Modelling and analysis systems in State Space

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Abstract: - This paper presents some considerations on the mathematical description as well as the assay and modelling and analysis systems in state space. The mathematical model of the systems consists of a set of equations describing the system trajectory and how the system behaves from some points of view. After obtaining the mathematical model and transfer matrix, we analysed the system both in terms of controllability and observability. This analysis was facilitated by the use of the Matlab programming environment and interfaces Labview.

Key-Words: - state space, mathematical model, controllability, observability.

1 Introduction

The output of a system is affected by the inputs before the time t_0. Taking into account the inputs from the time point t=-∞ is difficult to achieve, so a new concept, namely the state variable, will be introduced. Status variables represent a group of sizes that completely define the status of the system at a time. These variables also fulfil the role of initial conditions for the evolution of the previous system. By definition, the state $x(t_0)$ of the system at the time t_0 is the information available that together with the input u(t) for $t \geq t_0$, uniquely determines the output y(t) of the system for $t \geq t_0$. Thus, if the state of the system is known at the time t_0 of its determination, y(t) to $t \geq t_0$, it is no longer necessary to know the inputs applied before the time t_0. State variables are selected as output signal, along with its derivatives in relation to time.

We will write the general form of the system input-state-output equations with matrices A, B, C and D independent of time with the meaning of matrices [1-6].

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (1)$$

where:
A - the system matrix (nxn)
B - the input matrix (nxm)
C - the output matrix (pxn)
C - the direct connection matrix (pxn)

2 Mathematical model for the level liquid system

Using the transformed Laplace for the system (1) we obtain:

$$\begin{cases} sX(s) - x(0) = AX(s) + BU(s) \\ Y(s) = CX(s) + DU(s) \end{cases} \Rightarrow$$
$$\begin{cases} sX(s) - AX(s) = BU(s) + x(0) \\ Y(s) = CX(s) + DU(s) \end{cases} \Rightarrow$$
$$\Rightarrow \begin{cases} X(s) = \dfrac{BU(s) + x(0)}{sI - A} \\ Y(s) = CX(s) + DU(s) \end{cases} \Rightarrow \quad (2)$$
$$\begin{cases} X(s) = (sI - A)^{-1} BU(s) + (sI - A)^{-1} x(0) \\ Y(s) = CX(s) + DU(s) \end{cases} \Rightarrow$$

Replace equations X(s) in equation Y(s):

$$\Rightarrow Y(s) = C\left[(sI - A)^{-1} BU(s) + (sI - A)^{-1} x(0)\right] + DU(s) \Rightarrow$$
$$\Rightarrow Y(s) = C(sI - A)^{-1}\left[BU(s) + x(0)\right] + DU(s) \Rightarrow \quad (3)$$
$$\Rightarrow Y(s) = C(sI - A)^{-1} x(0) + \left[C(sI - A)^{-1} B + D\right]U(s)$$

If we consider x(0)=0 and know $G(s) = \dfrac{Y(s)}{U(s)}$ that we will determine the transfer function or the transfer matrix (it is also called because it is calculated on the basis of matrix operations) [7-15].

$$G(s) = C(sI - A)^{-1} B + D \quad (4)$$

where: I - is the unit matrix

There are industrial applications where the level of liquid in tanks used in the production process needs to be kept constant. A simplified model can be considered as the example in figure 1, for which it is required to determine the equivalent transfer function. It is assumed that under normal operating conditions, the inlet and outlet flow rates of the two tanks are equal to Q, and the liquid levels are H1 and H2. A u-disturbance of the inlet flow rate in the first tank is considered to cause variations in the liquid levels x1 and x2, as well as the flow rates of the two y1 and y tanks [16].

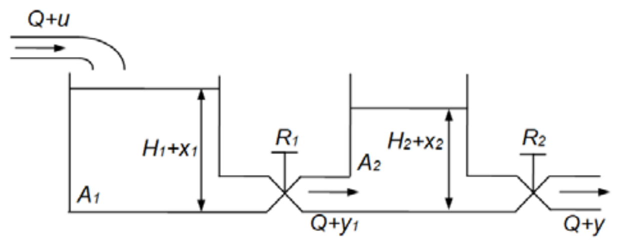

Fig.1 Liquid level system

If R1 and R2 are considered to be the resistances of the two valves in the system and they depend on the normal levels H1 and H2 we will have the relations:

$$y_1 = \frac{x_1 - x_2}{R_1} \quad y = \frac{x_2}{R_2} \qquad (5)$$

Changes in fluid levels can also be highlighted through the following equations:

$$\begin{cases} A_1 dx_1 = (u - y_1)dt \\ A_2 dx_2 = (y_1 - y)dt \end{cases} \Rightarrow \begin{cases} \frac{dx_1}{dt} = \frac{u - y_1}{A_1} \\ \frac{dx_2}{dt} = \frac{y_1 - y}{A_2} \end{cases} \qquad (6)$$

By replacing y and y1 in the above relations we obtain:

$$\begin{cases} \dot{x}_1 = \frac{u}{A_1} - \frac{x_1 - x_2}{A_1 R_1} \\ \dot{x}_2 = \frac{x_1 - x_2}{A_2 R_1} - \frac{x_2}{A_2 R_2} \end{cases} \qquad (7)$$

These equations are also written in matrix form as follows [16]:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{A_1 R_1} & \frac{1}{A_1 R_1} \\ \frac{1}{A_2 R_1} & -\left(\frac{1}{A_2 R_1} + \frac{1}{A_2 R_2}\right) \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix}}_{B} u \qquad (8)$$

$$y = \underbrace{\begin{bmatrix} 0 & \frac{1}{R_2} \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For examples, if $A_1=10$, $A_2=5$, $R_1=2$ and $R_2=1$, we obtain:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{20} & \frac{1}{20} \\ \frac{1}{10} & -\frac{3}{10} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{10} \\ 0 \end{bmatrix}}_{B} u \qquad (9)$$

$$y = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

3 Determining the transfer matrix and testing system controllability and observability

To determine the equivalent transfer function we will use the following Matlab code sequence:

```
syms s A1 A2 R1 R2
A=[-1/(A1*R1)    1/(A1*R1);    1/(A2*R1)    -(1/(A2*R1)+1/(A2*R2))]
B=[1/A1; 0]
C=[0 1/R2]
D=[0]
Phi=inv(s*eye(2)-A)
G=C*PHI*B+D
pretty(simple(G))
```

We will get matrices for the input-state-output system and transfer function:

$$G(s) = \frac{1}{A_1 A_2 R_1 R_2 s^2 + (A_1 R_1 + A_1 R_2 + A_2 R_2)s + 1} \qquad (10)$$

3.1 System controllability

A system is controllable if, for any initial state, an input vector can be found to determine the evolution of the state to any desired final value.

It is considered a system described by the equations (1). Because the output of the system has nothing to do with the controllability property, only the first equation will be referred. A system is fully controllable if there is an input vector u (t) that transfers the system from the initial state x (t0) to the final state, whatever the initial state x (t0) = x0 and the final state x (t1) x (t0) in a finite time. A system is a partially controllable or uncontrollable state if the x1 component of the state is controllable, and the x2 component of the state is uncontrollable.

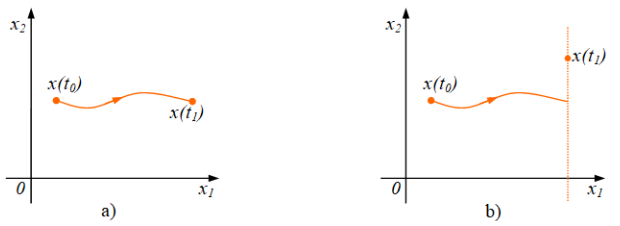

Fig. 2 System controllability [16].
a) fully controllable state system, b) partially controllable state system

Testing the controllability with formula (11).

$$C = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} \qquad (11)$$

where C is the controllability matrix formed by the sub-matrices A^kB, k = 0,1, ..., n-1

The system is fully controllable if and only if rank C=n or det C≠0.

3.1 System observability

A system is observable if the evolution of the inputs and outputs is known over a time interval and the state function can be deduced over the time interval considered. In other words, observability is the property of dynamic systems that highlight the possibility of estimating the state of the system by knowing its output [16].

It is considered a system described by the equations (1). A system is fully observable if, for any t0, the state vector x(t0) can be fully determined based on the knowledge of the input vector u (t) and the output vector y(t) on the interval [t0, t1] with t1> t0≥0.

Testing for a system observability is performed with formula (12).

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \qquad (12)$$

where O is the observability matrix formed by subclasses CA^k, k = 0,1, ..., n-1.

A system is fully observable and only if the rank O=n or det O ≠ 0.

We tested the system controllability and observability with the following Matlab program:

```
syms s t
A = [-1/20 1/20; 1/10 -3/10];
B=[1/10;0]
C=[0 1];
CO=ctrb(A,B)
OB=obsv(A,C)
d1=det(CO)
d2=det(OB)
if ((d1==0)&(d2==0))
disp ('The system not is controllable and nor is observable')
elseif ((d1==0)&(d2~=0))
disp('The system is observable, but not is controllable')
elseif ((d1~=0)&(d2==0))
disp('The system is controllable, but not is observable')
elseif ((d1~=0)&(d2~=0))
disp('The system is controllable and observable')
end
```

we will obtain d_1=0.001 and d_2=-0.1 and matrices that controllability and observability (13):

$$CO = \begin{bmatrix} 0.1 & -0.005 \\ 0.1 & -0.3 \end{bmatrix} \quad and \qquad OB = \begin{bmatrix} 0 & 1 \\ 0.1 & -0.3 \end{bmatrix} \qquad (13)$$

Resulting: *The system is controllable and observable*

To analyse and visualize controllability and observability following interface system we use Labview.

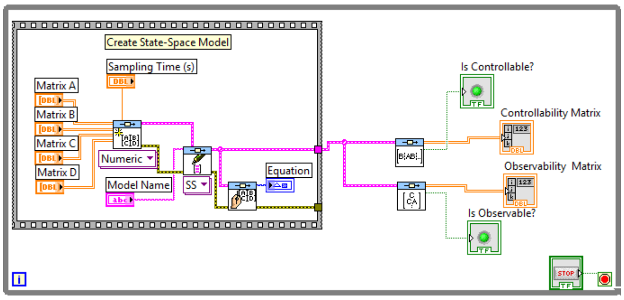

Fig. 3 Block Diagram

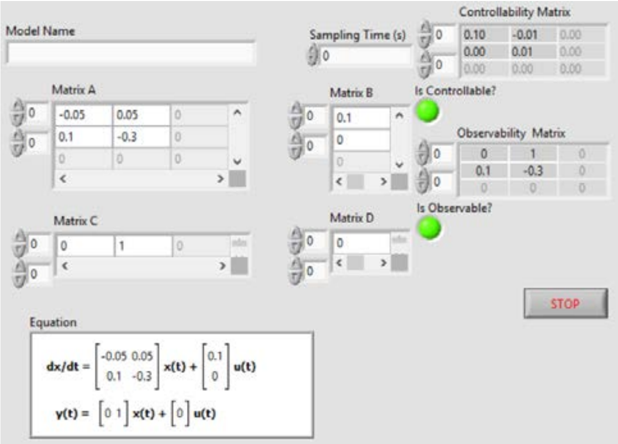

Fig. 4 Front panel

The LED's are on, so the system is controllable and observable.

5 Conclusion

The analysis of the trajectory of the electrical systems with the Matlab matrix calculation program has functions dedicated to the system analysis using the controllability matrix - ctrb and observability matrix - obsv. A system is controllable and observable in Labview if the LED's are on. The ranges of these matrices will give us information about the performance of the system. Thus, prototyping a system will have to go through the steps of analyzing the space of the states listed above in order to shorten the start-up time of the analyzed system.

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