Impact of the Horizontal and Vertical Electromagnetic Waves on Oscillations of the Surface of Horizontal Plane Film Flow

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Abstract: Problem on mathematical modeling of a plane film flow affected by alternating electromagnetic field (running horizontal EM wave) is considered. The parametric excitation and suppression of oscillations on a film surface at the essential inertia forces are studied using the developed model and computer simulation. In a linear approach some interesting new phenomena of the electromagnetic wave spreading in a plane film flow and its impact on excitation and suppression of oscillations on a film surface are investigated.

Key-Words: Plane, Film, Flow, Electromagnetic, Horizontal, Wave, Instability, Excitation, Suppression

1 Oscillations in the plane film flow

Earlier investigated impact of the vertical electromagnetic wave on surface waves on the plane film flow spreading on a solid plate has shown the features of the system (see Fig.1 and Fig.2) [1-3].

![Fig.1 Configuration of the induced electric current from vertical EM wave in a plane film flow](image1)

![Fig.2 Creation of the plane film flow](image2)

The established parameters of electromagnetic (EM) waves $H_z = H_m(z,t) \exp(i(kx + my))$ are important for analysis of their action on conductive medium and selection of the control fields, which supply an achievement of the requested behavior of the controlled liquid metal film flow. Analysis performed has shown that short electromagnetic wave with constant by $x$, $y$ amplitude is available if and only if magnetic field is rapidly decreasing in time, so that it is similar to the impulse packet having abrupt back front of type $\exp(-k^2 t/I Re_m)$.

Solution of the boundary problem by the model obtained has been done. The approximate solution has shown for $k \ll 1/\varepsilon$ ($k$- wave number, $\varepsilon$ - dimensionless film thickness):

$$
\zeta = \frac{H_0}{\sqrt{\varepsilon}} \left[ \exp \left( - \frac{8k^2}{R e_m} \left( 1 - \frac{\rho_m}{F r^2} \frac{8k^2}{W e} - \frac{1}{1 - i} - k^2 R e_m \right) \exp(2ikt) \right) \right] - \frac{1}{1 - i} - k^2 R e_m \exp(2ikt)
$$

Here $\zeta$ is amplitude of the film surface wave.

Analysis of the solution has shown that vertical electromagnetic wave causes perturbation of the film flow surface, which has a part similar to the exciting wave force and the other one, different. By real $k$ (speed of the wave spreading is equal to the velocity of film flow, $Re_m = 0$) the surface wave is alone (similar by form to the exciting force). By $\rho_m = 1 + \frac{8k^2}{Oh^2 Ga^2}$ the exponential growing of the amplitude of surface oscillations is available, by $\rho_m < 1 + \frac{8k^2}{Oh^2 Ga^2}$ influence of the surroundings is only quantitative. The amplitude of film surface
oscillations excited by EM wave is proportional to $H_0 \sqrt{AT}$, and parametric resonance is achieved by:
\[
\text{Re}_e = \frac{8\epsilon k \Re (1 - \rho_m) Ga^4 + 8k^4 / \Omega^4}{k \Re [e / Be 8 \Re (1 - \rho_m) Ga^4 + 8k^4 / \Omega^4]},
\]
Here $\beta_0, \gamma_0, \Omega_0$ are the Batchelor, Galileo and Ohnesorge numbers, respectively, $\text{Re}_e \gg 1$. The single dynamic criterion $\text{Re}_e$ is expressed through the parameters of system and wave number $k$, $\beta_0 = \nu / \nu_m$, $\gamma_0 = \sqrt{ \beta_0 } / \nu$, $\Omega_0 = \sqrt{ \beta_0 / \sigma }$ [2,3].

Now the horizontal EM wave impact on the plane film flow is in focus of the present paper.

2 Model for horizontal EM wave impact on plane film flow oscillation

Consider EM wave with only $y$-component (running by $x$ field or just alternating by $t$, if $k=0$):
\[
H_z = H_m(z) e^{i(kx - \omega t)},
\]
creating ponderomotive force (generally also running) on axis $z$ (unlike the previous case of vertical EM wave, where it was directed against the unperturbed flow). According to a stable state [2,3] the function $f(x)=1-4v/\nu_m \ (x/\nu - 1)(\partial u / \partial z)|_{z=0}$ is introduced, which is in the frame of model considered monotonous and weekly-gradient.

In a first approach, it is possible to put $f=1$ and further estimate an error of such simplification of a task. Then the system of linearized MHD-equations of the perturbed film for the considered case in non-inductive approach can be presented as follows:
\[
\begin{align*}
\frac{\partial u}{\partial t} + f \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} &= \frac{1}{2} \Re \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{Al}{2} \frac{\partial H^2}{\partial x} + \frac{Al}{2} \frac{\partial H^2}{\partial z} + \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} + \frac{\partial p}{\partial x} = \frac{1}{2} \Re \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right),
\end{align*}
\]
\[
\begin{align*}
\frac{\partial H_j}{\partial t} + f \frac{\partial H_j}{\partial x} + \frac{\partial H_j}{\partial x} + \frac{\partial H_j}{\partial z} + \frac{\partial H_j}{\partial z} &= \frac{1}{2} \Re \left( \frac{\partial^2 H_j}{\partial x^2} + \frac{\partial^2 H_j}{\partial z^2} \right) + \frac{1}{2} \Re \left( \frac{\partial^2 H_j}{\partial x^2} + \frac{\partial^2 H_j}{\partial z^2} \right).
\end{align*}
\]

Corresponding boundary conditions on solid plate:
\[
\begin{align*}
z = 0, & \quad u = w = 0; \quad \zeta \frac{\partial u}{\partial z} + \zeta \frac{\partial w}{\partial z} = 0, w = \frac{\partial \zeta}{\partial t} + f \frac{\partial \zeta}{\partial x},
\end{align*}
\]
\[
\begin{align*}
p &= \frac{\zeta}{Fr^2} + \frac{2}{Re} \frac{\partial \zeta}{\partial z} + \frac{Al}{2} \frac{H^2}{Fr^2} + \frac{1}{We} \frac{\partial^2 \zeta}{\partial x^2}.
\end{align*}
\]

As nature of parametric oscillations in a system in linear approach doesn't depend on initial amplitudes of the perturbed film, the initial conditions can be avoided, seeking for a linear system response in a wave mode (1) with the doubled frequency and wave number. Therefore from (2), (3) the following boundary task for the perturbation amplitudes is obtained:
\[
\begin{align*}
d^2 w / dz^2 + \Re \left( \frac{dp}{dz} + Al \frac{dH}{dz} \right) &= 2(i \omega \Re - 2k^2) w, \\
d^2 u / dz^2 + \left[ (\omega - f) i \Re - 2k^2 \right] u &= ik \Re (2p + Al \frac{H}{dz}), \\
2i \omega H_0 = H_m + i \Re \omega, \\
M = \omega - \omega_m = 0; & \quad M = \omega - \omega_m = 0; \\
z = \zeta, H = H_m, & \quad \Re M = \omega - \omega_m = 0; \\
2 \Re M &= \frac{Al}{2} H_m^2 + \frac{2}{Re} \frac{dH}{dz} + \left( \frac{1}{Fr^2} + \frac{4k^2}{We} \right) \zeta.
\end{align*}
\]

For simplification the amplitude perturbations of hydrodynamic parameters are designated by the same variables like namely hydrodynamic parameters as before. Ponderomotive force can be negative (pressing a film to a plane) only in case of existence of current in a film. Considering short-wave fluctuations of a film surface ($k \gg 1$) and assumed $k < 0, k \rightarrow -k$, (weak or moderate increase of perturbations on $x$), we carry out the terms’ assessment in the boundary task (4) and get its following approximate solution:
\[
\begin{align*}
u &= \frac{i}{2k} c_1 \exp(q_j \zeta), w = c_1 \exp(q_j \zeta), H_m = H_0 \exp \left( k \frac{1}{\zeta} \right), \\
p &= \frac{Al}{2} H_m^2 \exp \left[ 2k_m (\zeta - \zeta) \right] + 1 \exp (q_j \zeta) + \\
&\quad + c_j \left( \frac{q_j}{2} + \frac{2k_j - i \omega k}{Re} q_j \right),
\end{align*}
\]
where $c_j = const$, $q_j$ - the roots of characteristic equation, $k_m$ - electromagnetic wave numbers, $j = 1, 4$ - the summation index. Here:
\[
\begin{align*}
k_m &= \left( 1 + \frac{f^2 \Re c_j}{4k_c^2} \right) \left( k_j - f \Re c_j \right), \\
k_m &= \left( 1 + \frac{f^2 \Re c_j}{4k_c^2} \right) \left( k_j + f \Re c_j \right), c_j = \frac{\Delta_j}{\Delta}, \\
q_j &= \pm \sqrt{ \Re \left( f k - \omega \right) } \left( 1 \pm \frac{8k^2 \left( 2k^2 - i \omega \Re \right)}{2 \Re \left( f k - \omega \right)} \right)^{0.5}, \\
\Delta &= \sum_{i=1}^{n} \left[ \left( -1 \right)^{n_i} \left( g_1 + g_2 \right) + \left( -1 \right)^{n_2} g_2 \right] + \left( g_1 - g_2 \right) q_z.
\end{align*}
\]
\[ \frac{q_1^2 \exp(q_1 c) - q_2^2 \exp(-q_2 c)}{2q_1 (g_2 + g_4 - 2g_1) + q_1 (g_2 - g_4)} + \frac{k_n - k_m + ik_n}{1} \text{sh}(q_1 c), \quad \Delta_n = g_2 g_4 \left[ q_1 q_2 \text{sh}(q_1 c) + q_2^2 \text{ch}(q_1 c) \right] \pm q_1^2 \exp(q_1 c), \quad \Delta_n = \frac{g_2 q_1 \text{sh}(q_1 c) + q_2^2 \text{ch}(q_1 c) \pm q_2^2 \exp(q_1 c)}{4}, \]

where \( q_j \) with odd indexes \( j \) are obtained when in front of square root is "\( + \)", with even — when "\( - \). \( \Delta_1, \Delta_2 \) correspond the top signs in the formulas, \( \Delta_1, \Delta_2 \) - to bottom signs. Functions \( g_j \) in (6) are:

\[ g_j = \frac{2}{q_j} \left[ 1 - \exp(q_j c) \right] \left( \frac{1}{\text{Re}^2} \right) + \frac{q_j^2 \exp(q_j c) - 1}{\text{Re}} + \frac{\exp(q_j c) \left( \frac{1}{2(\omega - f_k)} + \frac{4k^2}{\text{Re}^2} \right)}{\text{Re}^2}, \quad g_j = \frac{A_1}{2} H_{\infty}^2 \exp(-k_0 c), \quad j = 1, 4. \]

With account of (4), the amplitude of perturbations:

\[ \zeta = \frac{2}{\Delta} A_1 \cdot \text{Re} \cdot H_{\infty}^2 \exp(-k_0 c) \left[ q_1 \text{sh}(q_1 c) \text{ch}(q_1 c) - q_2 \text{sh}(q_2 c) \text{ch}(q_2 c) \right], \tag{7} \]

where from is seen that, unlike the considered cases, at a horizontal cross field amplitude of a superficial wave is \( \sim A_1 \cdot \text{Re} \). It can be explained that direction of ponderomotive force action is vertical (up in case of lack of current in a film, except induced current) and therefore with increase of inertial forces also their contribution to energy pumping in waves.

### 2.1 Parametric resonance of surface waves

Parametric resonance of the system is possible by \( \omega' = f_k \cdot k \neq i f \text{Re}/2 \), when the phase speed of surface wave spreading coincides with velocity of the unperturbed (stable state) film flow. The Eigen oscillations are determined with characteristic equation \( \Delta = 0 \), where from dispersive equation yields

\[ 2 \left[ (\omega - f_k)^2 + q_1^2 \left( 1 - \exp(q_1 c) \right) - q_2 \text{sh}(q_1 c) - q_2 \right] + 2q_1 \left[ q_1^2 \left( 1 - \exp(q_1 c) \right) - q_1 \text{sh}(q_1 c) - q_1 \right] + \left( \exp(q_1 c) - 2q_1 - q_1 - q_1 \exp(q_1 c) \right) + 2q_1 \left[ q_1^2 \left( 1 - \exp(q_1 c) \right) - q_1 \text{sh}(q_1 c) - q_1 \right] + q_1^2 \text{ch}(q_1 c) - q_2 \text{sh}(q_1 c) + q_2 \text{ch}(q_1 c) \exp(q_1 c)] + q_1^2 \left[ q_1^2 \left( 1 - \exp(q_1 c) \right) - q_2 \text{sh}(q_1 c) + q_2 \text{ch}(q_1 c) \right] + \left. q_1^2 \left[ q_1 \text{sh}(q_1 c) + q_2 \text{ch}(q_1 c) \right] \right\} \right\} \frac{G_0 + 4k^2 \cdot \text{Oh}}{\text{Re} \left( \omega - f_k \right)} = 0. \]

Here can be seen that the Eigen oscillations generally spread on a film with dispersion, as \( \frac{d^2 \omega}{\text{d}k^2} \neq 0 \). The resonant pair \( (k_0', \omega') \), as it is easy to check, generally doesn't satisfy condition (8), i.e. the parametric resonance is excited not at Eigen frequency and, besides, the surface film oscillations are not dispersing in this case on a contrast to the Eigen oscillations because \( c' = \frac{\omega'}{i f k} = f_k \) and \( c c' \neq 0 \). The last, generally speaking, can be incorrect as far as friction of liquid on a plate depends on the mode of its flow; however within the accepted model it is right.

By \( \omega_n > 1/\text{Re} \) from (8) after the terms’ estimation the following dispersive equations result:

\[ 2 \left[ (\omega_n \text{Re} + 2k) \cos \theta \pm \omega_n \text{Re} \sin \theta \right] \left( \sqrt{\text{Re}} \left( \omega_n^2 + (f_k - \omega_n)^2 \right) \right] = \frac{(G_0 + 4k^2 \cdot \text{Oh}) \omega_n}{[\omega_n + (f_k - \omega_n)]^2} \left( \text{Re} \left( \omega_n^2 + (f_k - \omega_n)^2 \right) \omega_n (\text{Re} + 2k) \right), \tag{9} \]

where \( \phi = \frac{\text{Re}}{2 \text{Re}^2} (f_k - \omega_n) \). It is assumed \( |f_k - \omega_n| \ll \omega_n \).

It is easy to check that at the above-stated parameters of system noticeable increase of amplitude of the Eigen oscillations in time is possible only under condition \( \omega_n > 1 \), otherwise the characteristic time of perturbation development is less than existence time of a film, which edge (the rib for unloading from surface forces) breaks up owing to Rayleigh instability. Therefore from (9) the following system is obtained after further estimates and simplifications:

\[ \frac{\omega_n}{\alpha} = \text{Arctg} \frac{\omega_n}{\omega_n} + \pi n, \quad \alpha_n = \frac{2\omega_n}{\text{Re} (f_k - \omega_n)}, \]

where from

\[ \omega_n = \frac{\omega_n}{\alpha}, \quad \omega_n = f_k + \frac{1}{\alpha \text{Re}} \pm \sqrt{\frac{2f_k}{\alpha \text{Re}} + \frac{1}{\alpha^2 \text{Re}^2}}, \tag{10} \]

\( \alpha = 1.62; 3.426; \ 6.437, \ etc. (regular set). \)

(10) is satisfied by \( \omega_n > \alpha, \ k > \alpha \) (short-wave perturbations). By \( \text{Re} \gg 1 \) both solutions coincide and approximately it comes to \( \omega_n = f_k \), so that in this case \( \omega_n \approx \omega' \). One has to keep in mind that \( k \) cannot be big because in the small-amplitude linear approach the curvature of a surface cannot be big. The phase and group wave speeds are respectively

\[ c = f + \frac{1}{\alpha \text{Re}} \frac{1}{f_k} \left( 1 + \frac{2f_k}{\alpha \text{Re}} \right), \quad C = f + \frac{1}{\alpha \text{Re}} \frac{1}{f_k} \left( 1 + \frac{2f_k}{\alpha \text{Re}} \right). \]
where from follows that the Eigen oscillations are practically not dispersing, and phase and group speeds are approximately equal to unperturbed velocity of film flow (kinematic waves).

### 2.1.1 Action of the constant electromagnetic field

By \( \omega = 0 \), assumed \( \text{Re} \gg k_r \), with accuracy of \( (k/(f \text{ Re}))^2 \) from (6) follows

\[
q_{1.3} = \pm \sqrt{\text{Re} f k} (1+i), \quad q_{2.4} = \mp 2k(1-i) \sqrt{\frac{k}{f \text{ Re}}},
\]

And then, considering \( k_r \gg 1, k < 0, -k_r \ll k_r \), \( f \sim 1 \), results in

\[
q_{1.3} = \pm \sqrt{\text{Re} f k} (1+i), \quad q_{2.4} = \mp 2k(1-i) \sqrt{\frac{k}{f \text{ Re}}}.
\]

Similarly from (6), (7), with account of above-mentioned after estimation of the terms and further simplification yields

\[
\zeta = 2 \text{Al} \cdot \text{Re}^2 \cdot H_{\kappa}^2 \cdot \left[ \exp(-k_r x) - 1 \right] \frac{\alpha_r - \alpha_i \beta + i(\alpha_r + \alpha_i \beta)}{4(\beta_r^2 + \beta_i^2)}
\]

where:

\[
\alpha_r = 2 - \exp(-k_r x) \cos(k_r x) , \quad \alpha_i = \exp(-2k_r x) \sin(k_r x),
\]

\[
\beta_r = 2k_r \text{Re} - \epsilon(Ga \cdot 4k_r^2 / Oh^2) , \quad \beta_i = 4k_r(2k_r / Oh^2 + \text{Re}).
\]

Now real part of the film surface is expressed as

\[
\zeta_r = \text{Re} \cdot \frac{\text{Al}^2}{\text{Re}} \cdot H_{\kappa}^2 \cdot \left[ \exp(-k_r x) - 1 \right] \frac{\alpha_r - \alpha_i \beta + i(\alpha_r + \alpha_i \beta)}{4(\beta_r^2 + \beta_i^2)} \exp(2k_r x)
\]

\[
\left[ (\alpha_r - \alpha_i \beta \cos(2k_r x) - (\alpha_r + \alpha_i \beta \sin(2k_r x)) \right].
\]

The expression (11) thus obtained shows that by \( \text{Re} \rightarrow \infty \), \( \zeta_r \rightarrow -\text{Al} \cdot \text{Re}^2 \). Parametric resonance is impossible in case of constant electromagnetic field as seen from (11). By \( k_r \gg 1 \) follows \( \zeta_r \sim -\text{Al} \cdot \text{Re}^2 \cdot H_{\kappa}^2 / k_r^2 \), therefore for big enough values \( k_r \) (short waves) with increase of \( k_r \) the amplitude of the exciting electromagnetic field must grow proportionally. But this case is practically impossible due to technical impediments with creation of alternating field by \( x \) with a small wave length.

In general, electromagnetic excitation of oscillations of conductive film surface under transversal horizontal alternating by \( x \) field is determined by \( \text{Al} \cdot \text{Re}^2 \cdot Ga \cdot Oh \) and wave number \( k_r \). As an illustration of these results the calculation presented in Fig.3. It was computed by (11) for:

\[
k_r = 10, \quad k = 5, \quad \epsilon = 1, \quad u_0 = 4.43 m/s, \quad b = 10^{-3} m, \quad k_{mr} \approx k_r , k_{mr} \approx \text{Re}_{u0} f - 5, \quad \text{Re}_{u0} = 0.027, \quad \text{Re} = 3.67 \cdot 10^3.
\]

It corresponds to \( Ga \gg k_r / Oh \) and \( Ga^2 \gg k_r \), expression (11) was simplified:

\[
\zeta_r = -\text{Al} \cdot \text{Re}^2 \cdot H_{\kappa}^2 \cdot \left( 4k_r \text{Re} \cos(2k_r x) + Ga^2 \sin(2k_r x) \right),
\]

where \( \text{Al} \cdot \text{Re}^2 \) is just quadratic Reynolds number computed by the speed of the Alfvén waves as characteristic flow velocity.

Apparently from Fig.3, bending around oscillatory fronts have axes almost symmetric relatively the top and lower boundaries. Influence of magnetic Reynolds number in the considered model statement is insignificant and is shown generally on coefficient at \( \cos(2k_r x) \), which, however, is much less than a coefficient at \( \sin(2k_r x) \); here are \( Oh = 0.22, \quad Ga = 8.3 \cdot 10^4 \).

![Fig.3 Bending around oscillatory fronts of a film:](http://www.iaras.org/iaras/journals/ijce)

gainst longitudinal coordinate for 3 values of field

\[
H_{u0} = 9 \cdot 10^4, \quad 9 \cdot 10^3 \text{ and } 9 \cdot 10^2
\]

### 2.1.2 Falling of a film flow velocity with distance

On the distances of \( x = 8.20 \) and \( x = 16.4 \) velocity of the unperturbed film flow decreases, correspondingly, by \( 5\% \) and \( 1\% \), therefore with accuracy sufficient for practice follows that the assumption about constant unperturbed flow velocity is correct by \( H_{\kappa} < 10^{-3} \), when electromagnetic field is weak enough.

Calculation with account of the function \( f(x) \) is presented in Fig.3 by dashed lines. In case of film flow with current in external magnetic field of the same strength, when directions of the magnetic and gravitational forces coincide, the amplitude of perturbation on a film flow surface is lower about 20 times comparing to the only induction situation.

### 2.1.3 Influence of the fluid viscosity

For inviscid approach (\( \text{Re} \rightarrow \infty \)) from (2.3.11) reduction \( \zeta_r \) is approximately by one decimal order in comparison with a viscous case, and inviscid approach is correct at \( \text{Re} > 10^3 \) that is almost unreal. Thus, the electromagnetic field of the form
\( H = H_{\infty} \exp(ik_{x} - k_{y})x \) causes process of wave formation on a film surface with wavelength, twice bigger, than by \( H \), and small shift on \( x \). Increase on \( x \) happens under the law \( \exp(-2k_{x}x) \).

For example, the field with initial induction of \( B_0 \approx 0.023T \) (\( H_{\infty} \approx 9 \cdot 10^{-12}T \)) causes perturbation \( \zeta_x \approx 0.02 \varepsilon \), which already at \( x=19.5 \) reaches value \( \zeta_x \approx \varepsilon \) (film decay). At disintegration of a film the drops are formed under the Rayleigh law or at the action of mechanical flow dividers with formation the drops by diameter \( d \sim \varepsilon \).

2.1.4 The case of progressive (running) waves

For real \( k \), \( \omega \gg k \), \( \omega \gg k^{2}/\Re \), from (6) follows

\[ q_{n} = \pm 2 \sqrt{2 \Re \omega} \cos[\pi(n+0.25)] \cos[\pi(n+0.5)], \quad q_{n} = \pm 2k, \]

where \( n=0,1,2,3\ldots \) Then accounting \( (7) \) yields

\[ \zeta = g \Re e \exp \left( \frac{1}{\Re} \frac{2a}{q_{n}} \frac{2a}{2q_{n}} \exp(-q_{n}) \right), \]

where from in case of \( 1 \ll \omega \ll \Re \) it is got simpler expression for the perturbation amplitude

\[ \zeta = \frac{\Re e \cdot A \cdot H_{\infty}^{2}}{2(1+4)\omega^2} \cos[2(\omega x - \omega t)], \]

And a real part of the film surface perturbation is

\[ \zeta_r = \frac{\Re e \cdot A \cdot H_{\infty}^{2}}{2(1+4)\omega^2} \cos[2(\omega x - \omega t)]. \]

The second item of (12) in square brackets is small; therefore it can be omitted in engineering calculations, which simplifies (12) as follows

\[ \zeta = \frac{\Re e \cdot A \cdot H_{\infty}^{2}}{2(1+4)\omega^2} \sin[2(\omega x - \omega t)]. \]

where from follows that \( \zeta \sim 1/\omega \), so that with increase of the field frequency the phase speed of the wave is growing (the speeds of the electromagnetic and surface hydrodynamic waves coincide) and its amplitude is falling down.

The two surface waves differ in amplitude (0.6 times) and in a phase (small at \( \omega \gg \Re e /F r^{4} \)). The parametric resonance is absent owing to what, considering stated, it is possible to conclude that a horizontal electromagnetic wave can excite disintegration of a film flow only on the set wavelength, and unlike a case of a vertical wave an amplitude of wave perturbation is defined only by frequency. The excited oscillations on a film surface have a wavelength independent of \( x \), thus, they don't disperse. In a case of \( \omega \gg \Re e /F r^{4} \) the process of wave formation and wave spreading in a film is defined only by \( \Re e /Al \cdot k \cdot \omega \), and from film thickness independence is linear (\( \zeta = \varepsilon \)).

As seen from (12), in this case no film flow decay occur because an amplitude of superficial waves doesn't change in space-time, and the waves of \( \zeta = \varepsilon \) are beyond the linear theory.

2.2 Excitation of oscillations on the film flow surface by vertical electromagnetic wave

Dimensionless equations for this case are

\[ \frac{\partial u}{\partial t} + (2 - j) \frac{\partial u}{\partial x} + \rho_{v} \frac{\partial}{\partial x} \left( p_{j} + \frac{2 - j}{2} A \cdot H^{2} \right) = \frac{1}{\Re e} \Delta u, \]

\[ \frac{\partial v}{\partial t} + (2 - j) \frac{\partial v}{\partial x} + \rho_{v} \frac{\partial}{\partial y} \left( p_{j} + \frac{2 - j}{2} A \cdot H^{2} \right) = \frac{1}{\Re e} \Delta v, \]

\[ \frac{\partial w}{\partial t} + (2 - j) \frac{\partial w}{\partial x} + \frac{\partial}{\partial z} \left( p_{j} + \frac{1}{2} \Re e \cdot A \cdot H^{2} \right) = \frac{1}{\Re e} \Delta w, \quad \text{div} v = 0, \]

Boundary conditions are stated as follows:

\[ z = \varepsilon, \quad v_{i} = v_{i}, \quad \text{w} = \frac{\partial X}{\partial t} + \frac{\partial Z}{\partial x}; \]

\[ \mu_{i} \left( \frac{\partial v_{j}}{\partial x} + \frac{\partial u_{i}}{\partial z} \right) = \text{idem}, \quad \mu_{i} \left( \frac{\partial v_{j}}{\partial y} + \frac{\partial v_{j}}{\partial z} \right) = \text{idem}, \]

\[ \frac{p_i}{p} = \rho_0 \frac{1 - \rho_0}{F r^4} \chi + \frac{A H_{\infty}^{2}}{2} - \frac{1}{\Re e} \frac{\partial v_{j}}{\partial x} + \frac{\partial Z}{\partial y} + \frac{1}{\Re e} \frac{\partial u_{i}}{\partial z} \frac{\partial w_{j}}{\partial r} \frac{\partial w_{j}}{\partial r}, \]

\[ z = 0, \quad v_{i} = 0; \quad z = \chi, \quad v_{i} = 0; \]

\[ t = 0, \quad v_{i} = 0; \quad p_i = 0, \quad \text{z} = 0, \quad H = H_{\infty}, \]

where \( A = \mu \cdot H_{\infty}^{2}/(\rho u_{i}^{2}) \), \( Fe = b u^{2}/\sqrt{\sigma} \). We choose the characteristic scales of length, velocity, time, pressure and intensity of an electromagnetic field, respectively, \( b u_{i} /u_{i}, \rho u_{i}^{2}, H_{\infty} \) and put for simplicity \( u_{0} = \text{const} \).

The first boundary condition (15) corresponds to a film spreading on a solid surface. For free film flow instead of this condition by \( z=0 \) should be considered also boundary condition of conjugated
type on the interface with surroundings. Because \( u_\alpha = H_{\text{Alfven}} \mu_\alpha / \rho \) is a velocity of spreading the Alfven waves, the Alfven number can be presented as a ratio of characteristic velocities: \( A_l = u_\alpha / u_\gamma \).

2.2.1 Solution of the boundary problem

Assuming the small-amplitude perturbations, we seek solution of the boundary problem (13)-(16) as a linear response of the system to an external action. Following the superposition principle, the excited by electromagnetic wave \( H_\varsigma = H_{\text{alt}}(z,t) \exp j (kx + my) \) surface waves in a film flow are considered as proportional to the function \( \exp 2i(kx + my) \) with corresponding amplitudes – functions of time.

Solution of the boundary problem is convenient to search with use of the integral transformation methods [4]. We apply to the PDE array (13) the Laplace transformation by time. Accounting the above-mentioned and the initial conditions (16) we come to the following system of differential equations in the transforms:

\[
\begin{align*}
\frac{d^2 Q}{dz^2} - 4(k^2 + m^2)Q &= \text{Re} \left[ (s + 2ik)Q + iq \left( 2P + Al \cdot \mathcal{R}_n \right) \right], \\
\frac{dW}{dz} &= -2i(kU + mV), \\
\frac{d^2 Q}{dz^2} - 4(k^2 + m^2)Q &= \text{Re} \left[ (s + 2ik)Q + 2iq \rho \mu \cdot P \right], \\
\frac{d^3 W}{dz^2} - 4(k^2 + m^2)W &= \text{Re} \left[ (s + 2ik)W + \rho \mu \cdot dP / dz \right],
\end{align*}
\]

where \( \{ U, V, W \}(s), P(s), \mathcal{R}_n(s) \) are the Laplace-transforms of functions \( \{ u, v, w \}(t), p(t), \mathcal{H}_n(t) \), function \( H_\varsigma(t) \) is determined by

\[
H_n = H_{\text{alt}} \exp \left[ -ik + (m^2 + k^2) / \text{Re}_n \right] t.
\]

According to the above-mentioned assumptions and formulated equations, \( s \)- parameter of integral transformation.

The first and the third equations of (17) are presented for simplicity in a symbolic form: \( Q = \tilde{V}_1, Q = V_1 \) by \( q = m \), and \( Q = U_1, Q = U_2 \) by \( q = k \).

The boundary conditions (14), (15) in transforms:

\[
\begin{align*}
z &= 0, \quad U_1 = V_1 = W_1 = 0; \\
z &= \gamma, \quad U_2 = V_2 = W_2 = 0; \\
z &= \xi, \quad P_1 = P_2 = \frac{Al}{2} \mathcal{R}_n \left[ \frac{1 - \rho_\alpha}{Fr^2} + 4m^2 + k^2 \right] Z + 2 \frac{2 \text{Re} \mathcal{R}_n}{dz} \left[ -\mu \cdot \frac{dW}{dz} \right].
\end{align*}
\]

where \( Z(s) \) - the Laplace-transform of the amplitude of film flow perturbation \( \zeta'(t) \).

The solution of the boundary task in transforms (17)-(19) doesn't present principal difficulties, however it is so cumbersome that the return transformation to the functions' originals demands application of the numerical methods. Thus, for a free film (not restricted by a solid surface), for the film flow surrounded with the liquid or gaseous medium from the top and from the bottom the boundary conditions (18) are replaced with conditions of type (19) owing to what the solution is strongly complicated.

For convenience of the analysis of the considered class of problems by parametric excitation and suppression of oscillations on a surface of the conductive film flows spreading in non-conductive media, and for detection of the main regularities of such physical systems we consider the short-wave perturbations and suppose \( k = m \). Such case has important applications in practice of creation the perspective film MHD-granulators [1,2].

The written above allows significantly simplifying a task, and the system (17) gives

\[
\begin{align*}
\frac{dW_1}{dz} &= -(8k^2 + q^2) \frac{d^2 W_1}{dz^2} + 8k^2 q^3 W_1 = 0,
\end{align*}
\]

where \( q^2 = 8k^2 + (s + 2ik) \text{Re}_j, \quad j = 1,2 \). Solution of this system is \( W_j = d_{jn} \exp(g_{jn} z), \) where for each value of \( j \) the sum is taken by index \( n = 1,4 \). The values \( g_{jn} \) are

\[
g_{j1} = 2\sqrt{2k}, \quad g_{j2} = -g_{j1}, \quad g_{j3} = q_j, \quad g_{j4} = -q_j .
\]

Constants of integration \( d_{jn} \) are computed by substitution the function \( W_j(z) \) in the boundary conditions (18), (19), with account of (17). In a linear approach yields \( U_j = V_j - dW_j / dz \), and for \( d_{jn} \) the following linear algebraic equation array (LAEA) of the 8-th order is got:

\[
\begin{align*}
d_{wn} g_{wn} &= 0, \quad d_{wn} \exp (g_{wn} z) = (s + 2ik) Z, \quad \sum_{w=1}^d d_{wn} = 0, \\
d_{2n} \exp (g_{2n} z) &= (s + 2ik) Z, \quad d_{2n} g_{2n} \exp (g_{2n} z) = 0, \\
d_{4n} g_{4n} &= d_{2n} g_{2n} \exp \left[ (g_{2n} - g_{4n}) z \right], \\
d_{4n} \left( g_{4n}^2 + 8k^2 \right) &= \mu \mu d_{2n} \left( g_{4n}^2 + 8k^2 \right), \quad d_{4n} \exp (g_{4n} z) = 0.
\end{align*}
\]

After obtaining the \( d_{jn} \), the function \( Z(s) \) is computed from the last equation (19) with account of (17). Solution of the system (21) with account of (20) is going as follows. Two equations containing
both $d_{1n}$ and $d_{2n}$ are solved and then the remaining system of six equations is solved treating the values $d_{1n}, d_{2n}$ as parameters, afterwards $d_{1i}$ is determined from the system of two equations. Matrix of the first sub-system has the view $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$, where $A, B$ are non-zero quadratic matrices of the third order (two others are zero), and additionally $C$-column of the free terms. Such matrix allows easy computing the main as well as additional determinants through reduction of them to the product of corresponding determinants of the third order. Thus, the result is:

$$d_{ij} = \Delta_{ij} / \Delta, \quad i=1,2, \quad j=1,2,3; \quad (22)$$

$$d_{ii} = \frac{A(q_i^2 + 8k^2)\mu_0 - Aq_i\mu_0}{q_i\mu_0(q_i^2 + 8k^2) - q_i(q_i^2 + 8k^2)} \exp(q_i\epsilon),$$

where the main and minor determinants are:

$$\Delta_i = 2\left[ g_i \exp(q_i\epsilon) - ch(g_i\epsilon) \right] - q_i \cdot sh(g_i\epsilon),$$

$$\Delta_i = 2\left[ g_i \exp(q_i\epsilon) - ch(g_i\epsilon) \right] - q_i \cdot sh(g_i\epsilon),$$

$$\Delta_{1i} = 2\left[ g_i \exp(q_i\epsilon) - ch(g_i\epsilon) \right] - q_i \cdot sh(g_i\epsilon),$$

$$\Delta_{2i} = (q_i + g_i) \exp\left[ \left( q_i - g_i \right) \gamma \right] \left( s + 2ik \right) Z + \frac{2\left[ g_i \exp(-q_i\epsilon) - ch(-q_i\epsilon) \right] \exp(-q_i\epsilon)}{2\left[ \exp(q_i\epsilon) - ch(q_i\epsilon) \right] - q_i \cdot sh(q_i\epsilon)},$$

$$\Delta_{3i} = \left[ q_i \exp\left[ \left( q_i - g_i \right) \gamma \right] \left( s + 2ik \right) Z \right] + \frac{2\left[ q_i \exp(-q_i\epsilon) - ch(-q_i\epsilon) \right] \exp(-q_i\epsilon)}{2\left[ \exp(q_i\epsilon) - ch(q_i\epsilon) \right] - q_i \cdot sh(q_i\epsilon)},$$

$$\Delta_{4i} = 2\left[ g_i \exp(q_i\epsilon) - ch(q_i\epsilon) \right] - q_i \cdot sh(q_i\epsilon),$$

$$\Delta_{5i} = 2\left[ g_i \exp(q_i\epsilon) - ch(q_i\epsilon) \right] - q_i \cdot sh(q_i\epsilon),$$

$$\Delta_{6i} = 2\left[ g_i \exp(q_i\epsilon) - ch(q_i\epsilon) \right] - q_i \cdot sh(q_i\epsilon).$$

In view of the frequency of the received expressions (22), (23) at this stage it is expedient to carry out the assessment of terms allowing to simplify a task for each specific physical situation and further to do its solution in each case separately that allows applying the analytical methods effectively. We consider one of such situations for the example. Let $1/y \ll k, \ll 1/\epsilon$, where $k, \epsilon$ - real part of the wave number $k$. This condition covers quite bright region of the wave numbers and corresponding class of the wave processes has relation to many practical situations. The range of variation $\gamma$ is taken $\gamma = (10^2 + 10^4) \epsilon$, therefore $- (10^3 + 10^4) \ll k, \ll 1$. And as far as $\epsilon k, \ll 1, \left( \gamma - \epsilon \right) k, \gg 1$, then $\exp \left[ \left( \gamma - \epsilon \right) k \right] \approx 0.5 \exp\left[ \left( \gamma - \epsilon \right) k \right]$. Similar simplifications made in other expressions, then from (22), (23):

$$\Delta_{1} = g_i \epsilon^2 \left( s + 2ik \right) R_{ie}, \quad d_{ii} = \Delta_{ii} / \Delta, \quad d_{ij} = \Delta_{ij} / \Delta_i, \quad i=1,2, \quad j=1,2,3;$$

$$\Delta_{1} = g_i \epsilon^2 \left( s + 2ik \right) R_{ie}, \quad d_{ii} = \Delta_{ii} / \Delta, \quad d_{ij} = \Delta_{ij} / \Delta_i, \quad i=1,2, \quad j=1,2,3;$$

$$\Delta_{ii} = (q_i + g_i) \exp\left[ \left( q_i + g_i \right) \gamma \right] \left( s + 2ik \right) Z, \quad \Delta_{ii} = (q_i + g_i) \exp\left[ \left( q_i + g_i \right) \gamma \right] \left( s + 2ik \right) Z,$$

$$\Delta_{ii} = (q_i + g_i) \exp\left[ \left( q_i + g_i \right) \gamma \right] \left( s + 2ik \right) Z, \quad \Delta_{ii} = (q_i + g_i) \exp\left[ \left( q_i + g_i \right) \gamma \right] \left( s + 2ik \right) Z,$$

$$\Delta_{ii} = (q_i + g_i) \exp\left[ \left( q_i + g_i \right) \gamma \right] \left( s + 2ik \right) Z, \quad \Delta_{ii} = (q_i + g_i) \exp\left[ \left( q_i + g_i \right) \gamma \right] \left( s + 2ik \right) Z,$$

$$\Delta_{ii} = (q_i + g_i) \exp\left[ \left( q_i + g_i \right) \gamma \right] \left( s + 2ik \right) Z, \quad \Delta_{ii} = (q_i + g_i) \exp\left[ \left( q_i + g_i \right) \gamma \right] \left( s + 2ik \right) Z,$$

Further from the last equation of the system (19) with account of (24) the following equation for the Laplace-transform of the film flow perturbation $Z(s)$ is got:

$$\mu_0 \rho_0^2 \Re_i \left( g_i + q_i \right) \left( s + 2ik \right)^2 + 2q_i \left( 1 - \frac{\mu_0}{\Re_i} s + 2ik \right)^2 +$$

$$+ 2\left\{ \frac{16k^2 \rho_0^2 \left( g_i + q_i \right) \left( s + 2ik \right) Z}{8k^2 + i^2 q_i^2} \left[ \frac{\mu_0}{\Re_i} \frac{8k^2}{8k^2 + 8k^2} \right] \right\} = 0,$$

$$\frac{Z - \Re_i}{\epsilon^{\gamma}} = \frac{A}{\Re_i} \frac{k^2}{\epsilon^{\gamma}} + \frac{1 - \frac{\rho_0}{\Re_i}}{\frac{8k^2}{8k^2 + 8k^2}} Z.$$
2.2.2 Integral-differential equation for film surface oscillations

Now performed the reverse Laplace-transformation yields the integral-differential equation describing the parametric oscillations of the film surface excited by electromagnetic field having \( z \)-component:

\[
d\frac{\zeta}{dt} + 2ik\zeta + \frac{16k^2}{\varepsilon^2} \sqrt{\text{Re}_\varepsilon} \left( 1 - \frac{\rho_\varepsilon}{F_r^2} + \frac{8k^4}{\text{We}} \right) \zeta (t-\tau) dt = -\frac{8k^2}{\varepsilon \sqrt{\text{Re}_\varepsilon A}} H_\varepsilon^2 (t-\tau) \exp (2ikt)
\]

With the assumptions made the function under integrals in (26) are highly-oscillating, therefore these integrals are small and the equation (26) is averaged by the second scheme [5]. As a result, with account of the EM wave form, the averaged differential equation is got, the solution of which has the following form:

\[
\zeta = \frac{H_\alpha \sqrt{\text{Al}} \left[ \exp \left( \frac{8k^2}{\varepsilon \sqrt{\text{Re}_\varepsilon}} \left( 1 - \frac{\rho_\varepsilon}{F_r^2} + \frac{8k^4}{\text{We}} \right) t \right) \exp \left( -\frac{4k^2}{\text{Re}_\alpha} t \right) \right]}{\pm 2k \sqrt{\text{Re}_\varepsilon} \left( 1 - \frac{\rho_\varepsilon}{F_r^2} \right) \frac{8k^4}{\text{We}} (1-i) - k \frac{\varepsilon^2}{\text{Re}_\varepsilon} \exp (2ikt)}
\]

The approximate solution (27) of the equation (26) thus obtained is close enough to the exact solution with increase of \( k, \varepsilon \ll 1 \).

2.2.3 Suppression of the Eigen oscillations

The solution (27) accounting the above is transformed to the following form:

\[
\zeta = H_\alpha \sqrt{\text{Al}} \frac{4k^2}{\varepsilon^2 \pm (1-i) \omega_0 - 4k^2 \text{Re}_\alpha \omega_0} \exp (2ikt),
\]

where from one can see that suppression of the Eigen oscillations with the wave number \( k \) and frequency \( \omega_0 \) is available only by means of the running field having the distribution speed other than the speed of the movement of a film \( (k, \neq 0, \text{Re}_\alpha \neq 0) \). Otherwise both excitation and suppression of fluctuations demand big energy expenses (except a resonance).

2.2.4 Free film flow in non-conductive medium

Solution of these boundary task is similar to the considered task and does not have any principal impediments. But due to cumbersome expressions obtained the reverse integral transformation may be complicated a lot. Therefore for analysis of the features of considered class of tasks some specific physical situations are considered, such in particular, which are useful for practice of MHD-dispersers. Let for simplicity \( k = m \) as before and consider a case of negligibly small influence of the surrounding medium. Both assumptions are physically correct because the dispersers are often placed in a gaseous atmosphere or vacuum and equal wave numbers give more regular film decay into the drops of equal sizes. Then with account of the above similarly yields

\[
W_i = d_n \exp (g_s z), \quad n = 1,3
\]

\[
d = \frac{(g_s^2 + 4k^2)(s + 2ik)}{(g_s^2 - g_s^2)^2 \exp (2g_s e - 1)} \exp [g_s (3 - n) e] Z(s),
\]

\[
d_m = \frac{(g_s^2 + 4k^2)(s + 2ik)}{(g_s^2 - g_s^2)^2 \exp (2g_s e - 1)} \exp [g_s (3 - n) e] Z(s).
\]

The Laplace-transform for the free film surface is:

\[
\left[ \frac{s + 2ik}{g_s^2 - g_s^2} \left( s - 2ik \frac{k}{4k^2} + g_s^2 (g_s^2 + 4k^2) + g_s^2 (g_s^2 + 4k^2) \right) + \frac{1}{2k^2 + \text{We}} \right] Z(s) = -Al \cdot \tilde{H}_n^2.
\]

By \( \text{Re} \gg 1 \) the equation (2.2.23) can be simplified keeping the terms of order \( 1/\text{Re} \), which follows to:

\[
\left[ \frac{s + 2ik}{2k^2 + \sqrt{s^2 + 4k^2} + \frac{k (s + 2ik)^3}{\text{Re}_s (2ik - 5s) (s - 2ik)^2}} \right] Z(s) = \left( \frac{1}{2k^2 + \text{We}} \right) Z(s) + Al \cdot \tilde{H}_n^2,
\]

where from applying the reverse transformation is:

\[
\frac{d\zeta}{dt} + 2ik \left( 1 - \frac{5k}{\text{Re}_s} \right) \zeta + \frac{8k^2}{\text{Re}_s} \exp (2ikt) + \left[ 1 \frac{2k^2 + \text{We}}{\text{We}} \right] J_0 (2kt) \frac{d\zeta (t-\tau) dt}{dt} = Al \frac{d}{dt} J_0 (2kt) H_0^2 (t-\tau) dt,
\]

where \( J_0 (2kt) \) is the Bessel function of zero order.

Here at differentiation of integral with a variable top limit the additional initial condition is introduced: \( t = 0, \quad d\zeta / dt = 0 \). From here it is visible that at the real \( k \) influence of a field on a film perturbation is insignificant, especially at \( k \gg 1 \) as far as the integral in the first part (30) is fast-oscillating and thereof the right part quickly aspires to zero. Namely this part causes a parametric pumping of perturbations of a film (the left part defines the Eigen oscillations).
Solution of the integral-differential equation has been obtained by the method of averaging of integral-differential operator by the first scheme [5]. The averaged equation is got in the form

$$\gamma(t) \frac{d^2 \zeta}{dt^2} + 2ik \left( 1 - \frac{5k}{Re} \right) \zeta = A \frac{d}{dt} \int J_0(2k\tau)H_\nu^0(\tau - \tau) d\tau,$$

(31)

where is

$$\gamma(t) = 1 - \frac{1}{2k} \left( \frac{1}{Fr^2} + \frac{2k^2}{We} \right) \frac{1}{\tau} \int J_0(\tau) d\tau +$$

$$\frac{8k}{Re} \left[ 4k(1+2k\tau) \exp(2ikt) + 3.5 \left( 1 - \exp(2ikt) \right) \right].$$

Solution of the averaged equation has the form:

$$\zeta = A(t) \exp \left[ 2ik \left( 1 - \frac{5k}{Re} \right) \frac{d}{dt} \gamma(t) \right],$$

(32)

$$A(t) = \frac{B(t)}{\gamma(t)} \exp \left[ 2ik \left( \frac{5k}{Re} - 1 \right) \frac{d}{dt} \gamma(t) \right] dt,$$

$$B(t) = \left( \frac{d}{dt} \int J_0(2k\tau)H_\nu^0(\tau - \tau) d\tau \right).$$

3 Practical examples

To consider the solution obtained suppose monotonous intensity of an electromagnetic field, i.e. $k = ki$, where $\kappa$ - a real number (by $\kappa > 0$ intensity of a field decreases on $x$ and $y$ and grows in time, at $\kappa < 0$- on the contrary). Then function $\gamma(t)$ has only the real values:

$$\gamma(t) = 1 + \frac{8k}{Re} \left[ \frac{4k}{exp(2\kappa \tau)} \left[ (1+2\kappa t) + 3.5 \exp(-2\kappa t) - 1 \right] \right] \frac{1}{\kappa} \left[ \frac{\kappa}{\tau} \right] \frac{1}{\tau} \int J_0(\tau) d\tau,$$

where $J_0(\tau)$ is the modified Bessel function of zero order. Character of growing (decreasing) the surface wave is determined by sign and value of the real part of $d \gamma / dt$, or by value $\gamma(t)$ here.

The solution (32) is correct for large enough Reynolds numbers $Re \gg 1$, what is not always carried out in practice, therefore also a case of short waves ($k \gg 1$) is considered and an assessment of the terms in equation (29) is made based on this assumption. Then the approximate integral-differential equation is got as follows

$$\frac{d^2 \zeta}{dt^2} + 2ik \frac{8k + iRe}{Re} \zeta = 2 \sqrt{2k} \frac{d}{dt} \left\{ \frac{16k^2}{Re \sqrt{Re}} \frac{1}{\tau} \right\} \zeta(t - \tau) - 2k \left( 1 - \frac{4k}{Re} \right) \frac{d}{dt} \zeta,$$

(33)

where from seen that equations (30) and (33) are similar by structure, and parametric pumping of the energy perturbations is complicated with presence of the highly-oscillating terms by the external exciting force: by increase of $k$, the equation (33) reflects more precisely the investigated physical process and for the wave excitation higher and higher magnetic field is required.

By $Re \ll 1$, correct for many practical tasks, the equation (33) is solved using the averaged integral-differential operator by the second scheme [5]:

$$\frac{d^2 \zeta}{dt^2} + (8k + iRe) \frac{2k}{A} \zeta = \sqrt{\gamma} Ha \cdot H_\infty^0.$$

(34)

$$\left\{ i - \left( 1 + \frac{2k}{Re} \right) \exp \left( \frac{4k^2 + iRe}{A} \right) \right\} \exp(-2ikt),$$

where $Ha$ - Hartman number,

$$A = Re + \frac{\sqrt{\gamma} Re}{4k + iRe} \left( 1 + \frac{8k^2}{Re} \right) \frac{64k^2 \sqrt{\gamma}}{\sqrt{4k^2 + iRe \pi}}.$$

Solution of the averaged equation (34) is

$$\zeta = \frac{Ha \cdot H_\infty^0}{\sqrt{2A \cdot k \cdot D}} \left\{ \exp \left( -4k^2 + \frac{iRe}{A} \right) \right\} \exp(-2ikt).$$

(35)

where $D = 1 + \left( 1 + \frac{8k + iRe}{A} \right)$.

By the assumptions made, approximately $D = 1$. Solution (35) of averaged equation (34) is close to the solution of the integral-differential equation (33) on the interval $t \in [0, c \cdot Re]$, where $c=\text{const}$ and degree of proximity of these solutions raises with growth $Re$ [5].

Accounting in general case $k = k_1 + ik_2$, from the obtained solution for vertical EM wave comes

$$H = \frac{H_\infty}{exp \left[ k \left( x - y \right) + \frac{k^2 - k_1^2 + ik_1 \left( x + y - c\tau \right)}{Re} \right]} \left( 36 \right)$$

$$\zeta = \zeta_1 \exp \left[ 2ik \left( x + y - c\tau \right) \right] - \zeta_2 \exp \left[ 2ik \left( x + y - \tau \right) \right],$$

where $c'' = 1 + 4k / Re$ is phase speed of the wave spreading along the straightline $y = \text{const} - x$. Expression (36) shows that in this case the electromagnetic wave causes two waves on a film surface: similar to a wave of the revolving force and different from it spreading with velocity of unperturbed surface flow. Their amplitudes are:

$$\zeta_1 = \frac{Ha \cdot H_\infty^0}{\sqrt{2A \cdot k \cdot D}} \exp \left[ 2ik \left( x - y \right) \right].$$

(37)

$$\zeta_2 = \zeta_2 \exp \left[ k^2 - k_1^2 + \frac{4k}{Re} \right].$$
approach only the waves of two kind are available with the amplitudes (37). By \( c''=1 \) \((k,=0)\) when film flow velocity is zero, it is got \( \text{Re}_c = 0 \) and there is only one wave. Parametric resonance in a system as shown by (37) takes place under fulfillment of the condition \( A \cdot D = 0 \), or \( \text{Re}_c, (8k+i\text{Re}) = 2kA \), and further it follows:

\[
k' = 2\left(\frac{8\pi + i\text{Re}}{\pi \text{Ga}^2}\right)^{0.5}.
\]  

(38)

\[
\left\{1\pm \pi \text{Ga}^2 \left[4e^{-1} + \text{Re}_c/(4\pi)\right] \text{Re}_c - 2\pi/\text{Oh}^2 \right\}^{0.5},
\]

where \( \text{Ga}, \text{Be}, \text{Oh} \) - Galileo, Batchelor and Ohnesorge numbers determined by physical properties of media and film thickness (independent of flow regime), \( \text{Re}_c = 2\pi \text{Re}/k^* \) - Reynolds number defined by the length of corresponding resonance perturbation. Correlation (38) allows establishing the relation between resonant values \( k^* \) and corresponding Reynolds number \( \text{Re}_c \), as well as the other defining criteria of the system. Then the relation \( \text{Re} = k^* \text{Re}_c/(2\pi) \) can be established as well.

The formula (38) can be substantially simplified in many important cases, e.g. in MHD-granulation the following estimations are valid: \( \text{Re}_c \gg 1 \), \( \text{Ga}^2 \gg 1 \), \( \text{Be} \ll 1 \), \( \text{Oh}^2 \ll 1 \). This yields:

\[
k' = \frac{1}{\pi \text{Ga}^2} \left(\frac{2}{\text{Ga}^2} \frac{8\pi + i\text{Re}_c}{8\pi + i\text{Re}_c} \frac{2\pi/\text{Oh}^2}{8\pi + i\text{Re}_c}\right).
\]

The Eigen oscillations with account of (34) are

\[
\zeta = \zeta_0 \exp\left[-2(8k+i\text{Re})\frac{k}{A} t\right].
\]

(39)

where \( \zeta_0 = \zeta(0) \). Therefore \( \omega = -\frac{2k}{A} (8k+i\text{Re}) \).

Consider Eigen oscillations for real values \( k \) and \( \zeta_0 \), then after estimation of the terms and accounting the assumptions made and the relation

\[
\arctg\left(\frac{\text{Re}}{\sqrt{\text{Re}^2+16k^2}}\right) \approx \frac{\text{Re}}{\sqrt{\text{Re}^2+16k^2}}
\]

it is available to come at

\[
(32k^2-\text{Re}^2)\left[2\sqrt{2k} + \text{Re}\left(\frac{1}{\text{Fr}^2} + \frac{8k^2}{\text{We}^2}\right)\right] + 6\sqrt{4k^2\text{Re}^2-\text{Re}^2+6k\text{Re}^2} > 0
\]

(40)

based on condition \( re(\omega) > 0 \). The condition (40) answers increase of Eigen fluctuations of system in time (development of film instability). At big enough \( k \) condition (40) is not satisfied, i.e., starting from some value of \( k \) all subsequent fluctuations of smaller length become fading in time. At \( \text{Re} \gg k \) it possible to get the following approximate formula:

\[
re(\omega) = \frac{2k\text{Re}^2\left[\sqrt{2(\text{Ga}^2+8k/\text{Oh}^2)}-8\text{Re}\right]}{2(\text{Ga}^2+8k/\text{Oh}^2)+8\sqrt{2k}\text{Re}(\text{Ga}^2+8k/\text{Oh}^2)+\text{Re}^2}
\]

(41)

where from follows that decrease of perturbations in a film is possible only on condition that is extremely seldom in practice. Film is unstable, but, apparently from (41), the increase rate of perturbations is small at \( \text{Re} \gg 1 \) that practically means uselessness of suppression of Eigen modes by parametric excitation of film flow disintegration on the set wavelength.

### 4 Conclusions by the results obtained

The results obtained may be used for analysis of parametric excitation and suppression of films. Horizontal progressive electromagnetic wave excides on a film surface the wave with amplitude \( -A/\text{Re} \); parametric resonance in a film flow by action of constant magnetic field is impossible in contrast to the alternating field; viscous forces are important: application of inviscid approach lead to inaccuracy in calculation of a wave amplitude 3-4 decimal orders; parametric resonance is impossible by means of horizontal electromagnetic wave. The perturbation amplitude in case of \( \omega \gg k \), \( \omega \gg k^2/\text{Re} \) does not depend on the wave number and is determined only by frequency of a field.

### References:


