_m) = \frac{1}{m}\sum_{i=1}^{m} a_i.$

In [58] R.R. Yager introduced a class of mean operators called Ordered Weighed Averaging (OWA) operator.

DEFINITION 1 [58]: *An OWA operator of dimension* m *is mapping* $OWA: R^m \Rightarrow R^1$ *that has an associated weighting vector* W *of dimension* m *with* $w_j \in [0;1]$ *and* $\sum_{j=1}^{m} w_j = 1$, *such that*
$$OWA(a_1, ..., a_m) = \sum_{j=1}^{m} w_j b_j, \qquad (4)$$
where b_j *is the jth largest of the* $\{a_i\}$, $i = 1, 2, ..., m$.

Note that different properties could be studied such as the distinction between descending and ascending orders, different measures for characterizing the weighting vector and different families of the OWA operator ([1, 4, 27-34, 46, 48-53, 57, 58, 60, 61, 63] and others).

The OWA operator and its modifications are among the most known mean aggregation operators to the construction of DMUU valuation functions. These aggregations are generalizations of known instrument as Choquet Integral ([5, 7, 24, 39, 42, 52, 54, 55, 58] and others), Sugeno integral ([15, 18, 25, 26, 37, 43, 45] and others) or induced mean functions ([2, 13, 61, 63] and others).

The fuzzy numbers (FN) has been studied by many authors ([11, 20] and others). It can represented in a more complete way as an imprecision variable of the incomplete information because it can consider the maximum and minimum and the possibility that the interval values may occur.

DEFINITION 2 [20]: $\tilde{a}(t): R^1 \Rightarrow [0;1]$ *is called the FN which can be considered as a generalization of the interval number:*

$$\tilde{a}(t)=\begin{cases}1 & if \;\; t\in[a_2',a_2''] \\ \dfrac{t-a_1}{a_2'-a_1} & if \;\; t\in[a_1,a_2'] \\ \dfrac{a_3-t}{a_3-a_2''} & if \;\; t\in[a_2'',a_3] \\ 0 & otherwise\end{cases} \qquad (5)$$

where $a_1 \le a_2' \le a_2'' \le a_3 \in R^1$.

In the following, we are going to review the triangular FN (TFN) [20] arithmetic operation as follows (in (5) $a_2'=a_2''$). Let $\tilde{a}$ and $\tilde{b}$ be two TFNs, where $\tilde{a}=(a_1,a_2,a_3)$ and $\tilde{b}=(b_1,b_2,b_3)$. Then

1: $\tilde{a}+\tilde{b}=(a_1+b_1,\ a_2+b_2,\ a_3+b_3)$

2: $\tilde{a}-\tilde{b}=(a_1-b_3,\ a_2-b_2,\ a_3-b_1)$

3: $\tilde{a}\times k=(ka_1,\ ka_2,\ ka_3),\ k>0$

4: $\tilde{a}^k=(a_1^k,\ a_2^k,\ a_3^k),\ k>0,\ a_i>0$

5: $\tilde{a}\cdot\tilde{b}=(a_1b_1,\ a_2b_2,\ a_3b_3),\ a_i>0,\ b_i>0$ (6)

6: $\tilde{b}^{-1}=\left\{\dfrac{1}{b_3},\dfrac{1}{b_2},\dfrac{1}{b_1}\right\},\ b_i>0$

7: $\tilde{a}>\tilde{b}$ *if* $a_2>b_2$ and
if $a_2=b_2$ *then* $\tilde{a}>\tilde{b}$
if $\dfrac{a_1+a_3}{2}>\dfrac{b_1+b_3}{2}$ *otherwise* $\tilde{a}=\tilde{b}$.

The set of all TFNs is denoted by Ψ and positive TFNs ($a_i>0$) by Ψ^+.

Note that other operations and ranking methods could be studied ([20] and others).

Now we consider some extensions of the OWA operator, mainly developed by Merigo and others [27, 28, 30], because our future investigations concern with extensions of Merigo's aggregation operators constructed on the basis of the OWA operator.

DEFINITION 3 [30]: *Let* Ψ *be the set of TFNs. A fuzzy OWA operator - FOWA of dimension* m *is a mapping* $FOWA:\Psi^m\Rightarrow\Psi$ *that has an associated weighting vector* W *of dimension* m *with* $w_j\in[0,1]$, $\sum_{j=1}^{m}w_j=1$ *and*

$$FOWA(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_m)=\sum_{j=1}^{m}w_j\tilde{b}_j \qquad (7)$$

where $\tilde{b}_j$ *is the jth largest of the* $\{\tilde{a}_i\}_{i=1}^m$, *and* $a_i\in\Psi,\ i=1,2,...,m.$

The FOWA operator is an extension of the OWA operator that uses imprecision information in the arguments represented in the form of TFNs. The reason for using this aggregation operator is that sometimes the available information presented by the DMP and formalized in payoffs (valuations, utilities and others) can't be assessed with exact numbers and it is necessary to use other techniques such as TFNs. So, in this aggregation incomplete information is presented by imprecision variable of experts reflections and formalized in TFNs. Sometimes the available information presented by the DMP (or expert) also has an uncertain character, which is presented by the probability distribution on the states of nature consequents on the payoffs of the DMP.

The fuzzy-probability aggregations based on the OWA operator was constructed by J. M. Merigo and others. One of the variants we present here:

DEFINITION 4 [28]: *A probabilistic OWA operator - POWA of dimension* m *is a mapping* $POWA:R^m\Rightarrow R^1$ *that has an associated weighting vector* W *of dimension* m *such that* $w_j\in[0,1]$ *and* $\sum_{j=1}^{m}w_j=1$ *according to the following formula:*

$$POWA(a_1,a_2,...,a_m)=\sum_{j=1}^{m}\hat{p}_jb_j \qquad (6) \qquad (8)$$

where b_j *is the jth largest of the* $\{a_i\},\ i=1,2,...,m$; *each argument* a_i *has an associated probability* p_i *with* $\sum_{i=1}^{m}p_i=1,\quad 0\le p_i\le 1,\quad \hat{p}_j=\beta w_j+(1-\beta)p_j$ *with* $\beta\in[0,1]$ *and* p_j *is the probability* p_i *ordered according to* b_j, *that is according to the jth largest of the* a_i.

Note that if $\beta=0$, we get the usual probabilistic mean aggregation (mathematical expectation - E_p with respect to probability distribution $\{p_i\}_{i=1}^m$), and if $\beta=1$, we get the OWA operator. Equivalent representation of (8) may be defined as:

$$POWA(a_1,a_2,...,a_m)=\beta\sum_{j=1}^{n}w_jb_j+$$
$$+(1-\beta)\sum_{i=1}^{m}p_ia_i=\beta\cdot OWA(a_1,a_2,...,a_m)+$$
$$+(1-\beta)\cdot E_p(a_1,a_2,...,a_m)\ . \qquad (9)$$

We often use probabilistic information in the decision making systems and consequently in their aggregation operators. Many fuzzy-probabilistic aggregations have been researched in OWA and other operators ([5, 18, 19, 27-33, 36-43, 50-54, 60, 61, 63] and others). In the following we present one of them defined in [28]:

DEFINITION 5 [28]: *Let* Ψ *be the set of TFNs. A fuzzy-probabilistic OWA operator - FPOWA of dimension* m *is a mapping* $FPOWA:\Psi^m \Rightarrow \Psi$ *that associated a weighting vector* W *of dimension* m *such that* $w_j \in [0,1]$, $\sum_{j=1}^{m} w_j = 1$, *according to the following formula:*

$$FPOWA(\tilde{a}_1, \tilde{a}_2,..., \tilde{a}_m) = \sum_{j=1}^{n} \hat{p}_j \tilde{b}_j \tag{10}$$

where $\tilde{b}_j$ is the jth largest of the $\{a_i\}_{i=1}^{m}$ are TFNs and each one has an associated probability $p_i \equiv P(\tilde{a} = \tilde{a}_i)$, with $\sum_{j=1}^{m} p_j = 1$, $0 \le p_j \le 1$, $\hat{p}_j = \beta w_j + (1-\beta) p'_j$, $\beta \in [0,1]$ and p'_j is the probability ordered according to $\tilde{b}_j \left(p'_j = P(\tilde{a} = \tilde{b}_j)\right)$ that is according to the jth largest of the $\{\tilde{a}_i\}_{i=1}^{m}$.

Analogously to (9) we present the equivalent form of the FPOWA operator as a weighted sum of the OWA operator and the mathematical expectation - E_p:

$$FPOWA(\tilde{a}_1, \tilde{a}_2,..., \tilde{a}_m) = \beta \sum_{j=1}^{n} w_j \tilde{b}_j + (1-\beta)\sum_{i=1}^{m} p_i \tilde{a}_i = \beta \cdot OWA(\tilde{a}_1, \tilde{a}_2,..., \tilde{a}_m) + (1-\beta) \cdot E_p(\tilde{a}_1, \tilde{a}_2,..., \tilde{a}_m). \tag{11}$$

In [28] the Semi-boundary condition of the aggregation operator (11) was proved. Semi-boundary condition of some operator F if defined as:

$$\beta \times \min_i\{\tilde{a}_i\} + (1-\beta) \cdot E_p(\tilde{a}_1, \tilde{a}_2,..., \tilde{a}_m) \le F(\tilde{a}_1, \tilde{a}_2,..., \tilde{a}_m) \le \beta \times \max_i\{\tilde{a}_i\} + (1-\beta) \cdot E_p(\tilde{a}_1, \tilde{a}_2,..., \tilde{a}_m). \tag{12}$$

So the FPOWA operator is monotonic, bounded, idempotent, symmetric and semi-bounded.

3 On the Information Measures of the POWA and FPOWA Operators

As preliminary concepts of our investigation we present four probabilistic information measures of the POWA and FPOWA operators defined in [28] following similar methodology developed for the OWA operator ([1, 2, 3, 6, 48, 49, 51, 53] and others).

a) The *Orness* parameter classifies the POWA and FPOWA operators in regard to their location between *and* and *or*:

$$\alpha(\hat{p}_1, \hat{p}_2,..., \hat{p}_m) = \beta \sum_{j=1}^{m} w_j \left(\frac{m-j}{m-1}\right) + (1-\beta)\sum_{j=1}^{m} p'_j \left(\frac{m-j}{m-1}\right). \tag{13}$$

b) The *Entropy* (*dispersion*) measures the amount of information being used in the aggregation:

$$H(\hat{p}_1, \hat{p}_2,..., \hat{p}_m) = -\left\{\beta \sum_{j=1}^{m} w_j \ln w_j + (1-\beta)\sum_{i=1}^{m} p_i \ln p_i\right\} \tag{14}$$

c) The *divergence* of weighted vector W measures the divergence of the weights against the degree of *Orness*:

$$Div(\hat{p}_1, \hat{p}_2,..., \hat{p}_m) = \beta\left\{\sum_{j=1}^{m} w_j \left(\frac{m-j}{m-1} - \alpha(W)\right)^2\right\} + (1-\beta)\left\{\sum_{j=1}^{m} p'_j \left(\frac{m-j}{m-1} - \alpha(P)\right)^2\right\} \tag{15}$$

where $\alpha(W)$ is an *Orness* measure of the OWA or FOWA operators ($\beta = 1$):

$$\alpha(W) = \sum_{j=1}^{m} w_j \left(\frac{m-j}{m-1}\right) \tag{16}$$

and $\alpha(P)$ is an *Orness* measure of the fuzzy-probabilistic aggregation ($\beta = 0$):

$$\alpha(P) = \sum_{j=1}^{m} p'_j \cdot \left(\frac{m-j}{m-1}\right) \tag{17}$$

d) The *balance* parameter measures the balance of the weights against the *Orness* or the *andness*:

$$Bal(\hat{p}_1, \hat{p}_2,..., \hat{p}_m) = \beta\left\{\sum_{j=1}^{m} w_j \left(\frac{m+1-2j}{m-1}\right)\right\} + (1-\beta)\left\{\sum_{j=1}^{m} p'_j \left(\frac{m+1-2j}{m-1}\right)\right\}. \tag{18}$$

4 General Decision Making System (GDMS) and Its Information Structure (IS)

In the parts II and III of this work we will focus on the construction of new generalizations of the POWA and FPOWA fuzzy-probabilistic aggregation operators induced by the ME (Choquet Integral [5, 7, 24, 39, 42, 52, 54, 55, 58] and others), or the FEV (Sugeno integral [15, 18, 25, 26, 37, 43, 45] and others) with respect to different monotone measures (fuzzy measure [8, 15, 22, 23, 37-39, 44, 45, 54-56, 62] and others). When trying to functionally describe insufficient expert data, in many real situations the property of additivity remains unrevealed for a measurable representation of a set and this creates an additional restriction. Hence, to study such data, it is better to use monotone measures (estimators) instead of additive ones. So, we will construct new generalizations of the POWA and FPOWA operators with respect to different monotone measures (instead of the probability measure) and different mean operators.

We introduce the definition of a monotone measure (fuzzy measure) [45] adapted to the case of a finite referential.

DEFINITION 6: *Let* $S=\{s_1,s_2,...,s_m\}$ *be a finite set and* g *be a set function* $g:2^S\Rightarrow[0,1]$. *We say* g *is a monotone measure on* S *if it satisfies*

(i) $\quad g(\varnothing)=0;\ g(S)=1;$

(ii) $\quad \forall A,B\subseteq S,\ if\ A\subseteq B,\ then\ g(A)\le g(B).$

A monotone measure is a normalized and monotone set function. It can be considered as an extension of the probability concept, where additivity is replaced by the weaker condition of monotonicity. Non-additive but monotone measures were first used in the fuzzy analysis in the 1980s [45] and are well investigated ([8, 15, 22, 23, 37-39, 44, 45, 54-56, 62] and others). Therefore in order to classify OWA-type aggregation operators with probabilistic (POWA, FPOWA operators and others) or fuzzy uncertainty (defined in parts II and III) it is necessary to define an information structure of these operators. The different cases of incompleteness (uncertainty measure + imprecision variable) and objectivity (objective weighted function) will be considered in our new aggregation operators. Therefore from the point of view of systems approach it is necessary to describe and formally present the scheme of general decision making system (GDMS) in uncertain – objective environment. GDMS gives us the possibility to identify the different cases of levels of incompleteness and objectivity of available information which in whole defines the aggregation procedure.

Now we define the general decision making system and its information structure which will be considered in the aggregation problems of parts II and III.

DEFINITION 7: The general decision making system (GDMS) that will combine decision-making technologies and methods of construction of decision functions (aggregation operators) may be presented by the following 8-tuple

$$\langle D,\ S,\ a,\ g,\ W,\ I,\ F,\ \mathrm{Im}\rangle, \qquad (19)$$

where

1) $D=\{d_1,d_2,...,d_n\}$ is a set of all possible alternatives (decisions, diagnosis and so on) that are made by a Decision-Making Person (DMP).
2) $S=\{s_1,s_2,...,s_m\}$ is a set of systems states of nature (actions, activities, factors, symptoms and so on) that are act on the possible alternatives in the decision procedure.
3) a - is an imprecision on precision variable of payoffs (utilities, valuations, some degrees of satisfaction to a fuzzy set, prices and so on), which will by defined by DMP's subjective properties of preferences, dependences with respect to risks and other external factors. As a result variable a constructs some decision matrix (binary relation) on $D\times S$.
4) g is an uncertainty measure on 2^S $(g:2^S\Rightarrow[0,1])$. In our case it may be some monotone measure.
5) W is an objective weighted function (or vector) on the states of nature - S.
6) I is the Information Structure on the data of states of nature. Cases of different levels of information incompleteness (uncertainty measure + imprecision variable) and objectivity (objective weighted function) on the states of nature will be considered as:

   I = Information Structure (on S): =imprecision (on S) + uncertainty (on S) + objectivity (on S), where:

   a) Imprecision on S may be presented by some inexact (stochastic, fuzzy, fuzzy-stochastic or other) variable.
   b) Uncertainty on S may be presented by the levels of belief, credibility, probability, possibility and other monotone measures on 2^S. These levels identify the possibility of occurrence of some groups (events, focal elements and others) on the states of nature.
   c) Objectivity on S is defined by the objective importance of states of nature in the procedure of decision making. As usual the objective function is presented by a weighted function (vector) $W:S\Rightarrow R_0^+$.

   Now we may classify cases of the Information Structure – I:

   I1: The case:

   a) – imprecision is presented by the some exact variable $a:S\Rightarrow R^1$;

b) – the measure of uncertainty does not exists;
c) – Objectivity is presented by the weights $W=\{w_1,w_2,...,w_m\}$;

Examples: OWA and MEAN operators belong to I1.

I2: The case:

a) – imprecision is presented by the some fuzzy variable: $\tilde{a}\in\psi;\ \tilde{a}:S\Rightarrow[0,1]$;
b) – the measure of uncertainty does not exist;
c) – Objectivity is presented by the weights $W=\{w_1,w_2,...,w_m\}$;

Examples: FOWA operator belongs to I2.

I3: The case:

a) – imprecision is presented by the some stochastic variable: $a:S\Rightarrow R^1$;
b) – the measure of uncertainty is presented by concerning probability distribution on S ($\mathrm{P}:2^S\Rightarrow[0,1]$) $p_i=\mathrm{P}\{s_i\},\ i=1,2,...,m.$
c) – Objectivity is presented by the weights $W=\{w_1,w_2,...,w_m\}$;

Example: POWA operator belongs to I3.

I4: The case:

a) – imprecision is presented by the some fuzzy-stochastic variable: $\tilde{a}\in\psi;\ \tilde{a}:S\Rightarrow[0,1]$;
b) – uncertainty measure is presented by the concerning probability distribution on S ($\mathrm{P}:2^S\Rightarrow[0,1]$) $p_i=\mathrm{P}\{s_i\},\ i=1,2,...,m.$
c) – Objectivity is presented by the weights $W=\{w_1,w_2,...,w_m\}$;

Example: FPOWA operator belongs to I4.

I5: The case:

a) – imprecision is presented by the some exact variable: $a:S\Rightarrow R^1$;
b) – the measure of uncertainty defined by some monotone measure (possibility measure [11, 15, 22, 23], λ-additive measure ([45] and so on) $g:2^S\Rightarrow[0,1]$.
c) – Objectivity is presented by the weights $W=\{w_1,w_2,...,w_m\}$;

Examples: SEV (R.R. Yager [52]) operator belongs to I5; SEV-POWA, AsPOWA, SA-POWA, SA-AsPOWA (will be defined in the part II of this work) operators belong to I5.

I6: The case:

a) – imprecision is presented by the some fuzzy variable: $\tilde{a}\in\psi;\ \tilde{a}:S\Rightarrow[0,1]$;
b) – the measure of uncertainty is presented by some monotone measure $g:2^S\Rightarrow[0,1]$;
c) – Objectivity is presented by the weights $W=\{w_1,w_2,...,w_m\}$;

Examples: SEV-FOWA, AsFPOWA, and SA-AsFPOWA operators (will be defined in the part III of this work) belong to I6.

Note that some other cases may be considered in the Information Structure – I (for an example, the cases when the weights in structure are not present and others).

7) F – is an aggregation (in our case OWA-type) operator for ranking of possible alternatives by its outcome values calculated by the F. Following the Information Structure I on the states of nature for all possible alternatives $d\in D,\ F(d)$ is a ranking value. In general $F(d)$ is defined as converted (or condensed) information of imprecision values plus uncertainty measure and objective weights.

$$F(d)=aggregation(a(d),\ g,\ W)$$

We say – that alternative d_j is more prefered (dominated) than $d_k,\ d_j\succ d_k,\quad if\ F(d_j)>F(d_k),$ and d_j is equivalent to d_k, $d_j\equiv d_k$, if $F(d_j)=F(d_k)$. So the aggregation operator F induces some preference binary relation $\succeq$ on the all possible alternatives - D.

8) Im is a set of information measures of an aggregation operator F:

$$\mathrm{Im}=\{Orness,\ Dispersion,\ Divergence,\ Balance\} \tag{20}$$

In order to classify OWA-type aggregation operators $\{F\}$ it is necessary to investigate information measures (20). This analysis also gives us some information on the inherent subjectivity of the choice of the decision aggregation operator by DMP [6].

5 Conclusion

This paper has a conceptual and introductory character. The main preliminary concepts were presented. Definitions of the OWA operator and the POWA and FPOWA operators as some fuzzy-probabilistic extensions of the OWA operator were introduced. Their information measures as - *Orness, Enropy, Divergence* and *Balance* were considered. From the point of view of systems approach the scheme of general decision making system (GDMS) in uncertain – objective environment and its Information Structure was described and formally presented. New GDMS gives us the possibility to identify the different cases of levels of incompleteness and objectivity of available information which in whole defines the aggregation procedure. The main results on the constructions of new generalizations of the POWA and FPOWA operators will be presented in Parts II and III of this work.

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References:

[1] G. Beliakov, Learning Weights in the Generalized OWA Operators, *Fuzzy Optimization and Decision Making*, 2005; 4: pp. 119-130.

[2] G. Beliakov, A. Pradera, I. Calvo, Aggregation Functions: A Guide for Practitioners, Berlin-Heidelberg: Springer – Verlag, 2007, p 361.

[3] R.E. Bellman, L.A. Zadeh, Decision making in a fuzzy environment, Management Science, 1970, 17 (4): B-141- B-164.

[4] T. Calvo, G. Beliakov, Identification of weights in Aggregation Operators. In: Bustince H, Herrera I, Montero J. (Eds.). Fuzzy sets and their extensions: representation, aggregation and models: intelligent systems from decision making to data mining, web intelligence and computer vision. Berlin: Springer; 2008: pp. 145-162.

[5] De Campos Ibanez L.M., Bolanos Carmona MN, Representation of fuzzy measures through probabilities, Fuzzy sets and systems, 1989, 31 (1) : pp. 23–36.

[6] C. Carlson, R. Fuller, Fuzzy Reasoning in Decision Making and Optimization, Studies in fuzziness and Soft Computing 82, Heidelberg-New York: Physica - Verlag; 2002. p 341.

[7] G. Choquet, Theory of capacities, Annals d'Institute Fourier 1954; 5: pp. 131–295.

[8] D. Denneberg, Non-Additive Measure and Integral, Norwell, MA: Kluwer Academic; 1994. p 185.

[9] Y.C. Dong, Y.F. Xu, H.Y. Li, B. Feng, The OWA-Bases consensus operator under linguistic representation models using position indexes. Eurp. Journal of Operational Research 2010; 203(2): pp. 453-463.

[10] D. Dubois, J.L. Marichal, H. Prade, M. Roubens, R. Sabbadin, The use of the discrete Sugeno integral in decision-making: a survey. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 2001; 9 (5): pp. 539–561.

[11] D. Dubois, H. Prade, Possibility Theory. New York: Plenum Press; 1988.

[12] M. Ehrgott, Multicriteria optimization. Berlin-Heidelberg: Springer-Verlag; 2005. p. 323.

[13] T. Galvo, G. Magor, R. Mesier, Aggregation Operators: New Trends and Application. New York: Physica-Verlag; 2002. p. 368.

[14] A.M. Gil-Lafuente, J.M. Merigo, Computational Intelligence in Bussiness and Economics, Singapore: World Scientific; 2010. p. 835.

[15] M. Grabisch, T. Murofushi, M. Sugeno, Fuzzy measures and integrals: Theory and applications, Studies in Fuzziness and Soft Computing 40. Heidelberg: Physica-Verlag; 2000. p. 477.

[16] S. Greco, R.A. Marques Pereiza, M. Sguillante, R.R. Yager, J. Kacprzyk (Eds), Preferences and Decisions: Models and Applications, Studies in Fuzziness and Soft Computing 257, 1st Edition. Berlin, Heidelberg: Springer-Verlag; 2010. p. 430.

[17] J. Kacprzyk, S. Zadrozny, Towards a generalized and unified characterization of individual and collective choice functions under fuzzy and nonfuzzy preferences and majority via ordered weighted average operators. Int. Journal of Intelligent Systems 2009; 24 (1): 4-26.

[18] A. Kandel, On the control and evaluation of uncertain processes. IEEE Transactions on Automatic Control 1980; 25 (6) : 1182–1187.

[19] A. Kandel, Fuzzy statistics and forecast evaluation. IEEE Transactions on Systems, Man and Cybernetics 1978; SMC-8 (5): 396–401.

[20] A. Kaufman, M.M. Gupta, Introduction to fuzzy arithmetic. New York: Van Nostrad Reinhold Co; 1985. 384 p.

[21] G.J. Klir, D. Elias, Architecture of Systems Problem Solving, IFSR International Series on Systems Science and Engineering 21. New York: Kluwer Academic/Plenum; 2003. 354 p.

[22] G.J. Klir, T.A. Folger, Fuzzy Sets, Uncertainty and Information. New York: Prentice-Hall, Englewood Cliffs; 1988. 356 p.

[23] G.J. Klir, M.J. Wierman, Uncertainty-Based Information: Elements of Generalized Information Theory, Studies in Fuzziness and Soft Computing 15, 2nd ed. Heidelberg: Physica-Verlag; 1999. p. 170.

[24] J.L. Marichal, An axiomatic approach of the discrete Choquet integral as a tool to aggregate interacting criteria. IEEE Transactions on Fuzzy Systems 2000; 8(6): 800–807.

[25] J.L. Marichal, On Choquet and Sugeno Integrals as Aggregation Functions. In: Grabisch M, Murofushi T, Sugeno M (Eds).

Fuzzy Measures and Integrals. Heidelberg: Physica-Verlag; 2000. 247–272.

[26] J.L. Marichal, On Sugeno integral as an aggregation function. Fuzzy sets and systems 2000; 114 (3): 347–365.

[27] J.M. Merigo, The uncertain probabilistic weighted average and its application in the theory of expertons. African Journal of Business Management 2011; 5(15): 6092-6102.

[28] J.M. Merigo, Fuzzy Multi-Person Decision Making with Fuzzy Probabilistic Aggregation Operators. Int. Journal of Fuzzy Systems 2011; 13 (3): 163-174.

[29] J.M. Merigo, M. Casanovas, The uncertain induced quasi-arithmetic OWA operators, Int. Journal of Intelligent Systems 2011; 26 (1): 1-24.

[30] J.M. Merigo, M. Casanovas, Fuzzy Generalized Hybrid Aggregation Operators and its Application in Decision Making. Int. Journal of Fuzzy Systems 2010; 12 (1): 15-24.

[31] J.M. Merigo, M. Casanovas, The Fuzzy Generalized OWA Operators and its Application in strategic Decision Making. Cybernetics and Systems 2010; 41 (5): 359-370.

[32] J.M. Merigo, M. Casanovas, Induced Aggregation Operators in Decision Making with Dempster - Shafer Belief Structure. Int. Journal of Intelligent Systems 2009; 24 (8): 934-954.

[33] J.M. Merigo, M. Casanovas, L. Martiner, Linguistic Aggregation Operators for Linguistic Decision Making based on the Dempster-Shafer Theory of Evidence. Int. Journal of uncertainty, Fuzziness and Knowledge-Bases Systems 2010; 18 (3): 287-304.

[34] R. Mesiar, J. Spirkov, Weighted means and weighting functions. Kybernetika 2006; 42 (2): 151-160.

[35] G. Shafer, A mathematical theory of evidence. Princeton, NJ: Princeton University Press; 1976. p. 297.

[36] A. Sikharulidze, G. Sirbiladze, Average misbilief criterion in the minimal fuzzy covering problem. Proceedings of the 9th WSEAS International Conference on Fuzzy Systems (FS'08); 2008, 42–48.

[37] G. Sirbiladze, Extremal Fuzzy Dynamic Systems: Theory and Applications, IFSR International Series on Systems Science and Engineering 28, 1st Edition. Springer; 2012. p. 396.

[38] G. Sirbiladze, Modeling of extremal fuzzy dynamic systems. Parts I, II, III. International Journal of General Systems 2005, 34 (2): 107–198.

[39] G. Sirbiladze, T. Gachechiladze, Restored fuzzy measures in expert decision-making. Information sciences 2005; 169 (1/2): 71–95.

[40] G. Sirbiladze, B. Ghvaberidze, T. Latsabidze, B. Matsaberidze, Using a minimal fuzzy covering in decision-making problems. Information sciences 2009; 179: 2022–2027.

[41] G. Sirbiladze, A. Sikharulidze, Generalized Weighted Fuzzy Expected Values in Uncertainty Environment, Proceeding of the 9-th WSEAS International Conference on Artificial Intelligence, Knowledge Engineering and Data Bases; 2010. 54-64.

[42] G. Sirbiladze, A. Sikharulidze, Weighted fuzzy averages in fuzzy environment. Parts I, II. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 2003; 11(2) : 139–157, 159–172.

[43] G. Sirbiladze, A. Sikharulidze, B. Ghvaberidze, B. Matsaberidze, Fuzzy-probabilistic Aggregations in the Discrete Covering Problem. International Journal of General Systems 2011; 40 (2): 169 -196.

[44] G. Sirbiladze, N. Zaporozhets, About two probability representations of fuzzy measures on a finite set. Journal of fuzzy mathematics 2003; 11 (2): 1147–1163.

[45] M. Sugeno, Theory of fuzzy integrals and its applications, Thesis (PhD). Tokio Institute of Technology, 1974.

[46] V. Torra, The weighted OWA operator. Int. Journal of Intelligent Systems 1997; 12 (2): 153-166.

[47] V. Torra, Y. Narukawa, Modeling Decisions: Information Fusion and Aggregation Operators. Berlin: Springer - Verlag; 2007. p. 284.

[48] R.R. Yager, Weighted Maximum Entropy OWA Aggregation with Applications to Decision Making under Risk. IEEE Trans. On Systems, Man and Cybernetics 2009, Part A; 39 (3): 555-564.

[49] R.R. Yager, On the dispersion measures of OWA operators. Information Sciences 2009; 179 (22): 3908-3919.

[50] R.R. Yager, Aggregation of ordinal information. Fuzzy Optimization and Decision Making 2007; 6 (3): 199-219.

[51] R.R. Yager, Generalized OWA Aggregation Operators. Fuzzy Optimization and Decision Making 2004; 3 (1): 93-107.

[52] R.R. Yager, On the Evaluation of Uncertain Courses of Action. Fuzzy Optimization and Decision Making 2002; 1 (1): 13-41.

[53] R.R. Yager, Heavy OWA Operators. Fuzzy Optimization and Decision Making 2002; 1 (4): 379-397.
[54] R.R. Yager, On the cardinality index and attitudinal character of fuzzy measures. International Journal of General Systems 2002; 31 (3): 303-329.
[55] R.R. Yager, On the entropy of fuzzy measure. IEEE Transaction on Fuzzy Sets and Systems 2000; 8 (4): 453-461.
[56] R.R. Yager, A class of fuzzy measures generated from a Dempster-Safer Belief Structure. International Journal of Intelligent Systems 1999; 14 (12): 1239-1247.
[57] R.R. Yager, Families of OWA operators. Fuzzy Sets and Systems 1993; 59 (2): 125-148.
[58] R.R. Yager, On Ordered Weighted Averaging aggregation operators in multicriteria decision making. IEEE Trans. On Systems, Man and Cybernetics 1988; 18 (1): 183-190.
[59] R.R. Yager, J. Kacprzyk, M. Fedrizzi (Eds), Advances in the Dempster-Shafer Theory of Evidence, New York: John Wiley & Sons; 1994. p. 597.
[60] R.R. Yager, J. Kacprzyk (Eds), The Ordered Weighted Averaging Operators: Theory and Applications. Norwell: Kluwer Academic Publishers; 1997. p. 357.
[61] R.R. Yager, J. Kacprzyk, G. Beliakov (Eds), Recent Development in the ordered Weighted Averaging Operations: Theory and Practice, Studies in Fuzziness and Soft Computing 265, 1st Edition. Springer; 2011. p. 298.
[62] Z. Wang, G.J. Klir, Generalized Measure Theory. IFSR International Series of Systems Science and Engineering 25, 1st Edition. Springer; 2009. p. 384.
[63] Z.S. Xu, Q.L. Da, An Overview of operators for aggregating information. Int. Journal of Intelligent Systems 2003; 18 (9): 953-969.