Extended Relay Feedback Identification and Anisochronic Control

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Abstract: - This paper introduces a new simple technique extending possibilities of the relay feedback identification. The technique enables to estimate up to five parameters of the system transfer function without using the FFT algorithm. For this purpose it is used one conventional relay feedback experiment with an asymmetric relay. The suggested technique is used for estimating five parameters of the anisochronic model because this model is very universal and convenient for modeling time delay stable systems. The anisochronic model is then used for anisochronic controller design by the Desired Model Method. The practical applicability of the suggested identification technique and the designed anisochronic control is demonstrated on the physical laboratory model "Air Aggregate". All experiments were performed using Matlab/Simulink programing environment. The introduced relay feedback identification can be applied even for automatic tuning.

Key-Words: -Anisochronic control, estimation parameter, frequency response, relay feedback identification, time delay

1 Introduction

Control oriented system identification aims at obtaining a model that reflects the dynamic response relationship among manipulated variables and controlled variables of a system. System identification from the relay feedback belongs among the most popular methods applied in industrial and chemical practice. An important merit of using a relay test is that the process will not be propelled too far away from the set-point of system operation.

A plant under relay feedback is shown in Fig. 1, where w denotes the desired variable, y the controlled variable, u the manipulated variable, d the disturbance variable and e the control error. Let us consider a time invariant plant described by the plant frequency response function $G_P(j\omega)$, where ω is the angular frequency. Then we can determine one point $P_u=G_P(j\omega_u)$ on the Nyquist curve by relay feedback test, where ω_u is the angular frequency of the limit cycle that can be computed from the steady oscillation period T_u according to

$$\omega_u = \frac{2\cdot\pi}{T_u} \tag{1}$$

It was derived, e.g. [1],[3] that it holds for a plant under a symmetrical relay without hysteresis

$$P_u = G_P\left(j\omega_u\right) = -\frac{\pi\cdot y_A}{4\cdot u_A}, \tag{2}$$

where y_A is the harmonic oscillation amplitude of the plant output and u_A is the relay amplitude. This approach was also generalized for a relay with hysteresis, where by choosing the relation between the relay amplitude and the hysteresis width it is possible to determine a point on the Nyquist curve with a specified imaginary part, e.g. [2], [5].

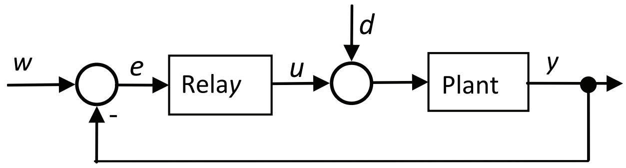

Fig. 1 Block diagram of a plant under relay feedback

If a plant model has more than two unknown parameters then it is necessary for system identification to find out more points on a frequency response function or to add some other information. This is solved mostly for low-order model structures [7], [16] or for example using more tests with relay feedback, where a known linearity (e.g. an integrator, a time delay) is connected in series with the plant [5], [11], [14] or by repeating the experiment with different relations between the relay amplitude and the hysteresis [6], [10]. There are also modified relay methods (e.g. with a parasitic relay and using the FFT algorithm) [8], [9] or it is assumed that the plant is initially in a steady state [15].

In the next chapter it is described the new technique enabling to estimate up to five parameters from sustained oscillations of the controlled variable observed during only one relay feedback experiment.

2 Relay Feedback Identification by Shifting Technique

The block diagram of the relay feedback experiment is shown in Fig. 1, where the plant is describable by the frequency response function $G_P(j\omega)$ and a biased (asymmetric) relay with hysteresis is used for this purpose, see Fig. 2. Let the stable oscillation is reached after the time t_L. The time courses of the biased relay output u and the plant output y after the time t_L are shown in Fig. 3.

The biased relay output u oscillating with the period T_p can be expanded in a Fourier series. If all higher harmonics are neglected and only the first harmonic (the fundamental frequency) is considered then the relay output u can be approximated by the function $u_{ap}(t)$

$$u(t) \approx u_{ap}(t) = a_u + A_u \cos(\omega_u (t - t_L) - \phi_u), t \geq t_L \quad (3)$$

where

$$a_u = \frac{u_A \cdot T_1 + u_B \cdot T_2}{T_p}, \quad (4)$$

$$\omega_u = \frac{2\pi}{T_p}, \quad (5)$$

$$\phi_u = \frac{\omega_u \cdot T_1}{2} = \frac{\pi \cdot T_1}{T_p}, \quad (6)$$

$$H = u_A + |u_B|, \quad (7)$$

$$A_u = \frac{2H}{\pi} \cdot \sin(\phi_u). \quad (8)$$

The time courses $u(t)$ and $u_{ap}(t)$ are illustrated in Fig. 3 where the parameters t_L, T_1, T_2, T_p, H, u_A and u_B are also depicted.

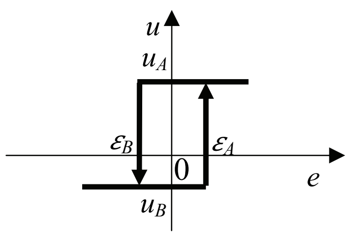

Fig. 2 The characteristic of a biased (asymmetric) relay with hysteresis

For many systems, where the manipulated signal is a square wave, the plant output is close to a sinusoid, which means, that the plant attenuates higher harmonics effectively. Therefore higher harmonics in y can be neglected. The point $G(j\omega_u)$ on the Nyquist curve of the plant can be estimated using the fundamental harmonics of the signal $u(t)$ and $y(t)$. This point is given by

$$G(j\omega_u) = \frac{y_A}{A_u} \cdot e^{j\phi_{uy}} \quad (9)$$

where y_A is an amplitude of the output y, ϕ_{uy} is a phase shift between u and y and

$$\phi_{uy} = -\omega_u \cdot t_{uy}, \quad (10)$$

see Fig. 3.

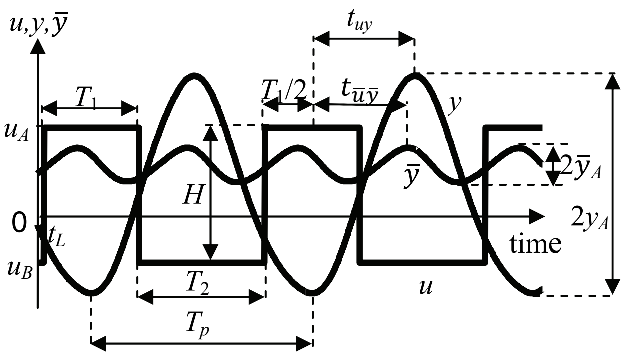

Fig. 3. The time courses u, y and $\bar{y}$

In the next step the simple technique called "shifting" is used. It is based on the assumption that the identified plant is time invariant. Then the auxiliary variables $\bar{u}$ and $\bar{y}$ are calculated using (11) and (12) to obtain a rectangular waveform of the variable $\bar{u}$ and a sinusoidal form of the variable $\bar{y}$.

$$\bar{u}(t) = u(t) + u\left(t - \frac{T_p}{2}\right), \quad (11)$$

$$\bar{y}(t) = y(t) + y\left(t - \frac{T_p}{2}\right). \quad (12)$$

The time course $\bar{y}$ is depicted in Fig. 3. Since the variable $\bar{y}$ is close to a sinusoidal form, higher harmonics of $\bar{y}$ can be neglected and the next point on the Nyquist curve of the plant can be estimated using the fundamental harmonics of the signal $\bar{u}(t)$ and $\bar{y}(t)$.

The signal $\bar{u}$ oscillating with the period $T_p/2$ can be expanded in a Fourier series. If all higher harmonics are neglected and only the first harmonic (the fundamental frequency) is considered then the signal $\bar{u}(t)$ can be approximated by the function $\bar{u}_{ap}(t)$ for $t \geq t_L$.

$$\bar{u}(t) \approx \bar{u}_{ap}(t) = a_{\bar{u}} + A_{\bar{u}} \cdot \cos(\omega_{\bar{u}} \cdot (t - t_L) - \phi_{\bar{u}}), \quad (13)$$

where

$$a_{\bar{u}} = \frac{2\left(u_A \cdot T_1 + u_B \cdot T_2\right)}{T_p}, \tag{14}$$

$$\omega_{\bar{u}} = 2 \cdot \omega_u , \tag{15}$$

$$\phi_{\bar{u}} = 2 \cdot \phi_u = \omega_u \cdot T_1 = \frac{\omega_{\bar{u}} \cdot T_1}{2} = \frac{2\pi \cdot T_1}{T_p}, \tag{16}$$

$$A_{\bar{u}} = \frac{2H}{\pi} \cdot \sin\left(\phi_{\bar{u}}\right). \tag{17}$$

The point $G(j\omega_{\bar{u}}\cdot)$ on the Nyquist curve of the plant can be estimated using the fundamental harmonics of the signals $\bar{u}(t)$ and $\bar{y}(t)$. This point is given by

$$G\left(j\omega_{\bar{u}}\right) = \frac{\bar{y}_A}{A_{\bar{u}}} \cdot e^{j\phi_{\bar{u}\bar{y}}}, \tag{18}$$

where $\bar{y}_A$ is an amplitude of the signal $\bar{y}$; $\phi_{\bar{u}\bar{y}}$ is a phase shift between $\bar{u}$ and $\bar{y}$ and

$$\phi_{\bar{u}\bar{y}} = -\omega_{\bar{u}} \cdot t_{\bar{u}\bar{y}}, \tag{19}$$

see Fig. 3.

The asymmetric relay enables to calculate the plant static gain K from

$$K = G(0) = \frac{\int_t^{t+T_p} y(\tau)\,d\tau}{\int_t^{t+T_p} u(\tau)\,d\tau}, t \geq t_L, \tag{20}$$

see [4],[7].

Summing up the previous results, the shifting method enables to determine the static gain K and two points $G(j\omega_u)$, $G(j\omega_{\bar{u}}\cdot)$ of the Nyquist curve from one relay feedback experiment without utilising the Fourier transform. For this purpose it is sufficient to use only the relay output u, the output y and the sum signal $\bar{y}$. The parameters, necessary for this goal, can be read out from the time courses of these signals, see Fig. 3.

The shifting method can be described by the following steps:

a) Control a plant by a biased relay.
b) Determine the period T_p of the stable oscillation.
c) Compute the plant static gain $K=G(0)$ using (20) and the angular frequency ω_u using (5).
d) Calculate the point $G(j\omega_u)$ from

$$G\left(j\omega_u\right) = \frac{y_A}{\frac{2H}{\pi} \cdot \sin\left(\frac{\pi \cdot T_1}{T_p}\right)} \cdot e^{-j\omega_u \cdot t_{uy}} \tag{21}$$

where y_A, T_1, u_A, u_B , t_{uy} can be determined directly from the time courses $u(t)$ and $y(t)$, see Fig. 3.

e) Calculate the course of the auxiliary variable $\bar{y}(t)$ using (12). (We need not calculate the auxiliary variable $\bar{u}(t)$).
f) Calculate the point $G(j\cdot\bar{\omega}_u)=G(2j\omega_u)$ according to

$$G\left(2j\omega_u\right) = \frac{\bar{y}_A}{\frac{2H}{\pi} \cdot \sin\left(\frac{2\pi \cdot T_1}{T_p}\right)} \cdot e^{-2j\omega_u \cdot t_{\bar{u}\bar{y}}} \tag{22}$$

where $\bar{y}_A$, $t_{\bar{u}\bar{y}}$ can be determined directly from the time courses $u(t)$ and $\bar{y}(t)$, see Fig. 3.
g) Estimate up to five parameters of mathematical models from the three points $G(0), G(j\omega_u)$ and $G(2j\omega_u)$ of the Nyquist curve.

Remark
From relations (21) and (22) it follows

$$\frac{G\left(2j\omega_u\right)}{G\left(j\omega_u\right)} = \frac{\bar{y}_A \cdot e^{j\omega_u\left(t_{uy} - t_{\bar{u}\bar{y}}\right)}}{y_A \cdot \cos\left(\frac{\pi \cdot T_1}{T_p}\right)}. \tag{23}$$

3 Anisochronic Control

The shifting technique enables to estimate up to five model parameters by a single relay feedback experiment. Therefore this approach can be used for the parameter estimation of the anisochronic model described by the transfer function

$$G_a(s) = \frac{K \cdot e^{-s\tau_u}}{\left(\tau_1 s + 1\right)\left(\tau_2 s + e^{-s\tau_y}\right)}, \tag{24}$$

where s is the complex variable in Laplace transform and K, τ_u, τ_1, τ_2, τ_y are the model parameters.

Although anisochronic model (24) has only five parameters it is very universal and convenient for modeling time delay plants and for description dynamics of high-order systems. It may be used for both oscillatory and nonoscillatory plants, see [12], [13]. Model (24) is stable if $\tau_y/\tau_2<\pi/2$, over-damped if $\tau_y/\tau_2<1/e$, critically damped if $\tau_y/\tau_2=1/e$ and under-damped if $\tau_y/\tau_2>1/e$.

The great universality of model (24) and the possibility to estimate by the shifting technique all the model parameters can be used for an anisochronic control design. A suitable method for direct synthesis using model (24) is the Desired Model Method (DMM) [17].

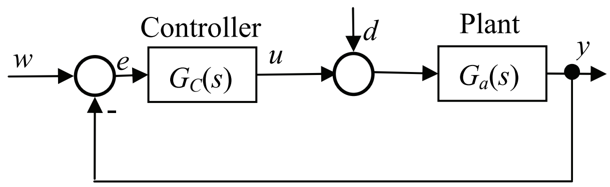

Fig. 4 Closed-loop system

The DMM uses the formula for direct synthesis (see Fig. 4)

$$G_C(s)=\frac{G_{wy}(s)}{G_a(s)\cdot\left(1-G_{wy}(s)\right)}, \tag{25}$$

where $G_C(s)$ is the controller transfer function, $G_a(s)$ is the plant transfer function, $G_{wy}(s)$ is the desired control system transfer function and it is selected in the form

$$G_{wy}(s)=\frac{k_0}{s+k_0\cdot e^{-\tau_u\cdot s}}e^{-\tau_u s}, \tag{26}$$

k_0 is the open-loop gain. The open-loop transfer function

$$G_0(s)=G_C(s)\cdot G_a(s)=\frac{k_0}{s}e^{-\tau_u s} \tag{27}$$

corresponds to desired control system transfer function (25).

After substitution of plant transfer function (24) to relationship (27) one obtains

$$G_0(s)=G_C(s)\cdot\frac{K\cdot e^{-s\tau_u}}{(\tau_1 s+1)\left(\tau_2 s+e^{-s\tau_y}\right)}=\frac{k_0}{s}e^{-\tau_u s} \tag{28}$$

hence

$$G_C(s)=\frac{k_0}{K}\cdot\frac{(\tau_1 s+1)\left(\tau_2 s+e^{-s\tau_y}\right)}{s}. \tag{29}$$

The open-loop gain k_0 can be easily determined analytically [17] assuming that the non-dominant poles and zeros of the control system have a negligible influence on its behaviour. The value of the open-loop gain k_0 can be decided according to

$$k_0=\frac{1}{\beta\cdot\tau_d} \tag{30}$$

where β is the coefficient depending on the relative overshoot κ; see Table I (copy from [17]).

Table 1 Values of coefficients β for given relative overshoot κ

κ	0	0.05	0.1	0.2	0.3	0.4	0.5
β	2.718	1.944	1.720	1.437	1.248	1.104	0.992

Transfer function (29) is completed by a low-pass filter with a steady-state gain of one to guarantee the physical realizable controller. The transfer function of the controller is then

$$G_C(s)=\frac{k_0}{K}\cdot\frac{(\tau_1 s+1)\left(\tau_2 s+e^{-s\tau_y}\right)}{\left(\tau_f s+1\right)^r s}, \tag{31}$$

where τ_f is the time constant of the filter and the natural number r can be chosen so that the order of the denominator is at least the same order as the numerator. The value of the time constant τ_f also allows restricting actions of the manipulated variable u.

4 Example

A laboratory model "Air Aggregate" consists of a ventilator located in a tunnel. The ventilator is fed by a controlled supply voltage (Fig. 5). In the tunnel there is also a sensor for measuring air flow. The manipulated variable (power to the ventilator) u and the controlled variable (air flow) y are provided via unified electrical signals (0-10 V). The task is to estimate by the shifting technique the parameters of model (24) and then to design a controller using the DMM.

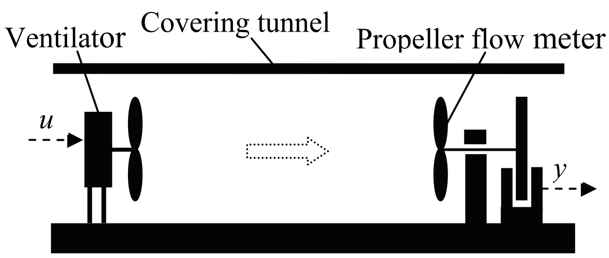

Fig. 5 The laboratory model "Air Aggregate"

Solution:

a) Extended relay feedback identification

After a single relay feedback test the parameters of anisochronic model (24) can be estimated from the sustained oscillations u and y. This part is denoted by the rectangle in Fig. 6.

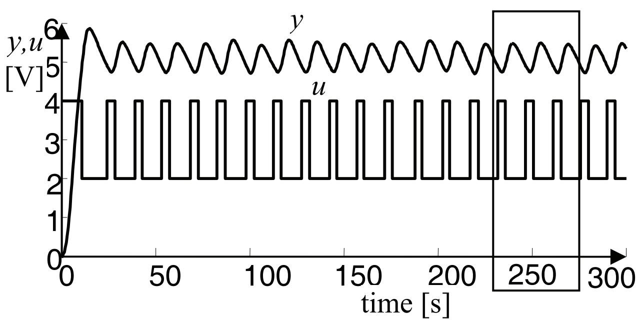

Fig. 6 The observed input u and the observed output y

The calculated sum signal $\bar{y}$ is depicted in Fig. 7. The signal $\bar{u}$ is not in Fig. 7 because it does not need to be calculated.

Using the shifting method it is was obtained

$$H = 2\ \mathrm{V}, \omega_u = \frac{2\pi}{T_p} = 0.45\ \mathrm{rad \cdot s^{-1}}. \quad (32)$$

$$K = G(0) = 2.02, \quad (33)$$

$$G(0.45j) = 0.35e^{-3.08j} = -0.35 - 0.02j, \quad (34)$$

$$G(0.9j) = 0.1e^{-3.95j} = -0.07 + 0.07j. \quad (35)$$

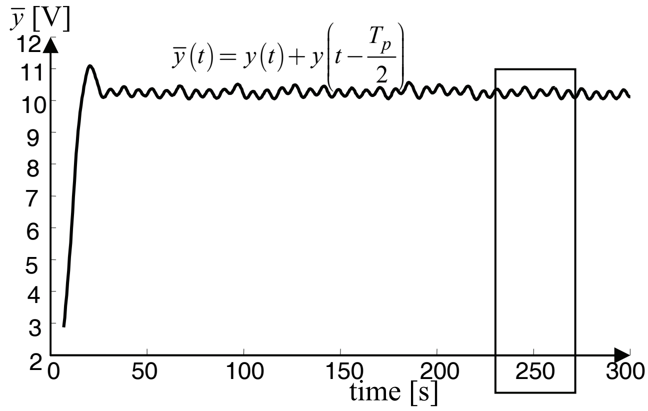

Fig. 7 The calculated output $\bar{y}$

The verification was performed by sinusoidal inputs with the following results:

$$G(0) = K = 1.99, \quad (36)$$

$$G(0.45j) = -0.38 - 0.01j, \quad (37)$$

$$G(0.9j) = -0.03 + 0.09j. \quad (38)$$

The points of the Nyquist diagram estimated by the relay feedback identification and by the sinosuidal inputs are depicted in the complex plain in Fig. 8.

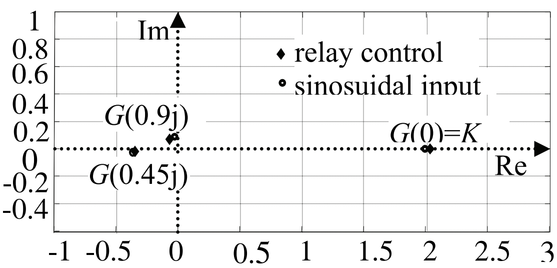

Fig. 8 The points of the Nyquist diagram estimated by the relay feedback identification and by the sinosuidal inputs

The time constants τ_u, τ_1, τ_2 and τ_y can be obtained by numerical fitting transfer function (24) to the points given by relations (34) and (35). The following time constants were estimated this way

$$\tau_u = 1.56\mathrm{s}, \tau_1 = 1.63\mathrm{s}, \tau_2 = 11.68\mathrm{s}, \tau_y = 5.55\mathrm{s}. \quad (39)$$

The frequency transfer function of the anisochronic model is then

$$G_a(j\omega) = \frac{2.02 \cdot e^{-1.56j\omega}}{(1.63j\omega + 1)\left(11.68j\omega + e^{-5.55j\omega}\right)}. \quad (40)$$

b) Anisochronic controller design

The transfer function $G_C(s)$ of the controller can be selected using the DMM with respect to (40) and (31) in the form

$$G_C(s) = k_0 \cdot \frac{(1.63s + 1) \cdot \left(11.68s + e^{-5.55s}\right)}{2.02 \cdot (0.2s + 1) \cdot s}, \quad (41)$$

where

$$\begin{aligned} &K = 2.02, \tau_1 = 1.63\ \mathrm{s}, \tau_2 = 11.68\ \mathrm{s}, \\ &\tau_y = 5.55\ \mathrm{s}, \tau_f = 0.2\ \mathrm{s}, r = 1. \end{aligned} \quad (42)$$

Therefore with respect to Tab. 1, the value of the open-loop gain is

$$k_0 = \frac{1}{\beta \cdot \tau_u} = \frac{1}{1.720 \cdot 1.56} = 0.37 \ \text{ for } \kappa = 0.1 \quad (43)$$

Fig. 9 shows the closed loop control of the laboratory model “Air Aggregate” for the step changes of the desired variable w and the disturbance variable d (the required relative overshoot κ=0.1).

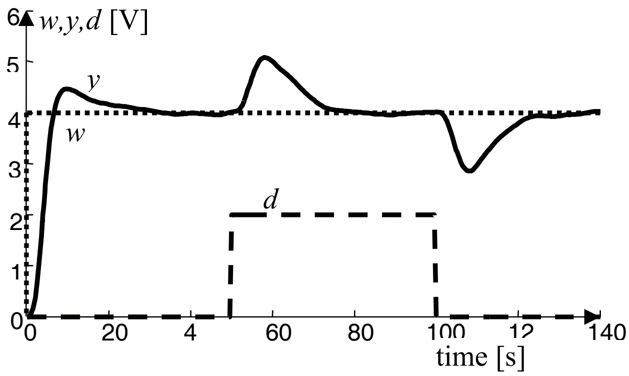

Fig. 9 The closed loop control of the laboratory model “Air Aggregate” for the step changes of the desired variable w and the disturbance variable d

5 Conclusion

This paper presents the shifting technique that significantly extends the capabilities of the relay feedback identification. This technique has been developed for estimation up to five parameters of mathematical models from a single relay feedback experiment without utilizing the Fourier transform. It can be applied for parameter tuning both isochronic and anisochronic models. For this purpose it uses a biased relay with a hysteresis. The necessary computations are simple and can be easily applied in practice. The introduced technique was demonstrated on the laboratory model called "Air Aggregate" in estimating five parameters of the anisochronic model. This very universal model was then used for the air flow control in the laboratory model "Air Aggregate". The introduced algorithm offers the possibilities to provide an automatic tuning tool for a large class of common control problems.

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