Robust integral terminal sliding mode control for quadrotor UAV with external disturbances

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Abstract: The purpose of this paper is to solve the problem of the controlling of a quadrotor exposed to external constant disturbances. The quadrotor system is partitioned into two parts: the attitude subsystem and the position subsystem. A new robust integral terminal sliding mode control law (RITSMC) is designed for the inner loop that insures the stability of this last and the quick tracking of the right desired values of the Euler angles. To estimate the disturbance displayed on the z-axis and to control the altitude position subsystem, an adaptive backstepping technique is proposed, while the horizontal position subsystem is controlled using the backstepping approach. The stability of each of the quadrotor subsystems is guaranteed by the Lyapunov theory. The effectiveness of the proposed methods is clearly comprehended through the obtained results of the various simulations effectuated on Matlab/Simulink, and a comparison with another technique is presented.

Key–Words: Quadrotor UAV, Integral terminal sliding mode control, adabtive laws, Newton-Euler

1 Introduction

1.1 Background and motivation

Over the last decade, the robotic field has attracted great attention from researchers in particular the unmanned aerial vehicle (UAV). The quadrotor is a type of drone, which consists of 4 rotors. This later has a simple mechanical structure. The quadrotor can land vertically and take-off. The quadrotor has been used in many applications such as military missions, journalism, disaster management, archaeology, geographical monitoring, taxi services, search/rescue missions, environmental protection, performing missions in oceans or other planets, mailing and delivery, and other miscellaneous applications [1, 2]. Therefore, the quadrotor is a strongly coupled and highly nonlinear system. In order to control of the quadrotor in closed loop, many techniques have been designed such as Backstepping control, Fuzzy logic based intelligent control, Sliding mode control, Adaptive control approaches, Neural network based on hybrid nonlinear and intelligent control, and Model predictive control [3], hybrid finite-time control [4], Proportional derivative-sliding mode control [5], Nonlinear PID controller [6], and Adaptive fractional order sliding mode control [7].

1.2 Literature review

In Ref. [8], nonlinear control techniques are designed for the quadrotor's position and attitude. The attitude loop use the regular sliding mode controller. The backstepping technique is combined with a sliding mode control to design robust controller for outer loop and the yaw angle. In order to estimate the Quadrotor UAV Fault, an observer is developed. In Ref. [9], a second order sliding mode control technique is developed to control an underactuated quadrotor UAV. In order to select the best coefficient of this proposed controller, the Hurwitz analysis is used. This control approach allows converging the state variables to their reference values and ensuring the stability of quadrotor system. In Ref [10], a global fast terminal sliding mode control technique is developed for a quadrotor UAV. This controller allows solving the chattering problem, stabilizing the vehicle and converging the all states variables to zero. In Ref. [11], an adaptive control method based on sliding mode technique is developed for tracking control and stability of uncertain quadrotor. The quadrotor unknown parameters are estimated at any moment. The proposed control laws guarantee the stability of the quadrotor system and recommend the states variables to converge of theirs origin values in finite time. A combination of sliding mode control and integral backstepping techniques is presented in the Ref. [12]. The control strategy allows tracking the flight trajectory in the presence of the disturbances and stabilizing the attitude of UAV. In Ref. [13], a nonsingular fast terminal sliding mode (NFTSM) method is designed to obtain the good performances of quadrotor attitude. The tracking errors of quadrotor are converged to zero through this controller. In Ref. [14], a high order sliding mode observer is combined with a nonsingular modified super-twisting algorithm to propose a solution for the trajectory tracking problem of unmanned aerial systems (UAS). The proposed approach techniques offer an estimation for the translational velocities of the quadrotor and improve the robustness performance ability for UAV system against external disturbances. In Ref. [15], a terminal sliding mode controller for the yaw and altitude subsystems is designed. A sliding mode technique is developed to control the quadrotor under-actuated subsystem. In Ref. [16], two nonlinear control techniques(backstepping and sliding mode controllers) are suggested to solve the tracking trajectory problem in the presence of a relatively high perturbations, problem of quadrotor system affected by the input delays, parametric uncertainties, unmolded uncertainties, time-varying state and external disturbances, a robust nominal controller and a robust compensator are proposed in the Ref. [17] to solve process.

1.3 Contribution

In this paper, a robust terminal integral sliding mode control method is proposed for a quadrotor attitude. The RITSMC scheme is designed based on the Lyapunov theory. However, an adaptive backstepping is designed to estimates the unknown external disturbance acting in z-axis and to control the altitude subsystem. The backstepping technique is used to control the horizontal position of the quadrotor in the presence of the external perturbations.

1.4 Paper organization

The rest of the paper is structured as follows: the formulation of the quadrotor system is given in Section 2 The new RITSMC and the adaptive backstepping techniques are presented in Section 3 The numerical simulation results is prepared in Section 4 Finally, the conclusions is exposed in Section 5

2 The system modeling

The quadrotor is equiped with for rotors as shown in Figure 1. The body-fixed (O_b, X_b, Y_b, Z_b) and the earth-fixed (O_e, X_e, Y_e, Z_e) frames are defined. The absolute position of the quadrotor is defined by the vector $\xi = [x, y, z]^T$. The vector $\eta = [\phi, \theta, \psi]^T$ represents the Euler angles. The angular and linear velocities of the quadrotor are defined by the vectors $\Omega = [p, q, r]^T$ and $V = [u, v, w]^T$, respectively.
In this context, the formalism of Euler-Newton is used to the obtained of the quadrotor model [16, 18]:

$$\begin{cases} \dot{\xi} &= V \\ m\dot{V} &= RFe_3 - mge_3 - k_i(i=1,2,3)\dot{\xi} \\ \dot{R} &= RS(\Omega) \\ J\dot{\Omega} &= -\Omega \times J\Omega + M_c - M_a + M_b \end{cases} \tag{1}$$

R is the rotation matrix

$$R = \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi S_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix} \tag{2}$$

The coordinates transformation is given by the rotation matrix:

$$T = \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & S_\phi C_\theta \\ 0 & -S_\phi & C_\phi C_\theta \end{bmatrix} \tag{3}$$

We use $C_{(.)}$ to $\cos(.)$ and $S_{(.)}$ to $\sin(.)$. m represents the total mass of quadrotor, $I_{3\times3}$ is a dentity matrix, $J = diag(I_x, I_y, I_z)$ denotes the matrix moment of the inertia. F is the total thrust, this exression is given by : $F = F_1 + F_2 + F_3 + F_4$ where F_i= $b\Omega_i^2$, Ω_i is the angular speed and b is a parameter depends on the air density and blades geometry. $S = [p, q, r]^T$ denots skew symmetric matrix. M_a,M_c, and M_b, are the resultant of aerodynamic friction torque, the resultant of gyroscopic effect torque, and the torque developed by the four rotor of quadrotor, respectevely. These expressions can be given, respectively as follows:

$$M_a = diag(k_4\dot{\phi}^2, k_5\dot{\theta}^2, k_6\dot{\psi}^2) \tag{4}$$

where k_4,k_5 and k_6 represent the friction aerodynamics parameters.

$$M_c = \sum_{i=0}^{4} \Omega \times J_r[0, 0, (-1)^{i+1}\Omega_i]^T \tag{5}$$

J_r denotes the rotor inertia.

$$M_b = \begin{bmatrix} l(f_4 - f_2) \\ l(f_3 - f_1) \\ d(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{bmatrix} \tag{6}$$

d is the drag coefficient, l is the distance between a propeller and the centers of the quadrotor. The complete quadrotor dynamics in the presence of external disturbances is as follow:

$$\begin{cases} \ddot{\phi} &=& \dot{\theta}\dot{\psi}\frac{(I_y-I_z)}{I_x} - \frac{I_r}{I_x}\Omega_r\dot{\theta} - \frac{k_4}{I_x}\dot{\phi}^2 + \frac{1}{I_x}U_2 + d_\phi \\ \ddot{\theta} &=& \dot{\phi}\dot{\psi}\frac{(I_z-I_x)}{I_y} + \frac{I_r}{I_y}\Omega_r\dot{\phi} - \frac{k_5}{I_y}\dot{\theta}^2 + \frac{1}{I_y}U_3 + d_\theta \\ \ddot{\psi} &=& \dot{\phi}\dot{\theta}\frac{(I_x-I_y)}{I_z} - \frac{k_6}{I_z}\dot{\psi}^2 + \frac{1}{I_z}U_4 + d_\psi \\ \ddot{x} &=& -\frac{k_1}{m}\dot{x} + \frac{1}{m}((C_\phi S_\theta C_\psi + S_\phi S_\psi)U_1 + d_x) \\ \ddot{y} &=& -\frac{k_2}{m}\dot{y} + \frac{1}{m}((C_\phi S_\theta S_\psi - S_\phi C_\psi)U_1 + d_y) \\ \ddot{z} &=& -\frac{k_3}{m}\dot{z} - g + \frac{1}{m}(C_\phi C_\theta U_1 + d_z) \end{cases} \tag{7}$$

The relationship between the quadrotor inputs and the propeller speeds of the quadrotor can be written as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -lb & 0 & lb \\ -lb & 0 & lb & 0 \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \tag{8}$$

The dynamic model of the quadrotor position subsystem has three outputs $z(t), x(t), y(t)$ and one control input U_1. In order to solve the underacted problem, three virtual control (v_2, v_2, v_3) inputs using adaptive backstepping and backstepping technique are designed. The virtual control inputs are given as follow:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} (C_\phi S_\theta C_\psi + S_\phi S_\psi)\frac{U_1}{m} \\ (C_\phi S_\theta S_\psi - S_\phi C_\psi)\frac{U_1}{m} \\ -g + (C_\phi C_\theta)\frac{U_1}{m} \end{bmatrix} \tag{9}$$

Therfore, the desired roll, pitch and yaw angles (ϕ_d, θ_d) and the total thrust U_1 can be obtained as follow:

$$\begin{cases} U_1 &=& m\sqrt{v_1^2 + v_2^2 + (v_3+g)^2} \\ \phi_d &=& \arctan(S_{\theta_d}(\frac{v_1 S_{\psi_d} - v_2 C_{\psi_d}}{v_3+g})) \\ \theta_d &=& \arctan(\frac{v_1 C_{\psi_d} + v_2 S_{\psi_d}}{v_3+g}) \end{cases} \tag{10}$$

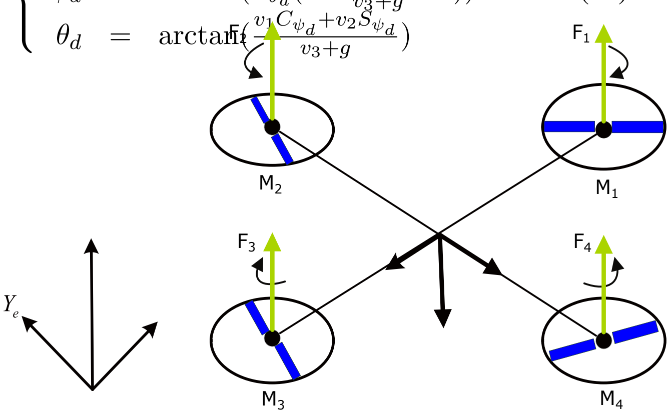

Figure 1: The quadrotor UAV with four rotors.

3 Controllers design and stability analysis

This section presents the robust nonlinear controllers for the quadrotor UAV in presence of the external perturbations. The proposed control of the quadrotor is divided into inner loop and the outer loop. For outer loop, an adaptive backstepping (AB) controller for altitude subsystem is designed. The AB method ensures the tracking of the desired altitude z_d and an accurate estimation of the unknown perturbation d_z acting of the altitude subsystem. The backstepping technique is used to obtain the virtual control (v_1, v_2). For inner loop, a new robust integral terminal sliding mode control (RITSMC) is proposed. The attitude controller generates the rolling, pitching and yawing torques to control the orientation of the quadrotor in the presence the external disturbances. Finally, the proposed control strategy solve the trajectory problem in short time with accuracy of the system performances. The general proposed control scheme is shown in Figure 2.

3.1 Adaptive backstepping control for altitude subsystem

Define tracking error of the altitude subsytem as

$$e_{z1} = z - z_d \tag{11}$$

Consider the Lyapunov candidate function

$$V_{z1} = \frac{1}{2}e_{z1}^2 \tag{12}$$

Taking the time derivative of Equation 12

$$\begin{aligned} \dot{V}_{z1} &= e_{z1}\dot{e}_{z1} \\ &= e_{z1}(\dot{z} - \dot{z}_d) \end{aligned} \tag{13}$$

The virtual control law as

$$\alpha_z = -c_{z1}e_{z1} + \dot{z}_d \tag{14}$$

with $c_{z1} > 0$ is a positive constant.
Define the tracking error of the step2

$$e_{z2} = \dot{z} - \alpha_z \tag{15}$$

The virtual control and adaptive laws of the altitude subsystem are given by the Equation 16 and Equation 17, respectively.

$$\begin{aligned} v_3 = -(e_{z1} + c_{z1}(e_{z2} - c_{z1}e_{z1}) + c_{z2}e_{z2} \\ + \frac{k_3}{m}\dot{z} - \ddot{z}_d + \hat{d}_z) \end{aligned} \tag{16}$$

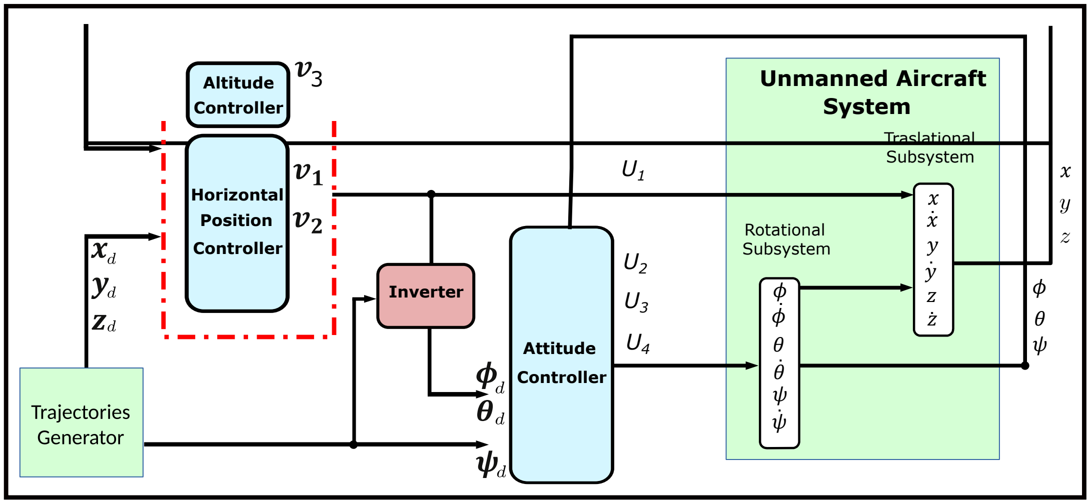

Figure 2: The control general structure for the quadrotor UAV.

$$\dot{\hat{d}}_z = \gamma_z e_{z2} \tag{17}$$

where c_{z2}, γ_z are the positive parameters and $\hat{d}_z$ denotes the estimate of d_z.
Proof. The Lyapunov function is considered as follows:

$$V_{z2} = \frac{1}{2}e_{z1}^2 + \frac{1}{2}e_{z2}^2 + \frac{1}{2\gamma_z}\widetilde{d}_z^2 \tag{18}$$

The time derivative of V_{z2} is given

$$\begin{aligned}\dot{V}_{z2} =& e_{z1}\dot{e}_{z1} + e_{z2}\dot{e}_{z2} + \frac{1}{\gamma_z}\widetilde{d}_z\dot{\widetilde{d}}_z \\ =& e_{z1}(e_{z2} - c_{z1}e_{z1}) + e_{z2}(\ddot{z} + c_{z1}\dot{e}_{z1} - \ddot{z}_d) \\ & - \frac{1}{\gamma_z}\widetilde{d}_z\dot{\hat{d}}_z \\ =& - c_{z1}e_{z1}^2 + e_{z2}[-\frac{k_3}{m}\dot{z} + v_3 + c_{z1}(ez2 - c_{z1}e_{z1}) \\ & - \ddot{z}_d] + e_{z2}\hat{d}_z + \widetilde{d}_z(e_{z2} - \frac{1}{\gamma_z}\dot{\hat{d}}_z)\end{aligned} \tag{19}$$

Considering the control law Equation 16 and adaptive law Equation 17, we get

$$\dot{V}_{z2} = -c_{z1}e_{z1}^2 - c_{z2}e_{z2}^2 \leq 0 \tag{20}$$

3.2 Backstepping control for horizontal position subsystem

In this part, the backstepping technique is used to control the horizontatl position of a quadrotor.
Introduce the tracking errors of the x and y positions

$$\begin{aligned} e_{x1} =& x - x_d \\ e_{y1} =& y - y_d \end{aligned} \tag{21}$$

Define the Lyapunov candidate function

$$\begin{aligned} V_{x1} =& \frac{1}{2}e_{x1}^2 \\ V_{y1} =& \frac{1}{2}e_{y1}^2 \end{aligned} \tag{22}$$

The time derivative of Equation 22

$$\begin{aligned} \dot{V}_{x1} =& e_{x1}\dot{e}_{x1} \\ =& e_{x1}(\dot{x} - \dot{x}_d) \\ \dot{V}_{y1} =& e_{y1}\dot{e}_{y1} \\ =& e_{y1}(\dot{y} - \dot{y}_d) \end{aligned} \tag{23}$$

From Equation 23 the virtual control laws are

$$\begin{aligned} \alpha_x &= -c_{x1}e_{x1} + \dot{x}_d \\ \alpha_y &= -c_{y1}e_{y1} + \dot{y}_d \end{aligned} \tag{24}$$

with $c_{x1}, c_{y1} > 0$ are the positive constants.
Consider the tracking errors of the step2 are given

$$\begin{aligned} e_{x2} &= \dot{x} - \alpha_x \\ e_{y2} &= \dot{y} - \alpha_y \end{aligned} \tag{25}$$

The virtual control laws of the horizontale subsystems are given by the Equation 26.

$$\begin{aligned} v_1 =& - (e_{x1} + c_{x1}(e_{x2} - c_{x1}e_{x1}) + c_{x2}e_{x2} \\ & + \frac{k_1}{m}\dot{x} - \ddot{x}_d + d_x) \\ v_2 =& - (e_{y1} + c_{y1}(e_{y2} - c_{y1}e_{y1}) + c_{y2}e_{y2} \\ & + \frac{k_2}{m}\dot{y} - \ddot{y}_d + d_y) \end{aligned} \tag{26}$$

Proof. Using the same procedure presented in the subsection 3.1 demonstrate the stability of horizontal subsystem.

3.3 Robust integral terminal sliding mode control laws for attitude subsystem

In order to solve the chattering phenomenon and to eliminate the singularity in a TSMC, a robust nonliner controller based on itegral terminal sliding mode technique is designed for the quadrotor attitude.
Introduce the tracking errors and its derivative, respectively of the attitude subsystem

$$\begin{aligned} e_\phi &= \phi - \phi_d \\ e_\theta &= \theta - \theta_d \\ e_\psi &= \psi - \psi_d \end{aligned} \tag{27}$$

$$\begin{aligned} \dot{e}_\phi &= \dot{\phi} - \dot{\phi}_d \\ \dot{e}_\theta &= \dot{\theta} - \dot{\theta}_d \\ \dot{e}_\psi &= \dot{\psi} - \dot{\psi}_d \end{aligned} \tag{28}$$

Consider the ITSM surface function as of the quadrotor attitude as [19, 20].

$$\begin{aligned} s_\phi &= \dot{e}_\phi + \int (\alpha_\phi \dot{e}_\phi^{(\frac{q_\phi}{p_\phi})} + \beta_\phi e_\phi^{(\frac{q_\phi}{2p_\phi - q_\phi})})dt \\ s_\theta &= \dot{e}_\theta + \int (\alpha_\theta \dot{e}_\theta^{(\frac{q_\theta}{p_\theta})} + \beta_\theta e_\theta^{(\frac{q_\theta}{2p_\theta - q_\theta})})dt \\ s_\psi &= \dot{e}_\psi + \int (\alpha_\psi \dot{e}_\psi^{(\frac{q_\psi}{p_\psi})} + \beta_\psi e_\psi^{(\frac{q_\psi}{2p_\psi - q_\psi})})dt \end{aligned} \tag{29}$$

where $\alpha_i(i = \phi, \theta, \psi)$ and $\beta_i(i = \phi, \theta, \psi)$ are the positive constants and $p_i(i = \phi, \theta, \psi)$ and $q_i(i = \phi, \theta, \psi)$ the positive integers with $p_i > q_i$.
The surface dynamics are

$$\begin{aligned} \dot{s}_\phi &= \ddot{\phi} - \ddot{\phi}_d + \alpha_\phi \dot{e}_\phi^{(\frac{q_\phi}{p_\phi})} + \beta_\phi e_\phi^{(\frac{q_\phi}{2p_\phi - q_\phi})} \\ \dot{s}_\theta &= \ddot{\theta} - \ddot{\theta}_d + \alpha_\theta \dot{e}_\theta^{(\frac{q_\theta}{p_\theta})} + \beta_\theta e_\theta^{(\frac{q_\theta}{2p_\theta - q_\theta})} \\ \dot{s}_\psi &= \ddot{\psi} - \ddot{\psi}_d + \alpha_\psi \dot{e}_\psi^{(\frac{q_\psi}{p_\psi})} + \beta_\psi e_\psi^{(\frac{q_\psi}{2p_\psi - q_\psi})} \end{aligned} \tag{30}$$

Using the exponential reaching laws, we get

$$\begin{aligned} \dot{s}_\phi &= -\lambda_\phi s_\phi - k_\phi sign(s_\phi) \\ \dot{s}_\theta &= -\lambda_\theta s_\theta - k_\theta sign(s_\theta) \\ \dot{s}_\psi &= -\lambda_\psi s_\psi - k_\psi sign(s_\psi) \end{aligned} \tag{31}$$

From Equation 30 and Equation 31, the control laws of attitude subsyetem are given as:

$$\begin{aligned} U_2 =& I_x(-(\dot{\theta}\dot{\psi}\frac{(I_y - I_z)}{I_x} - \frac{I_r}{I_x}\Omega_r\dot{\theta} - \frac{k_4}{I_x}\dot{\phi}^2) - d_\phi \\ &+ \ddot{\phi}_d - \alpha_\phi \dot{e}_\phi^{(\frac{q_\phi}{p_\phi})} - \beta_\phi e_\phi^{(\frac{q_\phi}{2p_\phi - q_\phi})} - \lambda_\phi s_\phi \\ &- k_\phi sign(s_\phi)) \end{aligned} \tag{32}$$

$$\begin{aligned} U_3 =& I_y(-(\dot{\phi}\dot{\psi}\frac{(I_z - I_x)}{I_y} + \frac{I_r}{I_y}\Omega_r\dot{\phi} - \frac{k_5}{I_y}\dot{\theta}^2) - d_\theta \\ &+ \ddot{\theta}_d - \alpha_\theta \dot{e}_\theta^{(\frac{q_\theta}{p_\theta})} - \beta_\theta e_\theta^{(\frac{q_\theta}{2p_\theta - q_\theta})} - \lambda_\theta s_\theta \\ &- k_\theta sign(s_\theta)) \end{aligned} \tag{33}$$

$$\begin{aligned} U_4 =& I_z(-(\dot{\phi}\dot{\theta}\frac{(I_x - I_y)}{I_z} - \frac{k_6}{I_z}\dot{\psi}^2) - d_\psi + \ddot{\psi}_d \\ &- \alpha_\psi \dot{e}_\psi^{(\frac{q_\psi}{p_\psi})} - \beta_\psi e_\psi^{(\frac{q_\psi}{2p_\psi - q_\psi})} - \lambda_\psi s_\psi \\ &- k_\psi sign(s_\psi)) \end{aligned} \tag{34}$$

Proof. in order to demonstrate the stability of attitude subsytem, the Lyaponov condidate function of the roll subsystem is given as follow:

$$V_\phi = \frac{1}{2}s_\phi^2 \tag{35}$$

The time derivative of V_ϕ is

$$\begin{aligned} \dot{V}_\phi =& s_\phi \dot{s}_\phi \\ =& s_\phi(\ddot{\phi} - \ddot{\phi}_d + \alpha_\phi \dot{e}_\phi^{(\frac{q_\phi}{p_\phi})} + \beta_\phi e_\phi^{(\frac{q_\phi}{2p_\phi - q_\phi})}) \\ =& s_\phi(\dot{\theta}\dot{\psi}\frac{(I_y - I_z)}{I_x} - \frac{I_r}{I_x}\Omega_r\dot{\theta} - \frac{k_4}{I_x}\dot{\phi}^2 + \frac{1}{I_x}U_2 \\ &+ d_\phi - \ddot{\phi}_d + \alpha_\phi \dot{e}_\phi^{(\frac{q_\phi}{p_\phi})} + \beta_\phi e_\phi^{(\frac{q_\phi}{2p_\phi - q_\phi})}) \\ =& s_\phi(\dot{\theta}\dot{\psi}\frac{(I_y - I_z)}{I_x} - \frac{I_r}{I_x}\Omega_r\dot{\theta} - \frac{k_4}{I_x}\dot{\phi}^2 \\ &+ \frac{1}{I_x}I_x(-(\dot{\theta}\dot{\psi}\frac{(I_y - I_z)}{I_x} - \frac{I_r}{I_x}\Omega_r\dot{\theta} - \frac{k_4}{I_x}\dot{\phi}^2) - d_\phi \\ &+ \ddot{\phi}_d - \alpha_\phi \dot{e}_\phi^{(\frac{q_\phi}{p_\phi})} - \beta_\phi e_\phi^{(\frac{q_\phi}{2p_\phi - q_\phi})} - \lambda_\phi s_\phi + d_\phi - \ddot{\phi}_d \\ &- k_\phi sign(s_\phi) + \alpha_\phi \dot{e}_\phi^{(\frac{q_\phi}{p_\phi})} + \beta_\phi e_\phi^{(\frac{q_\phi}{2p_\phi - q_\phi})}) \\ =& -k_\phi s_\phi sign(s_\phi) - \lambda_\phi s_\phi s_\phi \\ &= -k_\phi |s_\phi| - \lambda_\phi s_\phi^2 \leq 0 \end{aligned} \tag{36}$$

4 Simulation results

To validate the performances of the proposed controllers, numerical simulations will be presented in this section. Furthermore, In order to highlight the superiority of the proposed control laws, comparison with first order sliding mode control technique is done. The parameters of quadrotor used in the simulation are selected in Table 1.

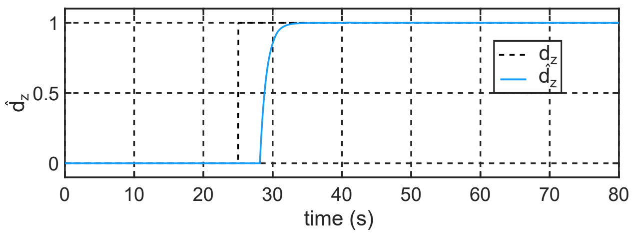

Figure 3: The estimate of d_z parameter

Table 1: Quadrotor parameters.

Parameter	Value	Parameter	Value
$g(m/s^2)$	9.81	$k_2(N/m/s)$	5.5670e-4
$m(kg)$	0.486	$k_3(N/m/s)$	5.5670e-4
$I_x(kg.m^2)$	3.827e-3	$k_4(N/m/s)$	5.5670e-4
$I_y(kg.m^2)$	3.827e-3	$k_5(N/m/s)$	5.5670e-4
$I_z(kg.m^2)$	7.6566e-3	$k_6(N/m/s)$	5.5670e-4
$I_r(kg.m^2)$	2.8385e-5	$b(N.s^2)$	2.9842e-3
$k_1(N/m/s)$	5.5670e-4	$k(N.m.s^2)$	3.2320e-2

The initial attitude and position of a quadrotor are chosen as $[0, 0, 0]$rad and $[0, 0, 0]$m. The desired trajectory of the yaw angle and the position are given in Table 2.

Table 2: The reference trajectories of the position and yaw angle

Variable	Value	Time(s)
$[x_d, y_d, z_d]$	$[0.6,0.6,0.6]m$	0
	$[0.3,0.6,0.6]m$	10
	$[0.3,0.3,0.6]m$	20
	$[0.6,0.3,0.6]m$	30
	$[0.6,0.6,0.6]m$	40
	$[0.6,0.6,0.0]m$	50
$[\psi_d]$	$[0.5]rad$	0
$[\psi_d]$	$[0.0]rad$	50

The external disturbances used in simulation are given as follows:
$d_x = N$ at t = 5 s; d_ϕ = 1 Nm at t = 10 s and $d_y = N$ at t = 15 s d_θ= 1 Nm at t = 20 s; $d_z = N$= 1 at t = 25 s; and d_ψ = 1 Nm at t = 30 s. Besides, the parameters of the proposed controllers are listed in Table 3.

Table 3: Control system parameters

Parameters	Value	Parameters	Value
c_{x1}	1.68	c_{x2}	2.34
c_{y1}	1	c_{y2}	2.25
c_{z1}	6.40	c_{z2}	20
α_i	59.78	β_i	974.57

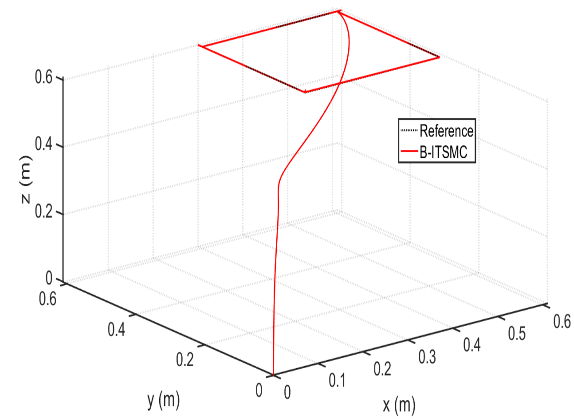

Figure 4: Flight trajectory tracking (RITSMC)

The simulations results are shown in Figures 3–9. The estimate force acting in direction z is shown in Figure 3. The desired and actual tracking position x, y and z are shown in Figure 5, where the proposed control strategy that drive the quadrotor to track the desired flight trajectory more rapidly and more accurately can be seen than classic sliding mode control method. The constant disturbances are added in the position subsystem at $t = 5$. It appears that the proposed controllers are managed to effectively hold the quadrotor's position in finite-time contrary to the SMC, the same behavior can be observed at t=15 for the y position and at $t = 25$ for the z position of the quadrotor system. Furthermore, trajectory attitude ϕ, θ and ψ are shown in Figure 6. It is shown that the quadrotor attitude track the desired angles in short finite time. The attitude sliding variables $(s_\phi, s_\theta, s_\psi)$ are shown in Figure 8, the convergece to zero in finite time of the sliding surfaces can be observed. The trajectory tracking errors of position are depicted in Figure 7. In order to demonstrate the superiority of the proposed control strategy, the quadrotor trajectory path in 3-D space via the proposed control method is shown in Figure 4, and via the SMC in Figure 9. Clearly, it can be seen that the proposed can accurately track the square trajectory. The proposed control ap-

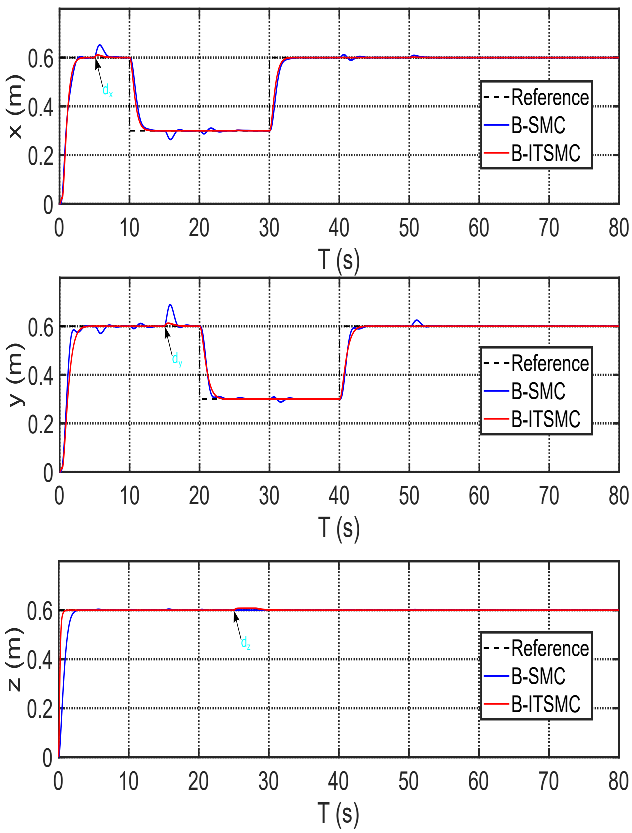

Figure 5: Response of quadrotor position

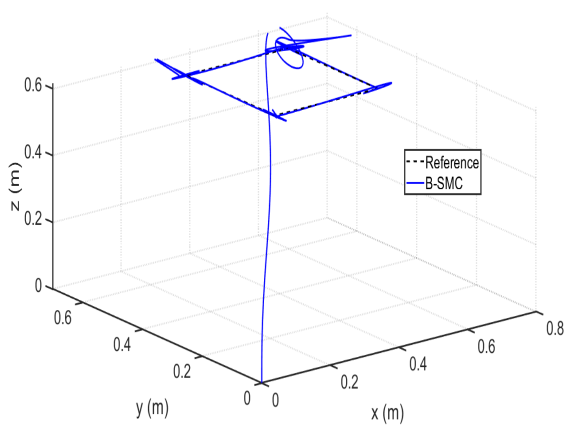

Figure 9: Flight trajectory tracking (SMC)

5 Conclusions

In this study, we've examined with the problem of the flight trajectory-tracking phenomenon of the quadrotor with disturbances. The proposed control scheme

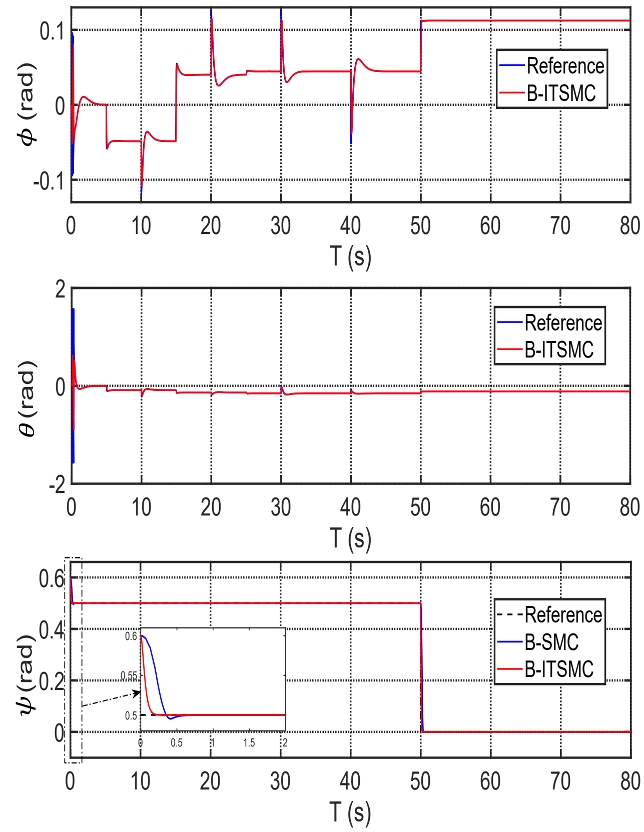

Figure 6: Response of quadrotor att

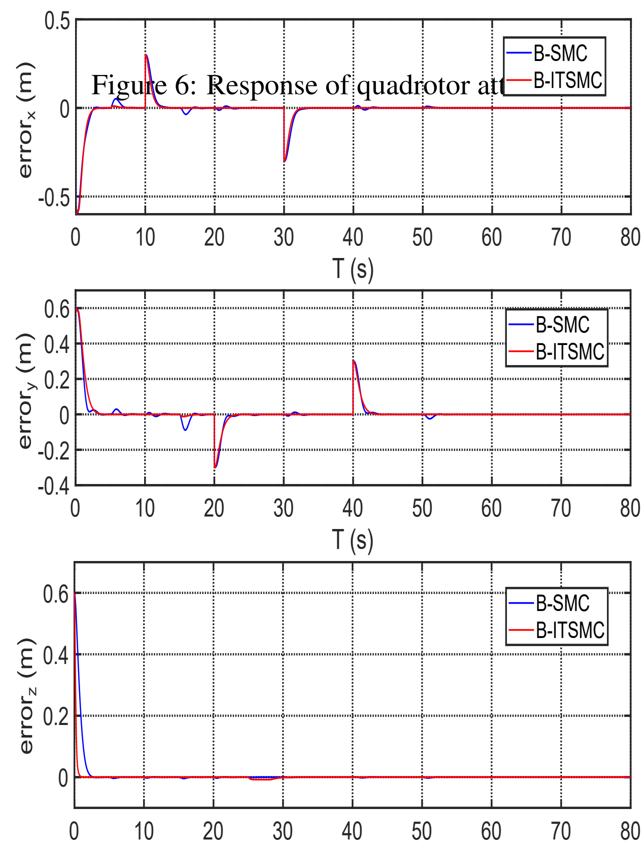

Figure 7: Position errors

Figure 8: Tracking errors $(s_\phi, s_\theta, s_\psi,)$, RITSMC

is made up of three different parts: control of an altitude, a horizontal position and an attitude subsystem. Firstly, the altitude subsystem is addressed based on the adaptive backstteping. Besides, the backstteping technique is designed to control the position x and y of a quadrotor. The control objectives of this loop are: (i) obtaining the desired roll and pitch angles; (ii) Tracking the desired flight trajectory in finite time; (iii) Generating the total thrust. Secondly, a new robust integral sliding mode controller has been constructed to stabilize the attitude subsystem. The objectives of the RITSMC are follows:

- Tracking the desired angles
- Stabilizing the attitude quadrotor.
- Generating the rolling, pitching and yawing torques.

In addition, the proposed controllers achieve the fast and the accurate tracking of the quadrotor trajectory. Finally, simulation results have demonstrated that the proposed controllers are able to improve control performance of the quadrotor UAV system in the presence of the external disturbances. The proposed control strategy has shown the effectiveness and superiority of the classical sliding mode strategy.

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