Optimal Control for Energy Harvesting: Electrical Model and Measurements

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Abstract: The energy harvesting of piezo-electric devices is not simple nor straightforward. The complex internal structure of the device makes the AC-DC conversion too involved. The buck-boost topology showed to be effective in discontinuous mode providing a constant input impedance for small amount of power. In this paper a closed-loop optimal control algorithm to deal with any kind of electrical voltage input of a buck-boost converter is considered. This optimality yields a much bigger output power when compared to the case of non-optimal control. Application to piezo-electric devices is focused with the energy harvesting in mind, in this case the sudden drop of energy exhibited by the nature of the piezo-device is significantly mitigated by the optimal algorithm. Some simulations as well as comparisons with real measurements using a commercial piezo-electric device are presented along with conclusions and future work.

Key–Words: Optimal Control, Energy Harvesting, Electrical Model, Piezo-Electric

1 Introduction

Energy haversting is an active research area nowadays (see for instance [1]). Many sources of energy can be depicted: thermal, light, vibration, etc. One of the most recently focused source is the well known piezo-electric haversting (see for instance [2]).

Ranging from microwatt to miliwatt, this small amount of energy must be optimized regarding it is input impedance befaore it can be applied. In this way, and taking into account that the internal model of a piezoelectric is far from being pure resistive, complex structures must be developed in order to extract as much energy as possible (see for instance [3]).

This optimization can be accomplished in several ways, however two main methodologies are researched at present:

- Optimal pure resistive load
- Trully optimal topology for any kind of load

The first case implies the simplest possibility: among all the pure resitive loads, find the one that extract the biggest amount of energy for the piezo-electric device, however the second case implies to find an optimal control algorithm to apply to some AC-DC converter in order to approximate, as much as possible, the well known *maximum power transfer theorem*. This is a very difficult task when implemented in hardware, but one approach to that, it is the use of buck-boost converters which, moreover, behave as a pure resistive input load in discontinuous mode (see [2]).

In this paper, following the research line depicted at [4], a novel optimal control technique is presented using a buck-boost converter. In fact, starting with a buck-boost circuit connected to a rectifier bridge after the piezo-electric device, a singular optimal control strategy is developed under the Pontryagin's principle.

This strategy, renders the design independent of the load connected thus improving and extending the applicability of energy harvesting beyond the scope of the pure resistive scenario. The singular optimal control algorithm is obtained as closed-loop, so it can be readily programmed in a microprocessor. As it is well known, singular optimal control is rather more difficult than traditional non-singular optimal control. However, this scenario allows to develop a control methodology that renders the solution and switching times (bang-bang control) independent of the load (only current measurements are required).

This paper is organized as follows: Section 2 presents simplified internal models of a piezo-devices available in the literature, Section 3 revisit the well-known maximum-power transfer theorem with simulations of a piezo-device to use later on, Section 4 recalls the Buck-Boost topology and presents a plete state-space formulation, Section 5 obtain optimal control law for the Buck-Boost, Secti presents some simulations using the model afore tioned along with the optimal control ans finally tion 7 presents some measurements with the algo implemented in hardware in a Texas' micro-proce Section 8 presents some conclusions and future v

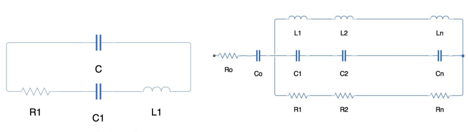
2 Electrical Model of a Pie Electric Device

A Piezo-Electric device it is an arrangement of such that they can generate electricity when sure is applied, or vibration if voltage is applie this manner, a possible simple electrical-mecha model was considered by Van Dyke model in [8 it is known that a piezoelectric device change th ternal model when is mounted on a structure to ex energy.

This phenomenon leads the idea to conside models instead:

- Unloaded model
- Loaded model

Figure 1 shows the two electrical models.



(a) Unloaded (b) Loaded

Figure 1: Two simple models of Van Dyke presented at [8]

Clearly, the determination of the physical parameters is not easy nor straightforward, so we consider the real model in [7] for the unloaded case (Figure 2 and Table 1).

$R_0(\Omega)$	5	$R_1(\Omega)$	115
$C_0(\mu F)$	0.15	$C_1(\mu F)$	0.277
	$L_1(\mu Hy)$	30.253	

Table 1: Estimated parameters of some real piezo-device given in [7]

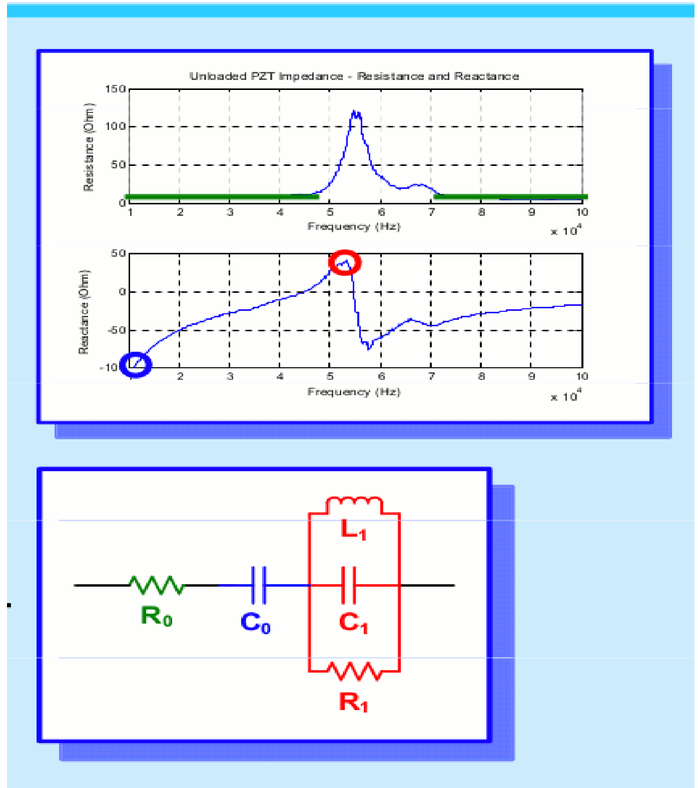


Figure 2: The real model presented in [7]

3 Maximum transfer power with passive loads

As it is well known, an equivalent R-C model can be derived from the previous one. Then the maximum transfer power theorem for sinusoidal sources yields the general two element model depicted in Figure 3.

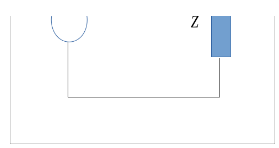


Figure 3: Maximum transfer power theorem.

with $Z_{th} = Z_L^*$ and with $*$. the complex conjugate. In the case of a simple electrical model of a piezo-electric device neglecting the internal inductance, Figure 4 is obtained with the nominal parameters for the unloaded case as shown in Table 2.

$R(\Omega)$	5
$C(\mu F)$	0.13

Table 2: Nominal parameters for the piezo-device neglecting the inductance.

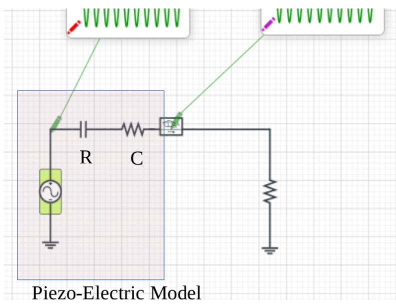
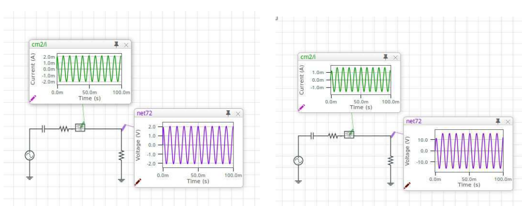


Figure 4: Sinusoidal waveforms for the maximum transfer power theorem.

3.1 Pure Resistive Loads

If only pure resistive loads are applied, low power is obtained as a result of the analysis with the parameters afore obtained in Table 2 (Figure 5).



(a) $R_L = 1K$ (b) $R_L = 10K$

Figure 5: Simulations with two different resistive loads.

with $R_L = 1K\Omega$, $P(RMS) = 1mW$ and $R_L = 10K\Omega$, $P(RMS) = 4.5mW$. For the simple R-C case, the maximum transfer power is obtained as:

$$R_{Load} = \frac{1}{2 \cdot \pi \cdot f \cdot C}$$

It is clear that a more sophisticated solution has to be developed in order to extract appropriate amount of energy to be used in a real application.

Discontinous Mode
$R_{IN} = \frac{2 \cdot L}{D^2 \cdot T_{SW}}$
Continuous Mode
$R_{IN} = \left(\frac{1-D}{D}\right)^2 \cdot R_0$

Table 3: Average input impedance for the Buck-Boost.

4 The Buck-Boost topology

Nowadays, the Buck-Boost converter is the most common topology due to its average pure resistive input impedance (see [3] and [6] along with Figure 6).

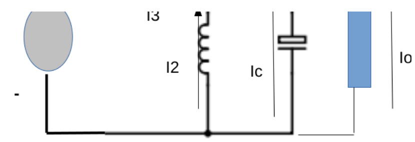


Figure 6: Buck-Boost topology

As indicated in [6], a Buck-Boost circuit can exhibit constant input average impedance depending on the mode of operation as depicted in Table 3.

On the other hand, the buck-boost topology possess and inverting nature between input and output voltage (see Figure 7 simulating a piezo-device with a buck-boost and resistive load).

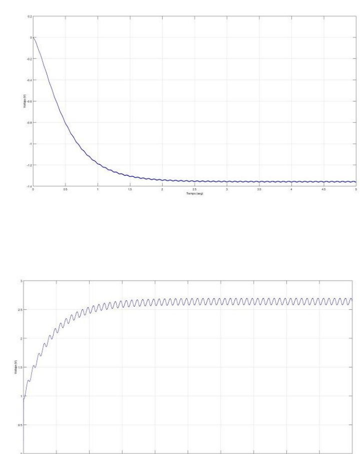


Figure 7: Simulation with buck-boost+piezo-device.

4.1 State-Space fomulation

Writing the state-space model of a buck-boost converter using the model of each component and modelling the diode and switch (Equation 1).

$$\begin{cases} F(v_1 - v_0) = C_L \cdot \dot{v_0}(t) + I_0(t) \\ (v_1 - v(t)) \cdot \frac{u}{R^*} = F(v_1 - v_0) + \\ -\frac{1}{L} \cdot \int_0^t v_1 \cdot d\sigma \\ v = \phi(t) + \Big[(v_1 - v) \cdot u \cdot \frac{R}{R^*} + \\ \frac{1}{C} \cdot \int_0^t (v_1 - v) \cdot \frac{u}{R^*} \cdot d\sigma\Big] \\ \phi(t) = I(t) \cdot R + \frac{1}{C} \cdot \int_0^t I(\sigma) \cdot d\sigma \end{cases} \quad (1)$$

Then, Figure 8 depicts the main currents and voltages with the diode model:

$$\begin{cases} I_D = \frac{V_D}{R_D} \cdot \frac{(1+sign(V_D))}{2} \\ u = 0, 1 \end{cases}$$

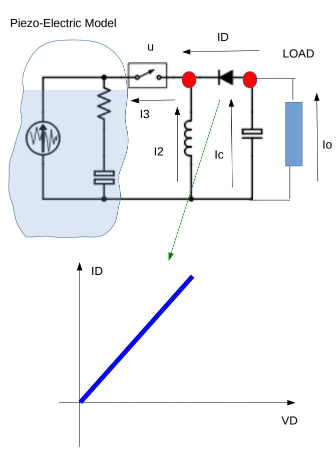


Figure 8: Electrical analysis with the diode-model.

This model shows a recursion that must be solved:

$$\begin{cases} \dot{x_1}(t) = \frac{1}{C_L} \cdot F(\dot{x_2} - x_1) + \\ -\frac{1}{C_L} \cdot I_0 \\ (\dot{x_2} - \dot{x_3}) \cdot \frac{u}{R^*} = F(\dot{x_2} - x_1) + \\ -\frac{1}{L} \cdot x_2 \\ \dot{x_3} = \phi(t) + (\dot{x_2} - \dot{x_3}) \cdot u \cdot \frac{R}{R^*} + \\ \frac{u}{R^* \cdot C} \cdot (x_2 - x_3) \end{cases}$$

then:

$$\dot{x_2} = \varphi_1 \cdot \frac{(1 + sign(\varphi_1 - x_1))}{2} + \\ +\varphi_2 \cdot \frac{(1 - sign(\varphi_2 - x_1))}{2}$$

where $x_1 = v_0, x_2 = \int_0^t v_{1(\sigma)d\sigma, x_3 = \int_0^t v(\sigma)d\sigma, R = R^*}$.
Finally:

$$\dot{x}(t) = f(x, u, I_0, \phi)$$

$$\varphi_1 = \frac{-(1+u) \cdot \left(\frac{x_1}{R_D} + \frac{x_2}{L}\right) + \frac{u}{R} \cdot (\phi + (x_2 - x_3) \cdot u)}{(1+u) \cdot \left(\frac{u}{R} - \frac{1}{R_D}\right) + u}$$

$$\varphi_2 = u \cdot R \cdot \left(-2 \cdot \frac{x_2}{L} + \frac{\phi}{R} + \frac{(x_2 - x_3)}{R}\right) + \\ + (1-u) \cdot \left(x_1 + \frac{R_D}{L} \cdot x_2\right)$$

with $x = [x_1, x_2, x_3]'$ and I_0 is considered as and extrnal perturbation.

5 Singular Optimal Control

Now it is possible to formulate the optimal control problem to solve:

$$min_{u=\{0,1\}} \quad x_1 \cdot I_0$$
such that:
$$\dot{x}(t) = f(x, u, I_0, \phi)$$
$$(x_1 - v_0^*)^2 \leq \Delta_v$$

where the last constraint is added to ensure a non trivial solution: $x_1 \cdot I_0 = 0$ with Δ_v a constant.

Then, defining: $\tau \in [0, t]$ and following [4] along with [5] pp.49-51:

- $(x_1 - v_0^*)^2 \geq 0$
- $(x_1 - v_0^*)^2 = \Delta_v$

Then:

$$min_{u=\{0,1\}} \quad \lambda(t)' \cdot f(x, u, I_0, \phi)$$
or
$$(v_0 - v_0^*)^2 = \Delta_v, \forall u = \{0, 1\}$$

where: $\lambda(\tau = t) = \frac{\partial(x_1 \cdot I_0)}{\partial x} = [I_0, 0, 0]'$. The synthesis of the problem leads:

$$min_{u=\{0,1\}} \frac{I_0 \cdot F(\dot{x_2} - x_1)}{C_L}, \quad \Rightarrow u = \frac{1 - sign(I_0 \cdot (v_1 - v))}{2}$$

Then, using Matlab/Simulink this closed-loop control law can be constructed (Figure 9).

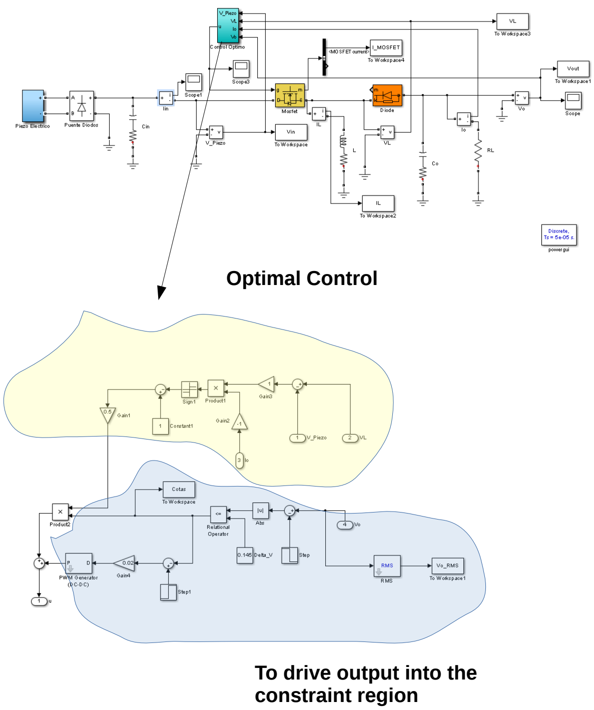


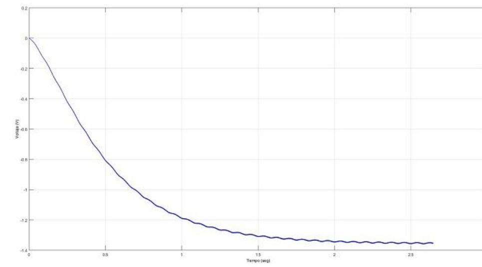
Figure 9: Matlab/Simulink model.

Notice that an extra block must be included to bring the initial conditions inside the optimal region.

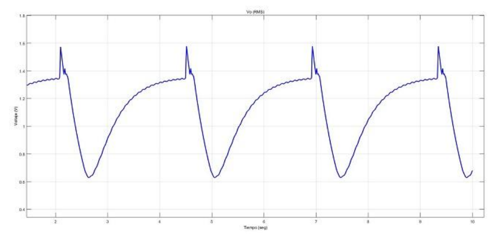
6 Simulation Results

Implementing in Matlab/Simulink the optimal control algorithm already developed, the simulations shown in Figure 10 can be obtained. However, several remarks are in order:

- The Domain of attraction changes with Δ_V
- Bigger voltage can be obtained than open-loop
- Non-linear regime is obtained with bigger output power



(a) $\Delta_v = 0.04, \quad v_0^* = 1.5V$



(b) $\Delta_v = 0.15, \quad v_0^* = 1.5V$

Figure 10: Simulations in Matlab for the optimal control algorithm.

7 Measurements

Finally, implementing the closed-loop control law obtained in previous sections it is straightforward using a micro-controller capable of handling floating point numbers. In this way, using the Texas' microcontroller MSP430G2253, the measurements shown in Figure 11 and Figure 12 were obtained.

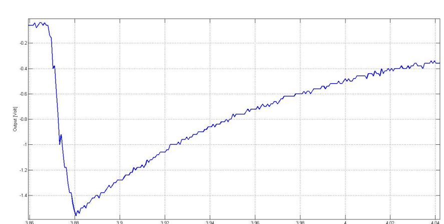


Figure 11: Measurement of the output voltage with $R_L = 1K\Omega$ and with 0.7 G of acceleration at the shaker.

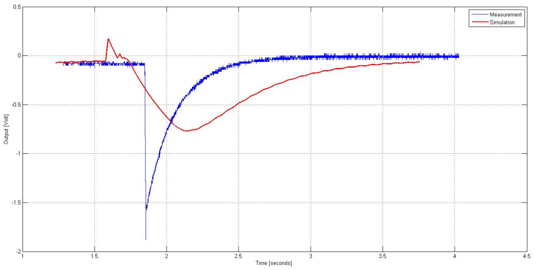


Figure 12: Measurement of the output voltage with $R_L = 10K\Omega$ and with 0.7 G of acceleration at the shaker.

It is very remarkable that under 0.7 G of acceleration at 6.6 Hz of the shaker (600mV peak at the leads of the piezo-device), 1.9V (peak) is obtained at a load of $1K\Omega$, this also means 1V or 1mW during 50 msecs, on the other hand, with $10K\Omega$ 1V is held during 90 msecs, whereas without the optimal control this is only possible whit an excitation (acceleration) three times bigger.

The comparison with the simulations obtained at Section 6 shows that the model and the optimal control are very precise.

Finally, both 50 msecs or 90 msecs (depending on the energy demand) it is enough to connect a low power micro-controller and a transmitter to use, for instance in remote applications.

8 Conclusions

In this paper, a simple internal model of a piezoelectric device is considered under a constant excitation and, after rectification, a buck-boost topology with a pure resistive load using optimal control.

The analysis of the optimal control, renders the problem as singular because of the formulation. However, this singular formulation allows to extract a closed-loop algorithm as opposed to the classical open-loop in the non-singular case using the Pontryagin's principle.

After obtaining some parameters to simulate in both scenarios: with and without optimal control in Matlab/Simulink, some measurements were obtained verifying a precise match between theory and practice but also a very big improvement in the amount of output energy.

It turns out that this closed-loop algorithm was programmed off-line in an 8-bit micro-controller becoming the result very appealing to use in remote applications were very low energy is available.

As a future work, other topologies rather than the buck-boost is going to be investigated in order to further extent the amount of energy at the output.

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