



















approach only the waves of two kind are available with the amplitudes (37). By  $c^H = 1$  ( $k_i = 0$ ) when film flow velocity is zero, it is got  $Re_{ms} = 0$  and there is only one wave. Parametric resonance in a system as shown by (37) takes place under fulfillment of the condition  $A \cdot D = 0$ , or  $Re_m(8k + iRe) = 2kA$ , and further it follows:

$$k^* = 2 \frac{(8\pi + iRe_k)^{0.5}}{\pi Ga^2}. \quad (38)$$

$$\left\{ 1 \pm \left[ 1 + \pi Ga^2 \left( \frac{4Be - 1 + Re_k / (4\pi)}{16\sqrt{2}\pi} Re_k - \frac{2\pi / Oh^2}{8\pi + iRe_k} \right) \right]^{0.5} \right\},$$

where  $Ga, Be, Oh$ - Galileo, Batchelor and Ohnesorge numbers determined by physical properties of media and film thickness (independent of flow regime),  $Re_k = 2\pi Re / k^*$ - Reynolds number defined by the length of corresponding resonance perturbation. Correlation (38) allows establishing the relation between resonant values  $k^*$  and corresponding Reynolds number  $Re_k$ , as well as the other defining criteria of the system. Then the relation  $Re = k^* Re_k / (2\pi)$  can be established as well.

The formula (38) can be substantially simplified in many important cases, e.g. in MHD-granulation the following estimations are valid:  $Re_k \gg 1$ ,  $Ga^2 \gg 1$ ,  $Be \ll 1$ ,  $Oh^2 \ll 1$ . This yields:

$$k^* = \frac{1}{\pi Ga} \left( \frac{2}{Ga} \sqrt{8\pi + iRe_k} \pm \sqrt{\frac{8\pi + iRe_k}{16\sqrt{2}\pi} Re_k^2 - \frac{8\pi^2}{Oh^2}} \right).$$

The Eigen oscillations with account of (34) are

$$\zeta = \zeta_0 \exp \left[ -2(8k + iRe) \frac{k}{A} t \right], \quad (39)$$

where  $\zeta_0 = \zeta(0)$ . Therefore  $\omega_* = -\frac{2k}{A}(8k + iRe)$ .

Consider Eigen oscillations for real values  $k$  and  $\zeta_0$ , then after estimation of the terms and accounting the assumptions made and the relation  $\arctg \left( \frac{Re}{\sqrt{Re^2 + 16k^2}} \right) \approx \frac{Re}{\sqrt{Re^2 + 16k^2}}$  it is available to come at

$$(32k^2 - Re^2) \left[ 2\sqrt{2}k + Re \left( \frac{1}{Fr^2} + \frac{8k^2}{We} \right) \right] + 6\sqrt{2}k^3 Re \sqrt{\frac{k}{\pi} \frac{32k^2 - Re^2 + 6k Re^2}{\sqrt{16k^2 + Re^2}}} > 0 \quad (40)$$

based on condition  $re(\omega_*) > 0$ . The condition (40) answers increase of Eigen fluctuations of system in

time (development of film instability). At big enough  $k$  condition (40) is not satisfied, i.e., starting from some value of  $k$  all subsequent fluctuations of smaller length become fading in time. At  $Re \gg k$  is possible to get the following approximate formula:

$$re(\omega_*) = \frac{2k Re^2 \left[ \sqrt{2}(Ga^2 + 8k^2 / Oh^2) - 8k Re \right]}{2(Ga^2 + 8k^2 / Oh^2)^2 + 8\sqrt{2}k Re(Ga^2 + 8k^2 / Oh^2) + Re^4}, \quad (41)$$

where from follows that decrease of perturbations in a film is possible only on condition that is extremely seldom in practice. Film is unstable, but, apparently from (41), the increase rate of perturbations is small at  $Re \gg 1$  that practically means uselessness of suppression of Eigen modes by parametric excitation of film flow disintegration on the set wavelength.

## 4 Conclusions by the results obtained

The results obtained may be used for analysis of parametric excitation and suppression of films. Horizontal progressive electromagnetic wave excites on a film surface the wave with amplitude  $\sim Al \cdot Re$ ; parametric resonance in a film flow by action of constant magnetic field is impossible in contrast to the alternating field; viscous forces are important: application of inviscid approach lead to inaccuracy in calculation of a wave amplitude 3-4 decimal orders; parametric resonance is impossible by means of horizontal electromagnetic wave. The perturbation amplitude in case of  $\omega \gg k$ ,  $\omega \gg k^2 / Re$  does not depend on the wave number and is determined only by frequency of a field.

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