A new delay-dependent absolute stability of Lur' e systems for neutral type with time-varying delays

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Abstract: This paper deals with the problem of absolute stability of neutral type Lur'e systems with time-varying delays. By constructing new Lyapunov-Krasovskii functional, a matrix-based on quadratic convex approach combining with some improved bounding techniques for integral terms such as Wirtinger-based integral inequality, new stability condition is much less conservative and more general than some existing results. New stability criteria is given in terms of linear matrix inequalities. Numerical examples are given to illustrate the effectiveness of the results.

Key–Words: absolute stability; neutral type; time-delay Lur'e system; combined quadratic convex technique; LMI approach

1 Introduction

In many practical systems, models of system are described by neutral differential equations, in which the models depend on the delays of state and state derivatives. Heat exchanges, distributed networks containing lossless transmission lines and population ecology are examples of neutral systems. Because of its wider application, several researchers have studied neutral systems and provided sufficient conditions to guarantee the stability of neutral time delay systems, see [1, 8, 19] and references cited therein.

It is well know that nonlinearities may cause instability and poor performance of practical systems, [5, 8, 14, 20, 25]. Many nonlinear control systems can be modeled as a feedback connection of a linear neutral system and a nonlinear element. One of the important classes of nonlinear systems is the Lur'e system whose nonlinear element satisfies certain sector constraints. Absolute stability of Lur'e systems with sector bounded nonlinearities has attracted several researcher [5, 7, 9, 15].

It is well known that the existence of time delay in a system may cause instability and oscillations. Examples of time delay systems are chemical engineering systems, biological modeling, electrical networks, physical networks and many others, [11, 12, 17]. The stability criteria for system with time delays can be classified into two categories: delay-independent and delay-dependent. Delay-independent criteria does not employ any information on the size of the delay; while delay-dependent criteria makes use of such information at different levels. Delay-dependent stability conditions are generally less conservative than delay-independent ones especially when the delay is small. In most of the existing results, the range of time-varying delay considered varies form 0 to an upper bound. In practice, the range of delay may vary in a range for which the lower bound is not restricted to be 0, i.e., interval time-varying delay. A typical example with interval time delay is the networked control system, which has been widely studied in the recent literature (see, e.g., [2, 11, 24]).

Recently, there are many research studies on the absolute stability of a class of neutral type Lur'e dynamical systems with time delay, see for examples [14, 16, 20, 22, 25]. The problems have been dealt with delay-dependent absolute and robust stability for time-delay Lur'e system [14]. Improved delay-dependent robust stability criteria for a class of uncertain mixed neutral and Lur'e dynamical systems with interval time-varying delays and sector-bounded nonlinearity were studied in [22]. On delay-dependent robust stability of a class of uncertain mixed neutral and Lur'e dynamical systems with interval time-varying delays were investigated in [25]. However, it is worth pointing out that, even though these results were elegant, there still exist some points waiting for the improvement. Firstly, most of the works above [5, 14], the augmented Lyapunov matrix P must be positive definite. We will remove this restriction by assuming that P are only real matrices. Secondly, By introducing new augmented Lyapunov-Kravoskii functional which have not been considered yet in stability analysis of Lur'e systems. Thirdly, by taking the time derivative of $\int_{t-h_1}^{t} h_1(h_1 - t + s)\dot{x}^T(s)W_1\dot{x}(s), \int_{t-h_1}^{t}(h_1 - t + s)^2\dot{x}^T(s)W_2\dot{x}(s)ds$, $\int_{t-h_2}^{t-h_1} h_{21}(h_2 - t + s)\dot{x}^T(s)R_1\dot{x}(s), \int_{t-h_2}^{t-h_1}(h_2 - t + s)^2\dot{x}^T(s)R_2\dot{x}(s)ds$, it is found that the integral terms $2\int_{t-h_2}^{t-h_1}(h_2 - t + s)\dot{x}^T(s)R_2\dot{x}(s)ds,\ 2\int_{t-h_1}^{t}(h_1 - t + s)\dot{x}^T(s)W_2\dot{x}(s)ds,\ h_{21}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_1\dot{x}(s)ds$, $-h_1\int_{t-h_1}^{t}\dot{x}^T(s)W_1\dot{x}(s)ds$, appear. For estimating these terms, techniques in [21, 27] are applied in this paper, called matrix-based quadradic convex optimization approach combined with some improved bounding techniques for integral terms such as Wirtinger-based integral inequality; as a result we obtain inequality encompassing the Jensen one and also goes to tractable LMI criteria to futher reduce the conservatism over the existing results [14, 16, 20, 22, 25]. Fourthly, most of the previous works did not consider the lower bound of the time-varying delay and its time-derivative. Factually, the lower bound can play an important role in reducing the conservatism when it can be available and fully tackled in [16, 20, 22, 25].

Based on the above discussions, we consider the problem of delay-dependent absolute stability of Lur'e systems of neutral type with time-varying delays, matrix-based quadratic convex approach will be used. The time delay is a continuous function belonging to a given interval, which means that the lower and upper bounds for the time varying delay are available. Based on the construction of improved Lyapunov-Krasovskii functionals combined with a quadratic convex approach, some new cross terms will be introduced which enhance the feasible stability criterion. New delay-dependent sufficient conditions for the neutral type Lur'e dynamical systems are established in terms of LMIs. The new stability condition is much less conservative and more general than some existing results. Numerical examples are given to illustrate the effectiveness of our theoretical results.

2 Problem statements and preliminaries

The following notation will be used in this paper: $\mathbb{R}^+$ denotes the set of all real non-negative numbers; $\mathbb{R}^n$ denotes the $n-$dimensional space and the vector norm $\|\ .\ \|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)-$dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\text{Re}\lambda; \lambda \in \lambda(A)\}$. $x_t := \{x(t+s) : s \in [-h, 0]\}, \| x_t \| = \sup_{s\in[-h,0]} \| x(t+s) \|$; $C([0,t], \mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n-$valued continuous functions on $[0,t]$; Matrix A is called semi-positive definite $(A \geq 0)$ if $x^TAx \geq 0$, for all $x \in \mathbb{R}^n$; A is positive definite $(A > 0)$ if $x^TAx > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$; diag$(c_1, c_2, ..., c_m)$ denotes block diagonal matrix with diagonal elements $c_i, i = 1, 2, ..., m$. The symmetric term in a matrix is denoted by $*$.

Consider the following Lur'e system of neutral type with interval time-varying delay:

$$\begin{aligned}
\dot{x}(t) &= A_1\dot{x}(t-\tau(t)) + Ax(t) \qquad (1)\\
&\quad + Bx(t-h(t)) + Cf(\omega(t)) + Dh(\sigma(t)),\\
\omega(t) &= Ex(t) = [E_1\ \ E_2\ \ ...\ \ E_{k_1}]^T x(t),\\
&\quad \forall t \geq 0, \qquad (2)\\
\sigma(t) &= Fx(t-h(t)) = [F_1\ \ F_2\ \ ...\ \ F_{k_2}]^T\\
&\quad \times x(t-h(t)),\ \ \forall t \geq 0, \qquad (3)\\
x(t+s) &= \phi(t+s),\ \dot{x}(t+s) = \varphi(t+s),\\
&\quad s \in [-m, 0],\ m = \max\{h_2, \tau_2\},
\end{aligned}$$

where $x(t) \in \mathbb{R}^n$, $\omega(t) \in \mathbb{R}^{k_1}$ and $\sigma(t) \in \mathbb{R}^{k_2}$ denote the state vector and output ones of the system, respectively; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{n\times k_1}$, $A_1 \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times k_2}$ are constant known matrices; $f(Ex(\cdot)) = [f_1(E_1^Tx(\cdot)), ..., f_{k_1}(E_{k_1}^Tx(\cdot))]^T$, $h(Fx(\cdot)) = [h_1(F_1^Tx(\cdot)), ..., h_{k_2}(F_{k_2}^Tx(\cdot))]^T$ are the nonlinear elements.

Assumption 1. *The delays $\tau(t)$ and $h(t)$ are time-varying continuous functions that satisfying*

$$\begin{aligned}
&0 \leq h_1 \leq h(t) \leq h_2, \quad \mu_1 \leq \dot{h}(t) \leq \mu_2, \qquad (4)\\
&0 \leq \tau(t) \leq \tau_2, \qquad\quad \dot{\tau}(t) \leq \delta < 1, \qquad (5)
\end{aligned}$$

in which $h_1, h_2, \tau_2, \mu_1, \mu_2$ and δ are constants.

Assumption 2. *For any $\epsilon_1, \epsilon_2 \in \mathbb{R}$, the nonlinear function $f_i(\cdot)$ and $h_j(\cdot)$ satisfy $f_i(0) = h_j(0) = 0$, and*

$$\begin{aligned}
&\sigma_i^- \leq \frac{f_i(\epsilon_1) - f_i(\epsilon_2)}{\epsilon_1 - \epsilon_2} \leq \sigma_i^+,\\
&\delta_j^- \leq \frac{h_j(\epsilon_1) - h_j(\epsilon_2)}{\epsilon_1 - \epsilon_2} \leq \delta_j^+,\\
&\epsilon_1 \neq \epsilon_2, i = 1, ..., k_1; j = 1, ..., k_2,
\end{aligned}$$

where $\sigma_i^+, \sigma_i^-, \delta_j^+$, and δ_j^- are given constants. Here, *we give*

$$\begin{aligned}\Upsilon_1 &= diag(\sigma_1^+\sigma_1^-, ..., \sigma_{k_1}^+\sigma_{k_1}^-),\\ \Upsilon_2 &= diag(\frac{\sigma_1^+ + \sigma_1^-}{2}, ..., \frac{\sigma_{k_1}^+ + \sigma_{k_1}^-}{2}),\\ \Upsilon_3 &= diag(\delta_1^+\delta_1^-, ..., \delta_{k_2}^+\delta_{k_2}^-),\\ \Upsilon_4 &= diag(\frac{\delta_1^+ + \delta_1^-}{2}, ..., \frac{\delta_{k_2}^+ + \delta_{k_2}^-}{2}),\\ \overline{\Upsilon}_1 &= diag(\sigma_1^+, ..., \sigma_{k_1}^+),\\ \overline{\Upsilon}_2 &= diag(\sigma_1^-, ..., \sigma_{k_1}^-),\\ \overline{\Upsilon}_3 &= diag(\delta_1^+, ..., \delta_{k_2}^+),\\ \overline{\Upsilon}_4 &= diag(\delta_1^-, ..., \delta_{k_2}^-). \qquad (6)\end{aligned}$$

Assumption 3. *All the eigenvalues of matrix A_1 are inside the unit circle.*

We introduce the following technical well-known propositions and Definition, which will be used in the proof of our results.

Lemma 4. *[21] For a given matrix $R > 0$, the following inequality holds for all continuously differentiable function ω in $[a, b] \to \mathbb{R}^n$:*

$$\begin{aligned}\int_a^b \dot{\omega}^T(u)R\dot{\omega}(u)du \;\geq\; & \frac{1}{b-a}(\omega(b)-\omega(a))^T R\\ & \times(\omega(b)-\omega(a)) \qquad (7)\\ & + \frac{3}{b-a}\tilde{\Omega}^T R\tilde{\Omega}\end{aligned}$$

where $\tilde{\Omega} = \omega(b) + \omega(a) - \frac{2}{b-a}\int_a^b \omega(u)du$.

Remark 5. *Clearly, the inequality (7) contains a tighter lower bound for $\int_a^b \dot{\omega}^T(u)R\dot{\omega}(u)du$ than Jensen's inequality, are applied in this paper that this resulting inequality encompasses the Jensen one and also goes to tractable LMI criteria to futher reduce the conservatism over the existing results [14, 16, 20, 22, 25].*

Lemma 6. *[27] Let $h(t)$ be a continuous function satisfying $0 \leq h_1 \leq h(t) \leq h_2$. For any $n \times n$ real matrix $R_1 > 0$ and a vector $\dot{x} : [-h_2, 0] \to \mathbb{R}^n$ such that the integration concerned below is well defined, the following inequality holds for any $2n \times 2n$ real matrices S_1 satisfying* $\begin{bmatrix} \tilde{R}_1 & S_1 \\ S_1^T & \tilde{R}_1 \end{bmatrix} \geq 0$

$$\begin{aligned}&-(h_2-h_1)\int_{t-h_2}^{t-h_1} \dot{x}^T(s)R_1\dot{x}(s)ds\\ &= 2\varphi_{11}^T S\varphi_{21} - \varphi_{11}^T\tilde{R}_1\varphi_{11} - \varphi_{21}^T\tilde{R}_1\varphi_{21}, \qquad (8)\end{aligned}$$

where $\tilde{R}_1 \triangleq \mathrm{diag}\{R_1, 3R_1\}$ *and*

$$\begin{aligned}\varphi_{11} &\triangleq \begin{bmatrix} x(t-h(t)) - x(t-h_2) \\ x(t-h(t)) + x(t-h_2) - 2\omega_1(t) \end{bmatrix},\\ \varphi_{21} &\triangleq \begin{bmatrix} x(t-h_1) - x(t-h(t)) \\ x(t-h_1) + x(t-h(t)) - 2\omega_2(t) \end{bmatrix},\end{aligned}$$

where

$$\begin{aligned}\omega_1 &\triangleq \frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)} x(s)ds,\\ \omega_2 &\triangleq \frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1} x(s)ds. \qquad (9)\end{aligned}$$

Lemma 7. *[27] Let $h(t)$ be a continuous function satisfying $0 \leq h_1 \leq h(t) \leq h_2$. For any $n \times n$ real matrix $R_2 > 0$ and a vector $\dot{x} : [-h_2, 0] \to \mathbb{R}^n$ such that the integration concerned below is well defined, the following inequality holds for any $\phi_{i1} \in \mathbb{R}^q$ and real matrices $Z_i \in \mathbb{R}^{q\times q}, N_i \in \mathbb{R}^{q\times n}$ satisfying* $\begin{bmatrix} Z_i & N_i \\ N_i^T & R_2 \end{bmatrix} \geq 0$ $(i = 1, 2)$

$$\begin{aligned}&-\int_{t-h_2}^{t-h_1}(h_2-t+s)\dot{x}^T(s)R_2\dot{x}(s)ds\\ &\leq \frac{1}{2}(h_2-h(t))^2\phi_{11}^T Z_1\phi_{11} + 2(h_2-h(t))\phi_{11}^T N_1\phi_{12}\\ &\quad + \frac{1}{2}[(h_2-h_1)^2 - (h_2-h(t))^2]\phi_{21}^T Z_2\phi_{21}\\ &\quad + 2\phi_{21}^T N_2[(h_2-h(t))\phi_{22} + (h(t)-h_1)\phi_{23}],\end{aligned}$$

where

$$\begin{aligned}\phi_{12} &\triangleq x(t-h(t)) - \omega_1(t),\\ \phi_{22} &\triangleq x(t-h_1) - x(t-h(t)),\\ \phi_{23} &\triangleq x(t-h_1) - \omega_2(t).\end{aligned}$$

Lemma 8. *[27] Let ξ_0, ξ_1 and ξ_2 be $m \times m$ real symmetric matrices and a continuous function h satisfy $h_1 \leq h \leq h_2$, where h_1 and h_2 are constants satisfying $0 \leq h_1 \leq h_2$. If $\xi_0 \geq 0$, then*

$$\begin{aligned}& h^2\xi_0 + h\xi_1 + \xi_2 < 0(\leq 0),\ \forall h \in [h_1, h_2],\\ \leftrightarrow\quad & h_i^2\xi_0 + h_i\xi_1 + \xi_2 < 0(\leq 0),\ (i = 1, 2), \qquad (10)\end{aligned}$$

or

$$\begin{aligned}& h^2\xi_0 + h\xi_1 + \xi_2 > 0(\geq 0),\ \forall h \in [h_1, h_2],\\ \leftrightarrow\quad & h_i^2\xi_0 + h_i\xi_1 + \xi_2 > 0(\geq 0),\ (i = 1, 2). \qquad (11)\end{aligned}$$

3 Main results

Now we present a Lyapunov-Krasovskii functional for the Lur'e system (1) satisfying the conditions (2), (3) with interval time-varying delay

$$V(t,x_t,\dot{x}_t)=\sum_{i=1}^{4}V_i(t), \tag{12}$$

where

$$\begin{aligned}
V_1(t) &\triangleq \eta^T(t)P\eta(t)+\int_{t-h_1}^{t}\dot{x}^T(s)Q_0\dot{x}(s)ds \\
&\quad+\int_{t-\tau(t)}^{t}\dot{x}^T(s)J\dot{x}(s)ds \\
V_2(t) &\triangleq \int_{t-h_1}^{t}[x^T(t)\ x^T(s)]Q_1[x^T(t)\ x^T(s)]^T ds \\
&\quad+\int_{t-h(t)}^{t-h_1}[x^T(t)\ \ x^T(s)]Q_2[x^T(t)\ \ x^T(s)]^T ds \\
&\quad+\int_{t-h_2}^{t-h(t)}[x^T(t)\ \ x^T(s)]Q_3[x^T(t)\ \ x^T(s)]^T ds \\
V_3(t) &\triangleq \int_{t-h_1}^{t}\Big\{h_1(h_1-t+s)\dot{x}^T(s)W_1\dot{x}(s) \\
&\quad+(h_1-t+s)^2\dot{x}^T(s)W_2\dot{x}(s)\Big\}\,ds \\
&\quad+\int_{t-h_2}^{t-h_1}\Big\{h_{21}(h_2-t+s)\dot{x}^T(s)R_1\dot{x}(s) \\
&\quad+(h_2-t+s)^2\dot{x}^T(s)R_2\dot{x}(s)\Big\}\,ds \\
V_4(t) &\triangleq 2\sum_{i=1}^{n}\int_{0}^{E_i^Tx}[k_i[f_i(s)-\sigma_i^-(s)] \\
&\quad+l_i[\sigma_i^+(s)-f_i(s)]]ds \\
&\quad 2\sum_{i=1}^{n}\int_{0}^{F_i^Tx}[g_i[h_i(s)-\delta_i^-(s)] \\
&\quad+t_i[\delta_i^+(s)-h_i(s)]]ds
\end{aligned} \tag{13}$$

where P are real matrices, $Q_0>0, Q_j>0, W_q>0, R_q>0, J>0 (j=1,2,3; q=1,2)$, $K=\text{diag}(k_1,...,k_n)>0, L=\text{diag}(l_1,...,l_n)>0, G=\text{diag}(g_1,...,g_n)>0, T=\text{diag}(t_1,...,t_n)>0$; and $h_{21}\triangleq h_2-h_1$,
$\eta(t) \triangleq \text{col}\{x(t),\ x(t-h_1),\ \int_{t-h_2}^{t-h(t)}x(s)ds,$ $\int_{t-h(t)}^{t-h_1}x(s)ds,$ $\int_{t-h_1}^{t}x(s)ds\ \}$.

Remark 9. *• This of [14] previous work only focused on the augment vector $\eta(t)=[x(t),\int_{t-\tau_0}^{t}x(s)ds]$ but our paper includes not only on $x(t),\int_{t-\tau_0}^{t}x(s)ds$ but also $x(t),\int_{t-h_2}^{t-h(t)}x(s)ds, x(t-h_1),\int_{t-h(t)}^{t-h_1}x(s)ds$. We can see that the adoption of new augmented variables, cross terms of variables and more multiple integral terms may reduce the conservatism.*
• Those of [5, 14] previous works, the augmented Lyapunov matrix P still need $P>0$, but for our paper does not need to be positive defiite, which can be seen in Lemma 10..

For simplicity of presentation, we set in the following
ω_1,ω_2 are defined in (9) and $\omega_3=\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds$. Denote by $\tilde{e}_i (i=1,\dots,5)$ the block-row vectors of the $5n\times 5n$ identity matrix. Then we have the following result.

Lemma 10. *[27] For the LKF (13), there exist scalars $\epsilon_1>0$ and $\epsilon_2>0$ such that*

$$\epsilon_1\|x\|^2\le V(t,x_t,\dot{x}_t)\le\epsilon_2\|x_t\|_W^2 \tag{14}$$

if the following LMIs are satisfied

$$\begin{aligned}
&\tilde{e}_1P\tilde{e}_1^T>0,\ \ P_0\ge 0,\ \ \Lambda_1(h_1)+\Lambda_2(h_1)\ge 0,\\
&\Lambda_1(h_2)+\Lambda_2(h_2)\ge 0,
\end{aligned} \tag{15}$$

where

$$\Lambda_1(h(t))\triangleq\begin{cases}\Delta, & h_1=0\\ \Delta+\frac{1}{h_1}\Gamma_2^T diag\{Q_0,3Q_0\}\Gamma_2, & h_1\neq 0\end{cases} \tag{16}$$

$$\begin{aligned}
\Lambda_2(h(t)) &\triangleq h_1[\tilde{e}_1^T\ \tilde{e}_5^T]Q_1[\tilde{e}_1^T\ \tilde{e}_5^T]^T+(h(t)-h_1)\\
&\quad\times[\tilde{e}_1^T\ \tilde{e}_4^T]Q_2[\tilde{e}_1^T\ \tilde{e}_4^T]^T+(h_2-h(t))\\
&\quad\times[\tilde{e}_1^T\ \tilde{e}_3^T]Q_3[\tilde{e}_1^T\ \tilde{e}_3^T]^T
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\Gamma_1 &= col\{\tilde{e}_1,\tilde{e}_2,(h_2-h(t))\tilde{e}_3,(h(t)-h_1)\tilde{e}_4,h_1\tilde{e}_5\},\\
\Gamma_2 &= col\{\tilde{e}_1-\tilde{e}_2,\tilde{e}_1+\tilde{e}_2-2\tilde{e}_5\},\\
P_0 &= (\tilde{e}_4^T\tilde{e}_4-\tilde{e}_3^T\tilde{e}_3)P(\tilde{e}_4^T\tilde{e}_4-\tilde{e}_3^T\tilde{e}_3),\\
\Delta &= \Gamma_1^TP\Gamma_1-\tilde{e}_1^T\tilde{e}_1P\tilde{e}_1^T\tilde{e}_1.
\end{aligned}$$

Theorem 11. *The system (1) satisfying the sector condition (2), (3), for given scalars h_1,h_2,μ_1,μ_2 and δ is absolutely stable if there exist P are real matrices to be determined, symmetric positive definite matrices $Q_0>0, Q_j>0, W_q>0, R_q>0, J>0 (j=1,2,3; q=1,2)$, and $n\times n$ diagonal matrices $K>0, L>0, G>0, T>0, U>0, V>0$ such that (15) and the following LMI holds:*

$$\sum_{i=1,h(t)=h_1,\mu(t)=\mu_1}^{4} \Sigma_i < 0, \quad \sum_{i=1,h(t)=h_1,\mu(t)=\mu_2}^{4} \Sigma_i < 0,$$
$$\sum_{i=1,h(t)=h_2,\mu(t)=\mu_1}^{4} \Sigma_i < 0, \quad \sum_{i=1,h(t)=h_2,\mu(t)=\mu_2}^{4} \Sigma_i < 0, \tag{18}$$

$$\begin{bmatrix} Z_i & N_i \\ N_i^T & R_2 \end{bmatrix} \geq 0, (i=1,2) \begin{bmatrix} Z_3 & N_2 \\ N_3^T & W_2 \end{bmatrix} \geq 0, \tag{19}$$

$$\begin{bmatrix} \tilde{R}_1 & S_1 \\ S_1^T & \tilde{R}_1 \end{bmatrix} \geq 0, Z_1 \geq Z_2, \tag{20}$$

where $\tilde{R}_1 = diag\{R_1, 3R_1\}$;*and*

$$\begin{aligned}
\Sigma_1(h(t),\dot{h}(t)) &\triangleq \Delta_1^T P\Delta_2 + \Delta_2^T P\Delta_1 + \Delta_0^T Q_0\Delta_0 \\
&- e_8^T Q_0 e_8 + \Delta_0^T J\Delta_0 \\
&- (1-\dot{\tau}(t))e_{11}^T J e_{11} \\
\Sigma_2(h(t),\dot{h}(t)) &\triangleq \Psi_{20} + [h(t)-h_1]\Psi_{21} + [h_2 - d(t)]\Psi_{22} \\
\Sigma_3(h(t)) &\triangleq \tilde{\varphi}_1^T S_1\tilde{\varphi}_2 + \tilde{\varphi}_2^T S_1^T\tilde{\varphi}_1 - \tilde{\varphi}_1^T\tilde{R}_1\tilde{\varphi}_1 \\
&+ (h_2 - h(t))^2(Z_1 - Z_2) \\
&+ (h_2 - h(t))\Psi_{31} + (h(t)-h_1)\Psi_{32} \\
&+ h_{21}^2 Z_2 - \tilde{\varphi}_2^T\tilde{R}_1\tilde{\varphi}_2 \\
\Sigma_4 &\triangleq -\tilde{\varphi}_3^T\tilde{W}_1\tilde{\varphi}_3 + \Delta_0^T(h_1^2 W_1 + h_1^2 W_2)\Delta_0 \\
&+ 2h_1 N_3(e_1 - e_7) + e_8^T(h_{21}^2 R_1 \\
&+ h_{21}^2 R_2)e_8 + 2h_1(e_1 - e_7)^T N_3^T + h_1^2 Z_3 \\
&+ e_{10}^T[K-L]E\Delta_0 + \Delta_0^T E^T[K-L]^T e_{10} \\
&+ e_1^T E^T[\bar{\Upsilon}_1 L - \bar{\Upsilon}_2 K]E\Delta_0 \\
&+ \Delta_0^T E^T[\bar{\Upsilon}_1 L - \bar{\Upsilon}_2 K]^T E e_1 \\
&+ e_9^T[G-T]F\Delta_0 + \Delta_0^T F^T[G-T]^T e_9 \\
&+ e_1^T F^T[\bar{\Upsilon}_3 G - \bar{\Upsilon}_4 T]F\Delta_0 \\
&+ \Delta_0^T F^T[\bar{\Upsilon}_3 G - \bar{\Upsilon}_4 T]^T F e_1 \\
&- [e_1^T E^T U\Upsilon_1 E e_1 - 2e_1^T E^T U\Upsilon_2 e_{10} \\
&+ e_{10}^T U e_{10}] - [e_1^T F^T V\Upsilon_3 F e_1 \\
&- 2e_1^T F^T V\Upsilon_4 e_9 + e_9^T V e_9]
\end{aligned} \tag{21}$$

with $e_i(i = 1,2,\ldots,11)$ *denoting the i-th row-block vector of the* $11n \times 11n$ *identity matrix* $\tilde{W}_1 = diag\{W_1, 3W_1\}$; *and*

$$\begin{aligned}
\Psi_{20} &\triangleq [e_1^T\ e_3^T](Q_2 - Q_1)[e_1^T\ e_3^T]^T \\
&+ h_1[\Delta_0^T\ 0]Q_1[e_1^T\ e_7^T]^T + h_1[e_1^T\ e_7^T]Q_1[\Delta_0^T\ 0]^T \\
&- (1-\dot{h}(t))[e_1^T\ e_2^T](Q_2 - Q_3)[e_1^T\ e_2^T]^T \\
&- [e_1^T\ e_4^T]Q_3[e_1^T\ e_4^T]^T + [e_1^T\ e_1^T]Q_1[e_1^T\ e_1^T]^T \\
\Psi_{21} &\triangleq [e_1^T\ e_6^T]Q_2[\Delta_0^T\ 0]^T + [\Delta_0^T\ 0]Q_2[e_1^T\ e_6^T]^T \\
\Psi_{22} &\triangleq [e_1^T\ e_5^T]Q_3[\Delta_0^T\ 0]^T + [\Delta_0^T\ 0]Q_3[e_1^T\ e_5^T]^T \\
\Psi_{31} &\triangleq 2N_1(e_2 - e_5) + 2N_2(e_3 - e_2) \\
&+ 2(e_3 - e_2)^T N_2^T + 2(e_2 - e_5)^T N_1^T \\
\Psi_{32} &\triangleq 2N_2(e_3 - e_6) + 2(e_3 - e_6)^T N_2^T \\
\tilde{\varphi}_1 &\triangleq col\{e_2 - e_4, e_2 + e_4 - 2e_5\} \\
\tilde{\varphi}_2 &\triangleq col\{e_3 - e_2, e_3 + e_2 - 2e_6\} \\
\tilde{\varphi}_3 &\triangleq col\{e_1 - e_3, e_1 + e_3 - 2e_7\} \\
\Delta_1 &\triangleq col\{e_1, e_3, (h_2 - h(t))e_5, (h(t)-h_1)e_6, h_1 e_7\} \\
\Delta_2 &\triangleq col\{\Delta_0, e_8, (1-\dot{h}(t))e_2 - e_4, e_3 \\
&- (1-\dot{h}(t))e_2, e_1 - e_3\}.
\end{aligned}$$

For simplicity of presentation, we denote $\Theta \triangleq col\{x(t), x(t-h(t)), x(t-h_1), x(t-h_2), \omega_1(t), \omega_2(t), \omega_3(t), \dot{x}(t-h_1), h(\sigma(t)), f(\omega(t)), \dot{x}(t-\tau(t))\}$, $\dot{x}(t) = \Delta_0\Theta(t), \Delta_0 \triangleq A_1 e_{11} + Ae_1 + Be_2 + Ce_{10} + De_9$.

Proof. Taking the derivative of V along the solution of system(1), we can be obtains as

$$\begin{aligned}
\dot{V}_1(t) =& 2\eta^T(t)P\dot{\eta}(t) + \dot{x}^T(t)Q_0\dot{x}(t) - \dot{x}^T(t-h_1)Q_0 \\
&\times \dot{x}(t-h_1) + \dot{x}^T(t)J\dot{x}(t) \\
&- (1-\dot{\tau}(t))\dot{x}^T(t-\tau(t))J\dot{x}(t-\tau(t)) \\
\dot{V}_2(t) =& [x^T(t)\ x^T(t)]Q_1[x^T(t)\ x^T(t)]^T \\
&- [x^T(t)\ x^T(t-h_1)]Q_1[x^T(t)\ x^T(t-h_1)]^T \\
&+ 2\int_{t-h_1}^{t}[x^T(t)\ x^T(s)]Q_1[\dot{x}(t)^T\ 0]^T ds \\
&+ [x^T(t)\ x^T(t-h_1)]Q_2[x^T(t)\ x^T(t-h_1)]^T \\
&- (1-\dot{h}(t))[x^T(t)\ x^T(t-h(t))]Q_2[x^T(t) \\
&x^T(t-h(t))]^T + 2\int_{t-h(t)}^{t-h_1}[x^T(t)\ x^T(s)]Q_2 \\
&\times [\dot{x}^T(t)\ 0]^T ds + [x^T(t)\ x^T(t-h_2)] \\
&\times Q_3[x^T(t)\ x^T(t-h_2)]^T \\
&- (1-\dot{h}(t))[x^T(t)\ x^T(t-h(t))]Q_3 \\
&\times [x^T(t)\ x^T(t-h(t))]^T \\
&+ 2\int_{t-h_2}^{t-h(t)}[x^T(t)\ x^T(s)]Q_3[\dot{x}^T(t)\ 0]^T ds
\end{aligned}$$

$$\begin{aligned}\dot{V}_3(t) =&\Theta^T(t)(h_1^2\Delta_0^T W_1\Delta_0 + h_1^2\Delta_0^T W_2\Delta_0)\Theta(t)\\ &- h_1\int_{t-h_1}^{t}\dot{x}^T(s)W_1\dot{x}(s)ds\\ &- 2\int_{t-h_1}^{t}(h_1-t+s)\dot{x}^T(s)W_2\dot{x}(s)ds\\ &+ h_{21}^2\dot{x}^T(t-h_1)R_1\dot{x}(t-h_1)\\ &+ h_{21}^2\dot{x}^T(t-h_1)R_2\dot{x}(t-h_1)\\ &- h_{21}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_1\dot{x}(s)ds\\ &- 2\int_{t-h_2}^{t-h_1}(h_2-t+s)\dot{x}^T(s)R_2\dot{x}(s)ds.\\ \dot{V}_4(t) =&2f^T(Ex(t))[K-L]E\dot{x}(t) + 2x^T(t)E^T[\bar{\Upsilon}_1 L\\ &- \bar{\Upsilon}_2 K]E\dot{x} + 2h^T(Fx(t))[G-T]F\dot{x}(t)\\ &+ 2x^T(t)F^T[\bar{\Upsilon}_3 G - \bar{\Upsilon}_4 T]F\dot{x}(t). \qquad (22)\end{aligned}$$

On the condition (6) and diagonal matrices $U > 0, V > 0$, then we have

$$\begin{aligned}&-[x^T(t)E^T U\Upsilon_1 Ex(t) - 2x^T(t)E^T U\Upsilon_2 f(Ex(t))\\ &+f^T(Ex(t))Uf(Ex(t))] - [x^T(t)F^T V\Upsilon_3 Fx(t)\\ &-2x^T(t)F^T V\Upsilon_4 h(Fx(t)) + h^T(Fx(t))Vh(Fx(t))]\\ &\geq 0.\end{aligned}$$

With the consideration of some term of $\dot{V}_2(t), \dot{V}_3(t)$, we obtained the following equality and inequality:

$$\begin{aligned}&\int_{t-h_1}^{t}[x^T(t)\ x^T(s)]Q_1[\dot{x}(t)^T\ 0]^T ds\\ &= [\int_{t-h_1}^{t}x^T(t)ds\ \ \int_{t-h_1}^{t}x^T(s)ds]Q_1[\dot{x}^T(t)\ \ 0]^T\\ &= h_1[x^T(t)\ \ \omega_3^T]Q_1[\dot{x}^T(t)\ \ 0]^T, \qquad (23)\end{aligned}$$

$$\begin{aligned}&\int_{t-h(t)}^{t-h_1}[x^T(t)\ x^T(s)]Q_2[\dot{x}(t)^T\ 0]^T ds\\ &= [\int_{t-h(t)}^{t-h_1}x^T(t)ds\ \ \int_{t-h(t)}^{t-h_1}x^T(s)ds]Q_2[\dot{x}^T(t)\ \ 0]^T\\ &= (h(t)-h_1)[x^T(t)\ \ \omega_2^T]Q_2[\dot{x}^T(t)\ \ 0]^T, \qquad (24)\end{aligned}$$

and

$$\begin{aligned}&\int_{t-h_2}^{t-h(t)}[x^T(t)\ x^T(s)]Q_3[\dot{x}(t)^T\ 0]^T ds\\ &= [\int_{t-h_2}^{t-h(t)}x^T(t)ds\ \ \int_{t-h_2}^{t-h(t)}x^T(s)ds]Q_3[\dot{x}^T(t)\ \ 0]^T\\ &= (h_2-h(t))[x^T(t)\ \ \omega_1^T]Q_3[\dot{x}^T(t)\ \ 0]^T. \qquad (25)\end{aligned}$$

By utilizing Lemma4, we can be estimated

$$\begin{aligned}&-\int_{t-h_1}^{t}\dot{x}^T(s)h_1W_1\dot{x}(s)ds\\ &\leq -[x(t)-x(t-h_1)]^T W_1[x(t)-x(t-h_1)]\\ &-3\tilde{\Omega_1}^T W_1\tilde{\Omega}_1, \qquad (26)\end{aligned}$$

where

$$\tilde{\Omega_1} = x(t) + x(t-h_1) - 2\omega_3.$$

And applying [27], we obtained the following

$$\begin{aligned}&-2\int_{t-h_1}^{t}(h_1-t+s)\dot{x}^T(s)W_2\dot{x}(s)ds\\ &\leq h_1^2\Theta^T(t)Z_3\Theta(t) + 2h_1\Theta^T(t)N_3[x(t)-\omega_3]\\ &+ 2h_1[x(t)-\omega_3]^T N_3^T\Theta(t), \qquad (27)\end{aligned}$$

$$\begin{aligned}&-\int_{t-h_2}^{t-h_1}\dot{x}^T(s)h_{21}R_1\dot{x}(s)ds\\ &\leq 2\varphi_{11}^T S_1\varphi_{21} - \varphi_{11}^T\tilde{R}_1\varphi_{11} - \varphi_{21}^T\tilde{R}_1\varphi_{21} \qquad (28)\end{aligned}$$

and

$$\begin{aligned}&-2\int_{t-h_2}^{t-h_1}(h_2-t+s)\dot{x}^T(s)R_2\dot{x}(s)ds\\ &\leq (h_2-h(t))^2\Theta^T(t)Z_1\Theta(t)\\ &+ 4(h_2-h(t))\Theta^T(t)N_1[x(t-h(t))-\omega_1]\\ &+ [(h_2-h_1)^2-(h_2-h(t))^2]\Theta^T(t)Z_2\Theta(t)\\ &+ 4\Theta^T(t)N_2[(h_2-h(t))[x(t-h_1)-x(t-h(t))]\\ &+ (h(t)-h_1)[x(t-h_1)-\omega_2(t)]]. \qquad (29)\end{aligned}$$

Hence, according to (22)-(29) we get

$$\begin{aligned}&\dot{V}(t,x_t,\dot{x}_t) \leq 2\eta^T(t)P\dot{\eta}(t) + \dot{x}^T(t)Q_0\dot{x}(t)\\ &- \dot{x}^T(t-h_1)Q_0\dot{x}(t-h_1) + \dot{x}^T(t)J\dot{x}(t)\\ &- (1-\delta)\dot{x}^T(t-\tau(t))J\dot{x}(t-\tau(t))\\ &+ [x^T(t)\ x^T(t)]Q_1[x^T(t)\ x^T(t)]^T\\ &- [x^T(t)\ x^T(t-h_1)]Q_1[x^T(t)\ x^T(t-h_1)]^T\\ &+ 2h_1[x^T(t)\ \ \omega_3^T]Q_1[\dot{x}^T(t)\ \ 0]^T\\ &+ [x^T(t)\ x^T(t-h_1)]Q_2[x^T(t)\ x^T(t-h_1)]^T\\ &- (1-\dot{h}(t))[x^T(t)\ x^T(t-h(t))]Q_2[x^T(t)\\ &x^T(t-h(t))]^T + 2(h(t)-h_1)[x^T(t)\ \ \omega_2^T]\\ &\times Q_2[\dot{x}^T(t)\ \ 0]^T + [x^T(t)\ x^T(t-h_2)]\\ &\times Q_3[x^T(t)\ x^T(t-h_2)]^T\\ &- (1-\dot{h}(t))[x^T(t)\ x^T(t-h(t))]Q_3\end{aligned}$$

$$\begin{aligned}
&\times [x^T(t)\ x^T(t-h(t))]^T \\
&+ 2(h_2 - h(t))[x^T(t)\ \ \omega_1^T]Q_3[\dot{x}^T(t)\ \ 0]^T \\
&+ \Theta^T(t)(h_1^2\Delta_0^T W_1\Delta_0 + h_1^2\Delta_0^T W_2\Delta_0)\Theta(t) \\
&- [x(t) - x(t-h_1)]^T W_1[x(t) - x(t-h_1)] \\
&- 3\tilde{\Omega}_1{}^T W_1\tilde{\Omega}_1 + h_1^2\Theta^T(t)Z_3\Theta(t) \\
&+ 2h_1\Theta^T(t)N_3[x(t) - \omega_3] + 2h_1[x(t) - \omega_3]^T \\
&\times N_3^T\Theta(t) + h_{21}^2\dot{x}^T(t-h_1)R_1\dot{x}(t-h_1) \\
&+ h_{21}^2\dot{x}^T(t-h_1)R_2\dot{x}(t-h_1) \\
&+ 2\varphi_{11}^T S_1\varphi_{21} - \varphi_{11}^T\tilde{R}_1\varphi_{11} - \varphi_{21}^T\tilde{R}_1\varphi_{21} \\
&+ (h_2 - h(t))^2\Theta^T(t)Z_1\Theta(t) \\
&+ 4(h_2 - h(t))\Theta^T(t)N_1[x(t-h(t)) - \omega_1] \\
&+ [(h_2 - h_1)^2 - (h_2 - h(t))^2]\Theta^T(t)Z_2\Theta(t) \\
&+ 4\Theta^T(t)N_2[(h_2 - h(t))[x(t-h_1) - x(t-h(t))] \\
&+ (h(t) - h_1)[x(t-h_1) - \omega_2(t)]] \\
&+ 2f^T(Ex(t))[K - L]E\dot{x}(t) + 2x^T(t)E^T \\
&\times [\bar{\Upsilon}_1 L - \bar{\Upsilon}_2 K]E\dot{x} + 2h^T(Fx(t))[G - T]F\dot{x}(t) \\
&+ 2x^T(t)F^T[\bar{\Upsilon}_3 G - \bar{\Upsilon}_4 T]F\dot{x}(t) \\
&- [x^T(t)E^T U\Upsilon_1 Ex(t) - 2x^T(t)E^T U\Upsilon_2 f(Ex(t)) \\
&+ f^T(Ex(t))Uf(Ex(t))] - [x^T(t)F^T V\Upsilon_3 Fx(t) \\
&- 2x^T(t)F^T V\Upsilon_4 h(Fx(t)) \\
&+ h^T(Fx(t))Vh(Fx(t))] \\
&\dot{V}(t, x_t, \dot{x}_t) \le \Theta^T(t)\Sigma(h(t), \dot{h}(t))\Theta(t) \qquad (30)
\end{aligned}$$

where $\Sigma(h(t), \dot{h}(t)) \triangleq \sum_{i=1}^{4}\Sigma_i$. Clearly, $\Sigma(h(t), \dot{h}(t))$ can be rewritten as $\Sigma(h(t), \dot{h}(t)) = h^2(t)\Pi_0 + h(t)\Pi_1 + \Pi_2$ where $\Pi = Z_1 - Z_2$ and Π_1 and Π_2 are $h(t)-$ independent real metrices. Now together with (8) and if $Z_1 - Z_2 \ge 0$ and the inequalities in (18) hold, then $\Sigma(h(t), \dot{h}(t)) < 0, \forall h(t) \in [h_1, h_2], \forall \dot{h}(t) \in [\mu_1, \mu_2]$. Then $\dot{V}(t, x_t) \le -\lambda\|x(t)\|$ for some $\lambda > 0, \forall x(t) \ne 0$. Thus the system (1) satisfy conditions (2),(3) is absolutely stable.

□

4 Numerical Example

In this section, we provide numerical examples to show the effectiveness of our theoretical results.

Example 4.1 Consider the following neutral system with time-varying delays which is studied in [16]:

$$\dot{x}(t) \;=\; A_1\dot{x}(t-\tau(t)) + Ax(t) + Bx(t-h(t))$$

with the following parameters:

$$A_1 = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \quad A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},$$
$$B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

By applying our proposed Theorem 11 to the above system, one can obtain maximum delay bounds as listed in Table II. It can be found that the maximum upper bounds on the allowable sizes to be $h(t) = \tau(t) = 4.2365$, which is larger than in [16]. This means that the proposed ideas in theorem 11 is effective in reducing the conservatism of stability criterion.

Table II: Upper bounds of interval time-varying delays with $h_1 = 0$ and $\tau(t) = h(t)$ for Example 4.2.

Methods	$h_2[\tau(t) = h(t)]$
[16]	0.985
Theorem 11	4.2365

5 Conclusion

In this paper, we have investigated new delay-dependent absolute stability of Lur' e systems for neutral type with time-varying delays. Based on Lyapunov-krasovskii theory combined with a quadratic convex approach. New delay-dependent sufficient conditions for absolute stability have been derived in terms of LMIs. Numerical examples are given to illustrate the effectiveness of the theoretic results which show that our results are much less conservative than some existing results in the literature.

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