Impact of the Horizontal and Vertical Electromagnetic Waves on Oscillations of the Surface of Horizontal Plane Film Flow

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Abstract: - Problem on mathematical modeling of a plane film flow affected by alternating electromagnetic field (running horizontal EM wave) is considered. The parametric excitation and suppression of oscillations on a film surface at the essential inertia forces are studied using the developed model and computer simulation. In a linear approach some interesting new phenomena of the electromagnetic wave spreading in a plane film flow and its impact on excitation and suppression of oscillations on a film surface are investigated.

Key-Words: - Plane, Film, Flow, Electromagnetic, Horizontal, Wave, Instability, Excitation, Suppression

1 Oscillations in the plane film flow

Earlier investigated impact of the vertical electromagnetic wave on surface waves on the plane film flow spreading on a solid plate has shown the features of the system (see Fig.1 and Fig.2) [1-3].

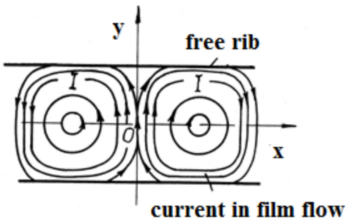

Fig.1 Configuration of the induced electric current from vertical EM wave in a plane film flow

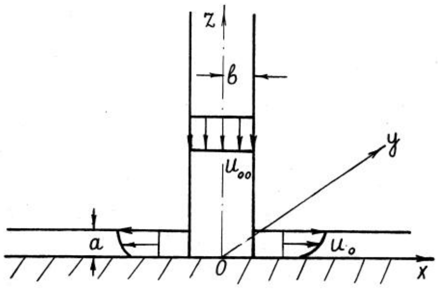

Fig.2 Creation of the plane film flow:
b, u_{00} - half-width of vertical jet and flow velocity,
l- distance from the nozzle to the horizontal plate

The established parameters of electromagnetic (EM) waves $H_z = H_m(z,t)\exp i(kx+my)$ are important for analysis of their action on conductive medium and selection of the control fields, which supply an achievement of the requested behavior of the controlled liquid metal film flow. Analysis performed has shown that short electromagnetic wave with constant by *x, y* amplitude is available if and only if magnetic field is rapidly decreasing in time, so that it is similar to the impulse packet having abrupt back front of type $\exp\left(-k_r^2 t/\mathrm{Re}_m\right)$.

Solution of the boundary problem by the model obtained has been done. The approximate solution has shown for $k_r \ll 1/\varepsilon$ (k_r- wave number, ε- dimensionless film thickness):

$$\zeta = \frac{Ha\sqrt{Al}\left\{\exp\left[-\frac{8k}{\varepsilon^2}\sqrt{k\,\mathrm{Re}_1}\left(\frac{1-\rho_{21}}{Fr^2}+\frac{8k^2}{We}\right)t\right]-\exp\left(-\frac{4k^2}{\mathrm{Re}_m}t\right)\right\}}{\left[\pm 2\sqrt{k\,\mathrm{Re}_1}\left(\frac{1-\rho_{21}}{Fr^2}+\frac{8k^2}{We}\right)(1-i)-k\frac{\varepsilon^2}{\mathrm{Re}_m}\right]\exp(2ikt)}.$$

Here ζ is amplitude of the film surface wave.

Analysis of the solution has shown that vertical electromagnetic wave causes perturbation of the film flow surface, which has a part similar to the exciting wave force and the other one, different. By real k (speed of the wave spreading is equal to the velocity of film flow, $\mathrm{Re}_{ms}=0$) the surface wave is alone (similar by form to the exciting force). By $\rho_{21} > 1+\frac{8k^2}{Oh^2 \cdot Ga^2}$ the exponential growing of the amplitude of surface oscillations is available, by $\rho_{21} < 1+\frac{8k^2}{Oh^2 \cdot Ga^2}$ influence of the surroundings is only quantitative. The amplitude of film surface oscillations excited by EM wave is proportional to $Ha\sqrt{Al}$, and parametric resonance is achieved by:

$$\mathrm{Re}_1=\frac{8Be\left[(1-\rho_{21})Ga^2+8k^2/Oh^2\right]^2}{k\varepsilon^2\left\{\varepsilon/Be\mp 8\left[(1-\rho_{21})Ga^2+8k^2/Oh^2\right]\right\}}.$$

Here Be, Ga, Oh are the Batchelor, Galileo and Ohnesorge numbers, respectively, $\mathrm{Re}_1\gg 1$. The single dynamic criterion Re_1 is expressed through the parameters of system and wave number k, $Be=\nu/\nu_m$, $Ga=\sqrt{gbb}/\nu$, $Oh=b\sqrt{\rho g/\sigma}$ [2,3].

Now the horizontal EM wave impact on the plane film flow is in focus of the present paper.

2 Model for horizontal EM wave impact on plane film flow oscillation

Consider EM wave with only *y*-component (running by *x* field or just alternating by *t*, if *k*=0):

$$H_y=H_m(z)\,e^{i(kx-\omega t)}\ , \qquad (1)$$

creating ponderomotive force (generally also running) on axis *z* (unlike the previous case of vertical EM wave, where it was directed against the unperturbed flow). According to a stable state [2,3] the function $f(x)=1-4\nu/u_{00}^2\cdot(x/b-1)(\partial u_s/\partial z)_{z=0}$ is introduced, which is in the frame of model considered monotonous and weekly-gradient.

In a first approach, it is possible to put $f=1$ and further estimate an error of such simplification of a task. Then the system of linearized MHD-equations of the perturbed film for the considered case in non-inductive approach can be presented as follows:

$$\frac{\partial u}{\partial t}+f\,\frac{\partial u}{\partial x}+\frac{\partial p}{\partial x}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{Al}{2}\,\frac{\partial H_y^2}{\partial x},\quad \frac{\partial w}{\partial z}=-\frac{\partial u}{\partial x},$$

$$\frac{\partial w}{\partial t}+\frac{\partial p}{\partial z}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{Al}{2}\,\frac{\partial H_y^2}{\partial z}, \qquad (2)$$

$$\frac{\partial H_y}{\partial t}+f\,\frac{\partial H_y}{\partial x}=\frac{1}{\mathrm{Re}_m}\left(\frac{\partial^2 H_y}{\partial x^2}+\frac{\partial^2 H_y}{\partial z^2}\right).$$

Corresponding boundary conditions on solid plate:

$$z=0,\ u=w=0;\quad z=\varepsilon,\ \frac{\partial u}{\partial z}=0,\ w=\frac{\partial\zeta}{\partial t}+f\,\frac{\partial\zeta}{\partial x},$$

$$p=\frac{\zeta}{Fr^2}+\frac{2}{\mathrm{Re}}\,\frac{\partial w}{\partial z}+\frac{Al}{2}H_y^2-\frac{1}{We}\,\frac{\partial^2\zeta}{\partial x^2}. \qquad (3)$$

As nature of parametric oscillations in a system in linear approach doesn't depend on initial amplitudes of the perturbed film, the initial conditions can be avoided, seeking for a linear system response in a wave mode (1) with the doubled frequency and wave number. Therefore from (2), (3) the following boundary task for the perturbation amplitudes is obtained:

$$\frac{d^2w}{dz^2}+\mathrm{Re}\left(\frac{dp}{dz}+AlH_m\,\frac{dH_m}{dz}\right)=2(\mathrm{i}\,\omega\,\mathrm{Re}-2k^2)\,w\,,$$

$$\frac{d^2u}{dz^2}+2\left[(\omega-fk)\mathrm{i}\mathrm{Re}-2k^2\right]u=ik\mathrm{Re}(2\mathrm{p}+\mathrm{Al}H^2_{\mathrm{m}}),$$

$$u=\frac{i}{2k}\,\frac{dw}{dz},\quad \frac{d^2H_m}{dz^2}+i\,\omega\,\mathrm{Re}_\mathrm{m}=k\left(k+if\mathrm{Re}_\mathrm{m}\right)H_m;$$

$$z=0,\ \ u=\mathrm{v}=0;\quad z=-\infty,\ \ H_\mathrm{m}=0; \qquad (4)$$

$$z=\varepsilon,\ \ H=H_{\mathrm{m}\,\varepsilon},\ \ \frac{du}{dz}=0,\ \ w=2i(kf-\omega)\zeta\,,$$

$$p=\frac{Al}{2}\,H^2_{\mathrm{m}\,\varepsilon}+\frac{2}{\mathrm{Re}}\,\frac{dw}{dz}+\left(\frac{1}{Fr^2}+\frac{4k^2}{We}\right)\zeta\,.$$

For simplification the amplitude perturbations of hydrodynamic parameters are designated by the same variables like namely hydrodynamic parameters as before. Ponderomotive force can be negative (pressing a film to a plane) only in case of existence of current in a film. Considering short-wave fluctuations of a film surface ($k_r\gg 1$) and assumed $k_i<0$, $k_r\gg -k_i$ (weak or moderate increase of fluctuations on *x*), we carry out the terms' assessment in the boundary task (4) and get its following approximate solution:

$$u=\frac{i}{2k}c_jq_j\exp(q_jz),\ \ w=c_j\exp(q_jz),\ \ H_m=H_m\exp\left[k_m(z-\varepsilon)\right],$$

$$p=\frac{Al}{2}\,H^2_{\mathrm{m}\,\varepsilon}\exp\left[2k_m(z-\varepsilon)\right]+1-\exp(q_jz)+$$

$$+\frac{c_j}{\mathrm{Re}}\left(q_j+2\,\frac{2k^2-i\omega\mathrm{Re}}{q_j}\right), \qquad (5)$$

where c_j=const, q_j- the roots of characteristic equation, k_m- electromagnetic wave numbers, $j=\overline{1,4}$- the summation index. Here:

$$k_{\mathrm{mr}}=\left(1+\frac{f^2\mathrm{Re}_m^2}{4k_r^2}\right)\left(k_r-f\,\mathrm{Re}_m\,\frac{k_i}{k_r}\right),$$

$$k_{\mathrm{mi}}=\left(1+\frac{f^2\mathrm{Re}_m^2}{4k_r^2}\right)\left(k_i+f\,\mathrm{Re}_m\right),\quad c_\mathrm{j}=\frac{\Delta_j}{\Delta},$$

$$q_\mathrm{j}=\pm\left\{\mathrm{Re}\left(fk-\omega\right)i\left[1\pm\sqrt{1+\frac{8k^2\left(2k^2-i\omega\mathrm{Re}\right)}{\mathrm{Re}^2\left(fk-\omega\right)^2}}\right]\right\}^{0,5},$$

$$\Delta=\sum_{i=1}^{2}\left\{q_1\left[(-1)^{n+1}(g_1+g_3)+(-1)^n g_{2n}\right]+(g_1-g_3)q_2\right\}.$$

$$\cdot\left\{q_1^2\exp(q_1\varepsilon)-q_2^2\exp\left[(-1)^n q_2\varepsilon\right]\right\}+\left[2q_1(g_2+g_4-2g_1)+\right.$$
$$\left.+q_1(g_2-g_4)\right]\text{sh}\,(q_1\varepsilon),\quad k_m=k_{mr}+i\,k_{mi},\qquad(6)$$
$$\Delta_{1,3}=2g_5q_2\left[q_1q_2sh(q_2\varepsilon)\mp q_2^2ch(q_2\varepsilon)\pm q_1^2\exp(\mp q_1\varepsilon)\right],$$
$$\Delta_{2,4}=2g_5q_1\left[q_1q_2sh(q_1\varepsilon)\mp q_1^2ch(q_1\varepsilon)\pm q_2^2\exp(\mp q_2\varepsilon)\right],$$

where q_j with odd indexes j are obtained when in front of square root is "+", with even – when "-", Δ_1,Δ_2 correspond the top signs in the formulas, Δ_3,Δ_4 - to bottom signs. Functions g_j in (6) are:

$$g_j=\frac{2}{q_j}\left[1-\exp(q_j\varepsilon)\right]\left(i\omega-\frac{2}{\text{Re}}k^2\right)+\frac{q_j}{\text{Re}}\left[3\exp(q_j\varepsilon)-1\right]+$$
$$\frac{i\exp(q_j\varepsilon)}{2(\omega-fk)}\left(\frac{1}{Fr^2}+\frac{4k^2}{We}\right),\;g_5=\frac{Al}{2}H^2_{m\varepsilon}\left[\exp(-k_m\varepsilon)-1\right],\quad\overline{j=1,4}.$$

With account of (4), the amplitude of perturbations:

$$\zeta=\frac{2}{\Delta}Al\cdot\text{Re}\cdot H^2_{m\varepsilon}\left[\exp(-k_m\varepsilon)-1\right]\sqrt{1+\frac{8k^2(2k^2-i\omega\text{Re})}{\text{Re}^2(fk-\omega)^2}}\cdot$$
$$\cdot\left[q_2sh(q_1\varepsilon)ch(q_2\varepsilon)-q_1sh(q_2\varepsilon)ch(q_1\varepsilon)\right],\qquad(7)$$

where from is seen that, unlike the considered cases, at a horizontal cross field amplitude of a superficial wave is $\sim Al\cdot\text{Re}$. It can be explained that direction of ponderomotive force action is vertical (up in case of lack of current in a film, except induced current) and therefore with increase of inertial forces also their contribution to energy pumping in waves.

2.1 Parametric resonance of surface waves

Parametric resonance of the system is possible by $\omega^*=fk$, $k\neq if\,\text{Re}/2$, when the phase speed of surface wave spreading coincides with velocity of the unperturbed (stable state) film flow. The Eigen oscillations are determined with characteristic equation $\Delta=0$, where from dispersive equation yields

$$2(i\omega_*\text{Re}-2k^2)\left\{sh(q_1\varepsilon)\left[q_2^2\exp(-q_2\varepsilon)\right]+2q_1\left[\frac{q_1}{q_2}(1-ch(q_2\varepsilon))-q_1sh(q_1\varepsilon)-q_2\right]+\right.$$
$$+q_1+\frac{q_2}{q_1}\left[q_1^2(\exp(q_1\varepsilon)-1)^2-2q_2^2(1-ch(q_1\varepsilon))ch(q_2\varepsilon)\right]+$$
$$+\left[\exp(q_1\varepsilon)(2q_2-q_1-q_1^2)+q_2^2\exp(q_2\varepsilon)\right]+\qquad(8)$$
$$\left.+2\frac{q_1}{q_2}\left[ch(q_2\varepsilon)-1\right]\left[q_2^2+q_1^2\exp(q_1\varepsilon)\right]\right\}+3q_1\{q_1\cdot$$
$$\cdot\left[2q_2(q_1ch(q_2\varepsilon)+2q_2sh(q_2\varepsilon))sh(q_1\varepsilon)-q_1\exp(q_1\varepsilon)\cdot\right.$$
$$\left.\cdot(\exp(-q_2\varepsilon)+q_2\exp(q_2\varepsilon))\right]+q_2\left[2q_1^2-q_2(1+q_2+2ch(q_1\varepsilon)ch(q_2\varepsilon))\right]\}+$$
$$\left\{ch(q_2\varepsilon)\left[q_1^2(q_1+q_2)sh(q_1\varepsilon)-q_2^2(sh(q_1\varepsilon)+q_1ch(q_1\varepsilon))\right]+\right.$$
$$\left.+q_1^3\left[ch(q_1\varepsilon)-sh(q_2\varepsilon)\right]\exp(q_1\varepsilon)\right]\right\}i\frac{Ga^2+4k^2/Oh}{\text{Re}(\omega-fk)}=0\cdot$$

Here can be seen that the Eigen oscillations generally spread on a film with dispersion, as $\partial^2\omega_*/\partial k^2\neq0$. The resonant pair (k,ω^*), as it is easy to check, generally doesn't satisfy condition (8), i.e. the parametric resonance is excited not at Eigen frequency and, besides, the surface film oscillations are not dispersing in this case on a contrast to the Eigen oscillations because $c^*=\partial\omega^*/\partial k=f$ and $\partial c^*/\partial k\equiv0$. The last, generally speaking, can be incorrect as far as friction of liquid on a plate depends on the mode of its flow; however within the accepted model it is right.

By $\omega_{*i}\gg1/\text{Re}$ from (8) after the terms' estimation the following dispersive equations result:

$$2\left[(\omega_{*i}\text{Re}+2k^2)\cos\Phi\pm\omega_{*r}\text{Re}\cdot\sin\Phi\right]\exp\left\{\sqrt{2\text{Re}}\left[\omega_{*i}^2+(fk-\omega_{*r})^2\right]^{0,25}\right\}=$$
$$=\frac{(Ga+4k/Oh)\omega_{*i}}{\left[\omega_{*i}^2+(fk-\omega_{*r})^2\right]\text{Re}},\;tg\Phi=\frac{\omega_{*i}\omega_{*r}\text{Re}+(\text{Re}\,\omega_{*i}+2k^2)(\omega_{*r}-fk)}{\omega_{*i}(\text{Re}\,\omega_{*i}+2k^2)-\omega_{*r}\text{Re}(\omega_{*r}-fk)},\quad(9)$$

where $\Phi=\sqrt{\frac{\text{Re}}{2\omega_{*i}}}(fk-\omega_{*r})$. It is assumed $|fk-\omega_{*r}|\ll|\omega_{*i}|$.

It is easy to check that at the above-stated parameters of system noticeable increase of amplitude of Eigen oscillations in time is possible only under condition $\omega_{*i}\gg1$, otherwise the characteristic time of perturbation development is less than existence time of a film, which edge (the rib for unloading from surface forces) breaks up owing to Rayleigh instability. Therefore from (9) the following system is obtained after further estimates and simplifications:

$$\frac{\omega_{*r}}{\omega_{*i}}=Arctg\frac{\omega_{*i}}{\omega_{*r}}+\pi n,\quad\omega_{*i}=\frac{2\omega_{*r}^2}{\text{Re}(fk-\omega_{*r})^2},$$

where from

$$\omega_{*i}=\frac{\omega_{*r}}{\alpha},\quad\omega_{*r}=fk+\frac{1}{\alpha\text{Re}}\pm\sqrt{\frac{2fk}{\alpha\text{Re}}+\frac{1}{\alpha^2\text{Re}^2}},\qquad(10)$$

$\alpha=1{,}162;\;3{,}426;\;6{,}437$, etc. (regular set). Solution (10) is satisfied by $\omega_{*r}\gg\alpha,\;k\gg\alpha$ (short-wave perturbations). By $\text{Re}\gg1$ both solutions coincide and approximately it comes to $\omega_{*r}=fk$, so that in this case $\omega_*\approx\omega^*$. One has to keep in mind that k cannot be big because in the small-amplitude linear approach the curvature of a surface cannot be big. The phase and group wave speeds are respectively

$$c=f+\frac{1}{\alpha k\text{Re}}\pm\frac{1}{k}\sqrt{\frac{2fk}{\alpha\text{Re}}+\frac{1}{\alpha^2\text{Re}^2}},\;C=f\left(1\pm\frac{1}{\alpha\text{Re}}/\sqrt{\frac{2fk}{\alpha\text{Re}}+\frac{1}{\alpha^2\text{Re}^2}}\right),$$

where from follows that the Eigen oscillations are practically not dispersing, and phase and group speeds are approximately equal to unperturbed velocity of film flow (kinematic waves).

2.1.1 Action of the constant electromagnetic field

By $\omega = 0$, assumed $\mathrm{Re} \gg k_r$, with accuracy of $\left(k/(f\,\mathrm{Re})\right)^2$ from (6) follows

$$q_{1,3} = \pm\sqrt{\mathrm{Re}\, fk}\,(1+i), \quad q_{2,4} = \mp 2k(1-i)\sqrt{\frac{k}{f\,\mathrm{Re}}},$$

And then, considering $k_r \gg 1$, $k_i < 0$, $-k_i \ll k_r$, $f \sim 1$, results in

$$q_{1,3} = \pm\sqrt{\mathrm{Re}\, fk_r}\,(1+i), \quad q_{2,4} = \mp 2k_r(1-i)\sqrt{\frac{k_r}{f\,\mathrm{Re}}}.$$

Similarly from (6), (7), with account of above-mentioned after estimation of the terms and further simplification yields

$$\zeta = \varepsilon Al\cdot \mathrm{Re}^2\cdot H_{m\varepsilon}^2\left[\exp(-k_m\varepsilon)-1\right]\frac{\alpha_1\beta_2-\alpha_2\beta_1+i(\alpha_1\beta_1+\alpha_2\beta_2)}{4(\beta_1^2+\beta_2^2)},$$

where:

$$\alpha_1 = 2-\exp(-k_{mr}\varepsilon)\cos(2k_{mi}\varepsilon), \quad \alpha_2 = \exp(-2k_{mr}\varepsilon)\sin(2k_{mi}\varepsilon),$$

$$\beta_1 = 2k_i\,\mathrm{Re}-\varepsilon\left(Ga^2+4k_r^2/Oh^2\right), \quad \beta_2 = 4k_r\left(2\varepsilon k_i/Oh^2+\mathrm{Re}\right).$$

Now real part of the film surface is expressed as

$$\frac{\zeta_r}{Al} = \mathrm{Re}^2\cdot\frac{\varepsilon H_{m\varepsilon}^2\left[\exp(-k_{mr}\varepsilon)-1\right]}{4(\beta_1^2+\beta_2^2)\exp(2k_ix)}\cdot \qquad (11)$$

$$\cdot\left[(\alpha_1\beta_2-\alpha_2\beta_1)\cos(2k_rx)-(\alpha_1\beta_1+\alpha_2\beta_2)\sin(2k_rx)\right].$$

The expression (11) thus obtained shows that by $\mathrm{Re}\to\infty$, $\zeta_r \sim \mathrm{Re}\,Al$, while by $\mathrm{Re} \sim Ga, 1/Oh^2$ it is got $\zeta_r \sim \mathrm{Re}^2\cdot Al$. Parametric resonance is impossible in case of constant electromagnetic field as seen from (11). By $k_r \gg 1$ follows $\zeta_r \sim Al\cdot H_{m\varepsilon}^2/k_r^2$, therefore for big enough values k_r (short waves) with increase of k_r the amplitude of the exciting electromagnetic field must grow proportionally. But this case is practically impossible due to technical impediments with creation of alternating field by x with a small wave length.

In general, electromagnetic excitation of oscillations of conductive film surface under transversal horizontal alternating by x field is determined by $Al\cdot\mathrm{Re}^2$, Ga, Oh and wave number k. As an illustration of these results the calculation presented in Fig.3. It was computed by (11) for: $k_r = 10$, $k_i = -5$, $\varepsilon = 1$, $u_{00} = 4{,}43$m/s, $b = 10^{-3}$m, $k_{mr} \approx k_r$, $k_{mi} \approx \mathrm{Re}_m f - 5$, $\mathrm{Re}_m = 0{,}027$, $\mathrm{Re} = 3{,}67\cdot 10^3$.

It corresponds to $Ga \gg k_r/Oh$ and $Ga^2 \gg k_r\,\mathrm{Re}$, expression (11) was simplified:

$$\zeta_r = -Al\cdot\mathrm{Re}^2\cdot H_{m\varepsilon}^2\frac{4k_r\,\mathrm{Re}\cdot\cos(2k_rx)+Ga^2\cdot\sin(2k_rx)}{Ga^4\exp(2k_ix)},$$

where $Al\cdot\mathrm{Re}^2$ is just quadratic Reynolds number computed by the speed of the Alfven waves as characteristic flow velocity.

Apparently from Fig.3, bending around oscillatory fronts have axes almost symmetric relatively the top and lower boundaries. Influence of magnetic Reynolds number in the considered model statement is insignificant and is shown generally on coefficient at $\cos(2k_rx)$, which, however, is much less than a coefficient at $\sin(2k_rx)$; here are $Oh = 0{,}22$, $Ga = 8{,}3\cdot 10^4$.

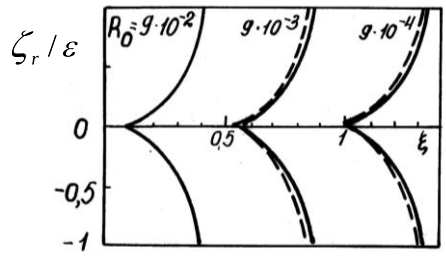

Fig.3 Bending around oscillatory fronts of a film: against longitudinal coordinate for 3 values of field $H_0=9*10^{-4}$, $9*10^{-3}$ and $9*10^{-2}$

2.1.2 Falling of a film flow velocity with distance

On the distances of x=82,0 and x=16,4 velocity of the unperturbed film flow decreases, correspondingly, by 5% and 1%, therefore with accuracy sufficient for practice follows that the assumption about constant unperturbed flow velocity is correct by $H_{m\varepsilon} < 10^{-2}$, when electromagnetic field is weak enough.

Calculation with account of the function *f(x)* is presented in Fig.3 by dashed lines. In case of film flow with current in external magnetic field of the same strength, when directions of the magnetic and gravitational forces coincide, the amplitude of perturbation on a film flow surface is lower about 20 times comparing to the only induction situation.

2.1.3 Influence of the fluid viscosity

For inviscid approach ($\mathrm{Re}\to\infty$) from (2.3.11) reduction ζ_r is approximately by one decimal order in comparison with a viscous case, and inviscid approach is correct at $\mathrm{Re} > 10^9$ that is almost unreal. Thus, the electromagnetic field of the form

$H = H_{m\varepsilon}\exp\left(ik_r - k_i\right)x$ causes process of wave formation on a film surface with wavelength, twice bigger, than by H, and small shift on *x*. Increase on *x* happens under the law $\exp\left(-2k_i x\right)$.

For example, the field with initial induction of $\mathrm{B}_0 \sim 0{,}023\mathrm{T}$ ($H_{m\varepsilon} \sim 9\cdot 10^{-2}$) causes perturbation $\zeta_r \sim 0{,}02\varepsilon$, which already at *x*=19,5 reaches value $\zeta_r \sim \varepsilon$ (film decay). At disintegration of a film the drops are formed under the Rayleigh law or at the action of mechanical flow dividers with formation the drops by diameter $d \sim \varepsilon$.

2.1.4 The case of progressive (running) waves

For real k, $\omega \gg k$, $\omega \gg k^2/\mathrm{Re}$, from (6) follows

$$q_{1,3} = \pm\sqrt{2\mathrm{Re}\,\omega}\left\{\cos\left[\pi\left(n+0{,}25\right)\right] \mp \cos\left[\pi\left(n+0{,}5\right)\right]\right\},\quad q_{2,4} = \pm 2k,$$

where *n*=0,1,2,3… Then accounting (7) yields

$$\zeta = \frac{q_1 g_5 \varepsilon \,\mathrm{Re}}{\mathrm{Re}\left[\dfrac{1}{\omega Fr^2} + \dfrac{2\omega}{q_1} - \left(\dfrac{2\omega}{q_1} + \dfrac{1}{2\omega Fr^2}\right)\exp\left(-q_1\varepsilon\right)\right] + 4\left(q_1 i - \omega\varepsilon\right) + q_1 i\left[1 - 3\exp\left(-q_1\varepsilon\right)\right]}$$

where from in case of $1 \ll \omega \ll \mathrm{Re}$ it is got simpler expression for the perturbation amplitude

$$\zeta = \frac{\varepsilon\,\mathrm{Re}\cdot Al \cdot H_{m\varepsilon}^2\left[\exp\left(-k_m\varepsilon\right) - 1\right]}{i\left(2\omega \mp 8\omega - \sqrt{\dfrac{\mathrm{Re}}{\omega}}\dfrac{1}{Fr^2}\right) \pm \sqrt{\dfrac{\mathrm{Re}}{\omega}}\dfrac{1}{Fr^2}},\quad \omega \gg \sqrt{\frac{\mathrm{Re}}{\omega}}\frac{1}{Fr^2}\cdot$$

And a real part of the film surface perturbation is

$$\zeta_r = \frac{\varepsilon\,\mathrm{Re}\cdot Al \cdot H_{m\varepsilon}^2}{2(1\mp 4)\omega^2}\left\{\omega\sin\left[2\left(kx - \omega t\right)\right] \pm \sqrt{\frac{\mathrm{Re}}{\omega}}\frac{\cos\left[2\left(kx - \omega t\right)\right]}{2(1\mp 4)Fr^2}\right\}. \quad (12)$$

The second item of (12) in square brackets is small; therefore it can be omitted in engineering calculations, which simplifies (12) as follows

$$\zeta_r = \frac{\varepsilon\,\mathrm{Re}\cdot Al \cdot H_{m\varepsilon}^2}{2(1\mp 4)\omega}\sin\left[2\left(kx - \omega t\right)\right],$$

where from follows that $\zeta_r \sim 1/\omega$, so that with increase of the field frequency the phase speed of the wave is growing (the speeds of the electromagnetic and surface hydrodynamic waves coincide) and its amplitude is falling down.

The two surface waves differ in amplitude (0,6 times) and in a phase (small at $\omega^3 \gg \mathrm{Re}/Fr^4$). The parametric resonance is absent owing to what, considering stated, it is possible to conclude that a horizontal electromagnetic wave can excite disintegration of a film flow only on the set wavelength, and unlike a case of a vertical wave an amplitude of wave perturbation is defined only by frequency. The excited oscillations on a film surface have a wavelength independent of *x*, thus, they don't disperse. In a case of $\omega^3 \gg \mathrm{Re}/Fr^4$ the process of wave formation and wave spreading in a film is defined only by $\mathrm{Re}\cdot Al$, k, ω, and from film thickness dependence is linear ($\zeta_r \sim \varepsilon$).

As seen from (12), in this case no film flow decay occur because an amplitude of superficial waves doesn't change in space-time, and the waves of $\zeta_r = \varepsilon$ are beyond the linear theory.

2.2 Excitation of oscillations on the film flow surface by vertical electromagnetic wave

Dimensionless equations for this case are

$$\frac{\partial u_j}{\partial t} + (2-j)\frac{\partial u_j}{\partial x} + \rho_{1j}\frac{\partial}{\partial x}\left(p_j + \frac{2-j}{2}Al\cdot H^2\right) = \frac{1}{\mathrm{Re}_j}\Delta u_j,$$

$$\frac{\partial v_j}{\partial t} + (2-j)\frac{\partial v_j}{\partial x} + \rho_{1j}\frac{\partial}{\partial y}\left(p_j + \frac{2-j}{2}Al\cdot H^2\right) = \frac{1}{\mathrm{Re}_j}\Delta v_j,$$

$$\frac{\partial w_j}{\partial t} + (2-j)\frac{\partial w_j}{\partial x} + \rho_{1j}\frac{\partial p_j}{\partial x} = \frac{1}{\mathrm{Re}_j}\Delta w_j,\quad div\vec{v}_j = 0, \quad (13)$$

$$\frac{\partial H}{\partial t} + \frac{\partial H}{\partial x} = \frac{1}{\mathrm{Re}_m}\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right);$$

Boundary conditions are stated as follows:

$$z = \varepsilon,\ \vec{v}_1 = \vec{v}_2,\ w = \frac{\partial \chi}{\partial t} + \frac{\partial \chi}{\partial x};$$

$$\mu_{j1}\left(\frac{\partial w_j}{\partial x} + \frac{\partial u_j}{\partial z}\right) = idem,\quad \mu_{j1}\left(\frac{\partial w_j}{\partial y} + \frac{\partial v_j}{\partial z}\right) = idem, \quad (14)$$

$$p_1 = p_2 + \frac{1-\rho_{21}}{Fr^2}\chi + \frac{Al}{2}H_m^2 - \frac{1}{We}\left(\frac{\partial^2\chi}{\partial x^2} + \frac{\partial^2\chi}{\partial y^2}\right) + 2\left(\frac{1}{\mathrm{Re}_1}\frac{\partial w_1}{\partial z} - \frac{\rho_{21}}{\mathrm{Re}_2}\frac{\partial w_2}{\partial z}\right);$$

$$z = 0,\quad \vec{v}_1 = 0;\qquad z = \gamma,\quad \vec{v}_2 = 0; \qquad (15)$$

$$t = 0,\qquad \vec{v}_j = 0,\qquad p_j = 0,\qquad \chi = 0,\qquad H = H_0, \quad (16)$$

where $Al = \mu_m H_0^2/\left(\rho_1 u_0^2\right)$, $Fr = u_0/\sqrt{gb}$, $We = \rho_1 b u_0^2/\sigma$, $\mathrm{Re}_j = u_0 b/\nu_j$, $\mathrm{Re}_m = u_0 b/\nu_m$ are the Alfven, Froude, Weber, Reynolds and magnetic Reynolds numbers, respectively; $H_0 = H(0)$, and γ - dimensionless total thickness of the layers of conductive and non-conductive media, $\varepsilon = a/b$, $\rho_{ij} = \rho_i/\rho_j$, $\mu_{j1} = \mu_j/\mu_1$, *j*=1,2. We choose the characteristic scales of length, velocity, time, pressure and intensity of an electromagnetic field, respectively, $b, u_0, b/u_0, \rho_1 u_0^2, H_0$ and put for simplicity $u_0 = const$.

The first boundary condition (15) corresponds to a film spreading on a solid surface. For free film flow instead of this condition by *z=0* should be considered also boundary condition of conjugated type on the interface with surroundings. Because $u_a = H_0\sqrt{\mu_m/\rho_1}$ is a velocity of spreading the Alfven waves, the Alfven number can be presented as a ratio of characteristic velocities: $Al = u_a^2/u_0^2$.

2.2.1 Solution of the boundary problem

Assuming the small-amplitude perturbations, we seek solution of the boundary problem (13)-(16) as a linear response of the system to an external action. Following the superposition principle, the excited by electromagnetic wave $H_z = H_m(z,t)\exp i(kx+my)$ surface waves in a film flow are considered as proportional to the function $\exp 2i(kx+my)$ with corresponding amplitudes – functions of time.

Solution of the boundary problem is convenient to search with use of the integral transformation methods [4]. We apply to the PDE array (13) the Laplace transformation by time. Accounting the above-mentioned and the initial conditions (16) we come to the following system of differential equations in the transforms:

$$\frac{d^2Q_1}{dz^2} - 4(k^2+m^2)Q_1 = \mathrm{Re}_1\left[(s+2ik)Q_1 + iq\left(2P_1 + Al\cdot\bar{H}_m^2\right)\right],$$

$$\frac{dW_j}{dz} = -2i(kU_j + m\cdot V_j), \qquad (17)$$

$$\frac{d^2Q_2}{dz^2} - 4(k^2+m^2)Q_2 = \mathrm{Re}_2\left[(s+2ik)Q_2 + 2iq\rho_{12}P_2\right],$$

$$\frac{d^2W_j}{dz^2} - 4(k^2+m^2)W_j = \mathrm{Re}_j\left[(s+2ik)W_j + \rho_{1j}\frac{dP}{dz}\right],$$

where $\{U,V,W\}(s), P(s), \bar{H}_m(s)$ are the Laplace-transforms of functions $\{u,v,w\}(t),\ p(t), H_m(t)$, function $H_m(t)$ is determined by

$$H_m = H_{m0}\exp\left\{-\left[ik + \left(m^2+k^2\right)/\mathrm{Re}_m\right]t\right\}$$

According to the above-mentioned assumptions and formulated equations, *s*- parameter of integral transformation.

The first and the third equations of (17) are presented for simplicity in a symbolic form: $Q_1 = V_1$, $Q_2 = V_2$ by $q = m$, and $Q_1 = U_1$, $Q_2 = U_2$ by $q = k$. The boundary conditions (14), (15) in transforms:

$z=0$, $U_1 = V_1 = W_1 = 0$; $\quad z = \gamma$, $U_2 = V_2 = W_2 = 0$; (18)

$z = \varepsilon$, $P_1 = P_2 + \dfrac{Al}{2}\bar{H}_m^2 + \left(\dfrac{1-\rho_{21}}{Fr^2} + 4\dfrac{m^2+k^2}{We}\right)Z + \dfrac{2}{\mathrm{Re}_1}\left(\dfrac{dW_1}{dz} - \mu_{21}\dfrac{dW_2}{dz}\right),$

$$\frac{dQ_1}{dz} + 2iqW_1 = \mu_{21}\left(\frac{dQ_2}{dz} + 2iqW_2\right), \qquad (19)$$

$$U_1 = U_2,\quad V_1 = V_2,\quad W_1 = W_2 = (s+2ik)Z,$$

where $Z(s)$- the Laplace-transform of the amplitude of film flow perturbation $\zeta(t)$.

The solution of the boundary task in transforms (17)-(19) doesn't present principal difficulties, however it is so cumbersome that the return transformation to the functions' originals demands application of the numerical methods. Thus, for a free film (not restricted by a solid surface), for the film flow surrounded with the liquid or gaseous medium from the top and from the bottom the boundary conditions (18) are replaced with conditions of type (19) owing to what the solution is strongly complicated.

For convenience of the analysis of the considered class of problems by parametric excitation and suppression of oscillations on a surface of the conductive film flows spreading in non-conductive media, and for detection of the main regularities of such physical systems we consider the short-wave perturbations and suppose *k*=*m*. Such case has important applications in practice of creation the perspective film MHD-granulators [1,2].

The written above allows significantly simplifying a task, and the system (17) gives

$$\frac{d^4W_j}{dz^4} - (8k^2+q_j^2)\frac{d^2W_j}{dz^2} + 8k^2q_j^2W_j = 0,$$

where $q_j^2 = 8k^2 + (s+2ik)\mathrm{Re}_j$, $j = 1,2$. Solution of this system is $W_j = d_{jn}\exp(g_{jn}z)$, where for each value of *j* the sum is taken by index $n = \overline{1,4}$. The values g_{jn} are

$$g_{j1} = 2\sqrt{2}k,\quad g_{j3} = -g_{j1},\quad g_{j2} = q_j,\quad g_{j4} = -q_j. \qquad (20)$$

Constants of integration d_{jn} are computed by substitution the function $W_j(z)$ in the boundary conditions (18), (19), with account of (17). In a linear approach yields $U_j = V_j \sim dW_j/dz$, and for d_{jn} the following linear algebraic equation array (LAEA) of the 8-th order is got:

$$d_{1n}g_{1n} = 0,\quad d_{1n}\exp(g_{1n}\varepsilon) = (s+2ik)Z,\quad \sum_{n=1}^{4} d_{jn} = 0,$$

$$d_{2n}\exp(g_{2n}\varepsilon) = (s+2ik)Z,\quad d_{2n}g_{2n}\exp(g_{2n}\gamma) = 0,$$

$$d_{1n}g_{1n} = d_{2n}g_{2n}\exp\left[(g_{2n}-g_{1n})\varepsilon\right], \qquad (21)$$

$$d_{1n}\left(g_{1n}^2 + 8k^2\right) = \mu_{21}d_{2n}\left(g_{2n}^2 + 8k^2\right),\quad d_{2n}\exp(g_{2n}\gamma) = 0.$$

After obtaining the d_{jn}, the function $Z(s)$ is computed from the last equation (19) with account of (17). Solution of the system (21) with account of (20) is going as follows. Two equations containing both d_{1n} and d_{2n} are solved and then the remaining system of six equations is solved treating the values d_{14}, d_{24} as parameters, afterwards d_{j4} is determined from the system of two equations. Matrix of the first sub-system has the view $\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$, where A, B are non-zero quadratic matrices of the third order (two others are zero), and additionally C- column of the free terms. Such matrix allows easy computing the main as well as additional determinants through reduction of them to the product of corresponding determinants of the third order. Thus, the result is:

$$d_{ij} = \Delta_{ij} / \Delta_i, \quad i = 1,2, \quad j = 1,2,3; \quad (22)$$

$$d_{i4} = \frac{A_1\left(q_{3-i}^2 + 8k^2\right)\mu_{2i} - A_2 q_{3-i}}{q_1\mu_{21}\left(q_2^2 + 8k^2\right) - q_2\left(q_1^2 + 8k^2\right)} \exp\left(q_i\varepsilon\right),$$

where the main and minor determinants are:

$$\Delta_1 = 2\left\{g_{11}\left[\exp\left(q_1\varepsilon\right) - ch\left(g_{11}\varepsilon\right)\right] - q_1 \cdot sh\left(g_{11}\varepsilon\right)\right\},$$

$$\Delta_2 = 2\left\{\exp\left(q_2\gamma\right)\left[g_{11}ch\left(g_{11}(\varepsilon-\gamma)\right) + q_2 sh\left(g_{11}(\varepsilon-\gamma)\right)\right] - g_{11} \cdot \exp\left(q_2\varepsilon\right)\right\},$$

$$\Delta_{11} = 2d_{14}\left[q_1 ch\left(q_1\varepsilon\right) - g_{11} sh\left(q_1\varepsilon\right) - q_1\exp\left(-g_{11}\varepsilon\right)\right] - \left(q_1 + g_{11}\right)\left(s + 2ik\right)Z,$$

$$\Delta_{12} = 2\left\{g_{11}\left[\left(s+2ik\right)Z + d_{14}\left(ch\left(g_{11}\varepsilon\right) - \exp\left(-q_1\varepsilon\right)\right)\right] - q_1 d_{14} sh\left(g_{11}\varepsilon\right)\right\},$$

$$\Delta_{13} = \left(q_1 - g_{11}\right)\left(s+2ik\right)Z + d_{14}\left[q_1\left(\exp\left(g_{11}\varepsilon\right) - ch\left(q_1\varepsilon\right)\right) - g_{11} sh\left(q_1\varepsilon\right)\right],$$

$$\Delta_{21} = (q_2 + g_{11})\exp\left[(q_2 - g_{11})\gamma\right](s+2ik)Z + \quad (23)$$

$$+2\left\{\left[g_{11} sh\left(q_2(\varepsilon-\gamma)\right) - q_2 ch\left(q_2(\varepsilon-\gamma)\right)\right]\exp\left(-\gamma g_{11}\right) + q_2\exp\left(-\varepsilon g_{11}\right)\right\} d_{24},$$

$$\Delta_{22} = 2\left\{\left[q_2\exp\left(-\gamma q_2\right) sh\left(g_{11}(\varepsilon-\gamma)\right) + g_{11}\left(\exp\left(-\varepsilon q_2\right) - \exp\left(-\gamma q_2\right)\cdot\right.\right.\right.$$

$$\left.\left.\left.\cdot ch\left(g_{11}(\varepsilon-\gamma)\right)\right]\cdot d_{24} - g_{11}\left(s+2ik\right)Z\right\},$$

$$\Delta_{23} = 2\left\{\exp\left(\varepsilon g_{11}\right)\left[q_2 ch\left(g_{11}(\gamma-\varepsilon)\right) - g_{11} sh\left(q_2(\gamma-\varepsilon)\right)\right] - q_2\exp\left(g_{11}\varepsilon\right)\right\}\cdot$$

$$\cdot d_{24} + \left(s+2ik\right)\exp\left[\left(q_2 + g_{11}\right)\gamma\right]\left(g_{11} - q_2\right)Z,$$

$$A_1 = g_{11}\left[\exp\left(g_{11}\varepsilon\right)\left(d_{11} - d_{12}\right) + \exp\left(-g_{11}\varepsilon\right)\left(d_{23} - d_{13}\right)\right] +$$
$$+ q_1\exp\left(q_1\varepsilon\right)d_{12} - q_2\exp\left(q_2\varepsilon\right)d_{22},$$

$$A_2 = 16k^2\left[\exp\left(g_{11}\varepsilon\right)\left(\mu_{21}d_{21} - d_{11}\right) + \exp\left(-g_{11}\varepsilon\right)\left(\mu_{21}d_{23} - d_{13}\right)\right] +$$

$$+\mu_{21}\left(q_2^2 + 8k^2\right)\exp\left(q_2\varepsilon\right)d_{22} - \left(q_1^2 + 8k^2\right)\exp\left(q_1\varepsilon\right)d_{12}.$$

In view of bulkiness of the received expressions (22), (23) at this stage it is expedient to carry out the assessment of terms allowing to simplify a task for each specific physical situation and further to do its solution in each case separately that allows applying the analytical methods effectively. We consider one of such situations for the example. Let $1/\gamma \ll k_r \ll 1/\varepsilon$, where k_r - real part of the wave number k. This condition covers quite bright region of the wave numbers and corresponding class of the wave processes has relation to many practical situations. The range of variation γ is taken $\gamma = \left(10^2 \div 10^4\right)\varepsilon$, therefore $\sim\left(10^{=3} \div 10^{-1}\right) \ll k_r \ll \sim 1 \cdot$ And as far as $\varepsilon k_r \ll 1$, $\left(\gamma - \varepsilon\right)k_r \gg 1$, then $ch\left(\gamma-\varepsilon\right)q \approx 0.5\exp\left[\left(\gamma-\varepsilon\right)q\right]$. Similar simplifications made in other expressions, then from (22), (23):

$$\Delta_1 = g_{11}\varepsilon^2\left(s+2ik\right)\mathrm{Re}_1, \quad d_{i4} = \Delta_{i4}/\Delta, \quad d_{ij} = \Delta_{ij}/\Delta_i,$$
$$i = 1,2, \quad j = 1,2,3;$$

$$\Delta = (1 - q_2\varepsilon)q_1 g_{11}\varepsilon^5\left[\frac{\mathrm{Re}_1}{2(g_{11} - q_2)}\left(2q_2 - g_{11}\right)\left(8k^2 + q_1^2\right)\left(s+2ik\right) +\right.$$

$$\left. + 8k^2\mu_{21}q_1^2 g_{11}\varepsilon\right], \quad \Delta_{14} = \left(1 + q_1\varepsilon\right)\left(1 - q_2\varepsilon\right)\varepsilon^2\left(s+2ik\right)Z\cdot$$

$$\cdot\left[8k^2\mu_{21}\left(g_{11} + q_1\right) + \frac{\mathrm{Re}_1}{2\left(g_{11} - q_2\right)}\left(2q_2 - g_{11}\right)\left(s+2ik\right)\right],$$

$$\Delta_{24} = q_1\varepsilon^3\left(1 + q_1\varepsilon\right)\left[\left(q_1 + g_{11}\right)\left(8k^2 + q_1^2\right) - g_{11}^2 q_2^2\varepsilon\right]\left(s+2ik\right)Z,$$

$$\Delta_2 = \left(g_{11} - q_2\right)\exp\left[\left(g_{11} + q_2\right)\gamma - g_{11}\varepsilon\right],$$

$$\Delta_{12} = g_{11}\left(s+2ik\right)\left(2Z - \mathrm{Re}_1\varepsilon^2 d_{14}\right), \quad (24)$$

$$\Delta_{11} = q_1\varepsilon^2\left(g_{11}^2 + q_1^2\right)d_{14} - \left(g_{11} + q_1\right)\left(s+2ik\right)Z,$$

$$\Delta_{13} = \left(q_1 - g_{11}\right)\left[\left(s+2ik\right)Z - q_1\left(q_1 + g_{11}\right)d_{14}\varepsilon^2\right],$$

$$\Delta_{21} = \left(q_1 + g_{11}\right)\exp\left[\left(q_2 - g_{11}\right)\gamma\right]\left[\left(s+2ik\right)Z + \left(q_2\varepsilon - 1\right)d_{24}\right] + 2q_2\left(1 - g_{11}\varepsilon\right)d_{24},$$

$$\Delta_{22} = \left[2g_{11}\exp\left(-q_2\varepsilon\right) + \left(g_{11}\varepsilon - 1\right)\left(q_2 + g_{11}\right)\exp\left[\left(g_{11} - q_2\right)\gamma\right]\right]d_{24} - 2g_{11}\left(s+2ik\right)Z,$$

$$\Delta_{23} = \left(q_2 - g_{11}\right)\exp\left[\left(g_{11} + q_2\right)\gamma\right]\left[\left(1 - q_2\varepsilon\right)d_{24} - \left(s+2ik\right)Z\right].$$

Further from the last equation of the system (19) with account of (24) the following equation for Laplace-transform of the film flow perturbation $Z(s)$ is got:

$$\left\{\frac{\mu_{12}\rho_{21}^2\,\mathrm{Re}_1\left(g_{11} + q_1\right)\left(s+2ik\right)^2}{8k^2\left(g_{11} - 2q_2\right)\left(g_{11} + q_2\right)} + 2q_1\left(\frac{1 - \mu_{21}}{\mathrm{Re}_1} + \frac{s+2ik}{8k^2}\right) +\right.$$

$$+2\frac{16k^2\rho_{21}\left(g_{11} + q_1\right) + \left(g_{11} - 2q_2\right)\left(g_{11} + q_2\right)}{\left(8k^2 + q_1^2\right)\left(g_{11} - 2q_2\right)\left(g_{11} + q_2\right)g_{11}\varepsilon}\cdot\left[\left(\mu_{21} - 1\right)\frac{8k^2}{\mathrm{Re}_1} - \right. \quad (25)$$

$$\left.\left. - \left(s+2ik\right) - \frac{\mathrm{Re}_1}{8k^2}\left(s+2ik\right)^2\right]\right\}Z = \frac{\mathrm{Re}_1}{\varepsilon^2}\left[\frac{Al}{2}\bar{H}_m^2\varepsilon + \left(\frac{1 - \rho_{21}}{Fr^2} + \frac{8k^2}{We}\right)Z\right].$$

The thin films are got by substantially high flow velocity when inertial forces strongly prevail the drag forces due to friction on a solid plane where the film flow spreads, therefore $\mathrm{Re}_1 \gg 1$ (by $b = 10^{-3}$ m, $u_0 = 1$ m/s, for most metal melts $\mathrm{Re}_1 \sim 10^3$) and expression (25) after estimation of the terms yields:

$$\left[\frac{s+2ik}{2\sqrt{\mathrm{Re}_1}} + \frac{8k^2}{\varepsilon^2\sqrt{s+2ik}}\left(\frac{1 - \rho_{21}}{Fr^2} + \frac{8k^2}{We}\right)\right]Z = -\frac{4k^2 Al\bar{H}_m^2}{\varepsilon^2\sqrt{s+2ik}}.$$

2.2.2 Integral-differential equation for film surface oscillations

Now performed the reverse Laplace-transformation yields the integral-differential equation describing the parametric oscillations of the film surface excited by electromagnetic field having z-component:

$$\frac{d\zeta}{dt}+2ik\zeta+\frac{16}{\varepsilon^2}k^2\sqrt{\mathrm{Re}_1}\left(\frac{1-\rho_{21}}{Fr^2}+\frac{8k^2}{We}\right)\int_0^t\frac{\zeta(t-\tau)d\tau}{\sqrt{\pi\tau}\exp(2ik\tau)}=$$

$$=-\frac{8k^2}{\varepsilon^2}\sqrt{\mathrm{Re}_1}Al\int_0^t\frac{H_m^2(t-\tau)d\tau}{\sqrt{\pi\tau}\exp(2ik\tau)}. \qquad (26)$$

With the assumptions made the function under integrals in (26) are highly-oscillating, therefore these integrals are small and the equation (26) is averaged by the second scheme [5]. As a result, with account of the EM wave form, the averaged differential equation is got, the solution of which has the following form:

$$\zeta=\frac{Ha\sqrt{Al}\left\{\exp\left[-\frac{8k}{\varepsilon^2}\sqrt{k\,\mathrm{Re}_1}\left(\frac{1-\rho_{21}}{Fr^2}+\frac{8k^2}{We}\right)t\right]-\exp\left(-\frac{4k^2}{\mathrm{Re}_m}t\right)\right\}}{\left[\pm2\sqrt{k\,\mathrm{Re}_1}\left(\frac{1-\rho_{21}}{Fr^2}+\frac{8k^2}{We}\right)(1-i)-k\frac{\varepsilon^2}{\mathrm{Re}_m}\right]\exp(2ikt)}. \qquad (27)$$

The approximate solution (27) of the equation (26) thus obtained is close enough to the exact solution on the infinite interval of time and its fidelity grows with increase of k_r (according to the assumptions made there is $k_r \ll 1/\varepsilon$).

2.2.3 Suppression of the Eigen oscillations

The solution (27) accounting the above is transformed to the following form:

$$\zeta=Ha\sqrt{Al}\,\frac{4k\left[\exp(-\omega_* t)-\exp\left(-4k^2t/\mathrm{Re}_m\right)\right]}{\varepsilon^2\left[\pm(1-i)\omega_*-4k^2/\mathrm{Re}_m\right]\exp(2ikt)}, \qquad (28)$$

where from one can see that suppression of the Eigen oscillations with the wave number k and frequency ω_* is available only by means of the running field having the distribution speed other than the speed of the movement of a film $(k_i\neq0, \mathrm{Re}_{ms}\neq0)$. Otherwise both excitation and suppression of fluctuations demand big energy expenses (except a resonance).

2.2.4 Free film flow in non-conductive medium

Solution of these boundary task is similar to the considered task and does not have any principal impediments. But due to cumbersome expressions obtained the reverse integral transformation may be complicated a lot. Therefore for analysis of the features of considered class of tasks some specific physical situations are considered, such in particular, which are useful for practice of MHD-dispersers. Let for simplicity $k=m$ as before and consider a case of negligibly small influence of the surrounding medium. Both assumptions are physically correct because the dispersers are often placed in a gaseous atmosphere or vacuum and equal wave numbers give more regular film decay into the drops of equal sizes. Then with account of the above similarly yields

$$W_1=d_n\exp(g_n z), \quad n=1,3$$

$$d_n=\frac{(g_2^2+4k^2)(s+2ik)}{(g_1^2-g_2^2)\left[\exp(2g_1\varepsilon)-1\right]}\exp\left[g_1(3-n)\varepsilon\right]\cdot Z(s),$$

$$d_{n+1}=\frac{(g_1^2+4k^2)(s+2ik)}{(g_1^2-g_2^2)\left[\exp(2g_2\varepsilon)-1\right]}\exp\left[g_2(3-n)\varepsilon\right]\cdot Z(s),$$

The Laplace-transform for the free film surface is:

$$\left[\frac{s+2ik}{g_2^2-g_1^2}\left\{\left(\frac{s-2ik}{4k^2}-\frac{3}{\mathrm{Re}_1}\right)\left[g_1(g_2^2+4k^2)+g_2(g_1^2+4k^2)\right]+\right.\right. \qquad (29)$$

$$\left.\left.+\frac{g_1^3(g_2^2+4k^2)+g_2^3(g_1^2+4k^2)}{4k^2\,\mathrm{Re}_1}\right\}+\frac{1}{Fr^2}+\frac{2k^2}{We}\right]Z(s)=-Al\cdot\bar{H}_m^2.$$

By $\mathrm{Re}_1 \gg 1$ the equation (2.2.23) can be simplified keeping the terms of order $1/\mathrm{Re}_1$, which follows to:

$$\left[\frac{s+2ik}{2ik}\sqrt{s^2+4k^2}+\frac{k(s+2ik)^{1,5}}{\mathrm{Re}_1(2ik-5s)(s-2ik)^{1,5}}\right]Z(s)=$$

$$=\left(\frac{1}{Fr^2}+\frac{2k^2}{We}\right)Z(s)+Al\cdot\bar{H}_m^2,$$

where from applying the reverse transformation is:

$$\frac{d\zeta}{dt}+2ik\left(1-\frac{5k}{\mathrm{Re}_1}\right)\zeta+\int_0^t\left[(8k\tau-7i)k^2\frac{8i}{\mathrm{Re}_1}\exp(2ik\tau)+\right.$$

$$\left.-\left(\frac{1}{Fr^2}+\frac{2k^2}{We}\right)J_0(2k\tau)\right]\frac{d\zeta(t-\tau)d\tau}{dt}=Al\frac{d}{dt}\int_0^t J_0(2k\tau)H_m^2(t-\tau)d\tau, \qquad (30)$$

where $J_0(2k\tau)$ is the Bessel function of zero order. Here at differentiation of integral with a variable top limit the additional initial condition is introduced: $t=0,\ \ d\zeta/dt=0$. From here it is visible that at the real k influence of a field on a film perturbation is insignificant, especially at $k\gg1$ as far as the integral in the first part (30) is fast-oscillating and thereof the right part quickly aspires to zero. Namely this part causes a parametric pumping of perturbations of a film (the left part defines the Eigen oscillations).

Solution of the integral-differential equation has been obtained by the method of averaging of integral-differential operator by the first scheme [5]. The averaged equation is got in the form

$$\gamma(t)\frac{d\zeta}{dt}+2ik\left(1-\frac{5k}{\mathrm{Re}_1}\right)\zeta = Al\frac{d}{dt}\int_0^t J_0(2k\tau)H_m^2(t-\tau)d\tau, \quad (31)$$

where is

$$\gamma(t)=1-\frac{1}{2k}\left(\frac{1}{Fr^2}+\frac{2k^2}{We}\right)\int_0^{2kt} J_0(\tau)d\tau+$$
$$+\frac{8k}{\mathrm{Re}_1}i\left[4k(i+2kt)\exp(2ikt)+3{,}5\left(1-\exp(2ikt)\right)\right]\cdot$$

Solution of the averaged equation has the form:

$$\zeta = A(t)\exp\left[2ik\left(1-\frac{5k}{\mathrm{Re}_1}\right)\int_0^t\frac{dt}{\gamma(t)}\right], \quad (32)$$

$$A(t)=\int_0^t\frac{B(t)}{\gamma(t)}\exp\left[2ik\left(\frac{5k}{\mathrm{Re}_1}-1\right)\int_0^t\frac{dt}{\gamma(t)}\right]dt,$$

$$B(t)=Al\frac{d}{dt}\int_0^t J_0(2k\tau)H_m^2(t-\tau)d\tau.$$

3 Practical examples

To consider the solution obtained suppose monotonous intensity of an electromagnetic field, i.e. $k=\kappa i$, where κ- a real number (by κ>0 intensity of a field decreases on *x* and *y* and grows in time, at κ<0- on the contrary). Then function $\gamma(t)$ has only the real values:

$$\gamma(t)=1+\frac{8\kappa}{\mathrm{Re}_1}\left[\frac{4\kappa}{\exp(2\kappa t)}(1+2\kappa t)+3{,}5\left(\exp(-2\kappa t)-1\right)\right]+\frac{1}{\kappa}\left(\frac{\kappa^2}{We}-\frac{1}{2Fr^2}\right)\int_0^{2\kappa t}I_0(\tau)d\tau,$$

where $I_0(\tau)$ is the modified Bessel function of zero order. Character of growing (decreasing) the surface wave is determined by sign and value of the real part of $d\chi/dt$, or by value $\gamma(t)$ here.

The solution (32) is correct for large enough Reynolds numbers $\mathrm{Re}_1 \gg 1$, what is not always carried out in practice, therefore also a case of short waves $(k_r \gg 1)$ is considered and an assessment of the terms in equation (29) is made based on this assumption. Then the approximate integral-differential equation is got as follows

$$\frac{d\zeta}{dt}+2k\frac{8k+i\mathrm{Re}_1}{\mathrm{Re}_1}\zeta = 2\sqrt{2}k\frac{d}{dt}\int_0^t\left\{\left[\frac{16k^2}{\mathrm{Re}_1\sqrt{\pi\mathrm{Re}_1\tau}}+\right.\right. \quad (33)$$
$$\left.\left.-\left(\frac{1}{Fr^2}+\frac{8k^2}{We}\right)\right]\zeta(t-\tau)-Al\cdot H_m^2(t-\tau)\right\}\exp\left[-2k\left(i+\frac{4k}{\mathrm{Re}_1}\right)\tau\right]d\tau,$$

where from seen that equations (30) and (33) are similar by structure, and parametric pumping of the energy perturbations is complicated with presence of the highly-oscillating terms by the external exciting force: by increase of k_r the equation (33) reflects more precisely the investigated physical process and for the wave excitation higher and higher magnetic field is required.

By $\mathrm{Re}_m \ll 1$, correct for many practical tasks, the equation (33) is solved using the averaged integral-differential operator by the second scheme [5]:

$$\frac{d\zeta}{dt}+(8k+i\mathrm{Re})\frac{2k}{A}\zeta = \frac{\sqrt{2}}{A}Ha^2\cdot H_{m0}^2\cdot$$
$$\cdot\left[i-\left(i+\frac{2k}{\mathrm{Re}_m}\right)\exp\left(-\frac{4k^2t}{\mathrm{Re}_m}\right)\right]\exp(-2ikt), \quad (34)$$

where Ha - Hartman number,

$$A=\mathrm{Re}+\frac{\sqrt{2}\,\mathrm{Re}^2}{4k+i\mathrm{Re}}\left(\frac{1}{Fr^2}+\frac{8k^2}{We}\right)-\frac{64k^2\sqrt{k}}{\sqrt{(4k+i\mathrm{Re})\pi}}\cdot$$

Solution of the averaged equation (34) is

$$\zeta = \frac{Ha^2\cdot H_{m0}^2}{\sqrt{2}A\cdot k\cdot D}\left[\exp\left(-4\frac{k^2t}{\mathrm{Re}_m}\right)-1\right]\exp(-2ikt), \quad (35)$$

where $D=1+\left(i-\frac{8k+i\mathrm{Re}}{A}\right)\frac{\mathrm{Re}_m}{2k}$.

By the assumptions made, approximately $D=1$. Solution (35) of averaged equation (34) is close to the solution of the integral-differential equation (33) on the interval $t\in[0,c\cdot\mathrm{Re}]$, where *c=const* and degree of proximity of these solutions raises with growth Re [5].

Accounting in general case $k=k_r+ik_i$, from the obtained solution for vertical EM wave comes

$$H=H_{m0}\exp\left[k_i(t-x-y)+2\frac{k_i^2-k_r^2}{\mathrm{Re}_m}t+ik_r\left(x+y-c^H t\right)\right], \quad (36)$$
$$\zeta=\zeta_2\exp\left[2ik_r\left(x+y-c^H t\right)\right]-\zeta_1\exp\left[2ik_r(x+y-t)\right],$$

where $c^H=1+4k_i/\mathrm{Re}_m$ is phase speed of the wave spreading along the straight line *y=const-x*. Expression (36) shows that in this case the electromagnetic wave causes two waves on a film surface: similar to a wave of the revolting force and different from it spreading with velocity of unperturbed film flow. Their amplitudes are:

$$\zeta_1=\frac{Ha^2\cdot H_{m0}^2}{\sqrt{2}A\cdot k\cdot D}\exp\left[2k_i(t-x-y)\right],$$

$$\zeta_2=\zeta_1\exp\left[\left(k_i^2-k_r^2\right)\frac{4t}{\mathrm{Re}_m}\right]. \quad (37)$$

Thus, the mathematical models developed and solutions obtained on their base for a set of physical situations show that in a linear small-amplitude approach only the waves of two kind are available with the amplitudes (37). By $c^H = 1 \quad (k_i = 0)$ when film flow velocity is zero, it is got $\mathrm{Re}_{ms} = 0$ and there is only one wave. Parametric resonance in a system as shown by (37) takes place under fulfillment of the condition $A \cdot D = 0$, or $\mathrm{Re}_m(8k + i\,\mathrm{Re}) = 2kA$, and further it follows:

$$k^* = 2\frac{(8\pi + i\,\mathrm{Re}_k)^{0,5}}{\pi Ga^2}\cdot \qquad (38)$$

$$\cdot\left\{1 \pm \left[1 + \pi Ga^2\left(\frac{4Be - 1 + \mathrm{Re}_k/(4\pi)}{16\sqrt{2}\pi}\mathrm{Re}_k - \frac{2\pi/Oh^2}{8\pi + i\,\mathrm{Re}_k}\right)\right]^{0,5}\right\},$$

where Ga, Be, Oh - Galileo, Batchelor and Ohnesorge numbers determined by physical properties of media and film thickness (independent of flow regime), $\mathrm{Re}_k = 2\pi\,\mathrm{Re}/k^*$ - Reynolds number defined by the length of corresponding resonance perturbation. Correlation (38) allows establishing the relation between resonant values k^* and corresponding Reynolds number Re_k, as well as the other defining criteria of the system. Then the relation $\mathrm{Re} = k^*\,\mathrm{Re}_k/(2\pi)$ can be established as well.

The formula (38) can be substantially simplified in many important cases, e.g. in MHD-granulation the following estimations are valid: $\mathrm{Re}_k \gg 1$, $Ga^2 \gg 1$, $Be \ll 1$, $Oh^2 \ll 1$. This yields:

$$k^* = \frac{1}{\pi Ga}\left(\frac{2}{Ga}\sqrt{8\pi + i\,\mathrm{Re}_k} \pm \sqrt{\frac{8\pi + i\,\mathrm{Re}_k}{16\sqrt{2}\pi}\mathrm{Re}_k^2 - \frac{8\pi^2}{Oh^2}}\right),$$

The Eigen oscillations with account of (34) are

$$\zeta = \zeta_0 \exp\left[-2(8k + i\,\mathrm{Re})\frac{k}{A}t\right], \qquad (39)$$

where $\zeta_0 = \zeta(0)$. Therefore $\omega_* = -\frac{2k}{A}(8k + i\,\mathrm{Re})\cdot$ Consider Eigen oscillations for real values k and ζ_0, then after estimation of the terms and accounting the assumptions made and the relation $arctg\left(\frac{\mathrm{Re}}{\sqrt{\mathrm{Re}^2 + 16k^2}}\right) \approx \frac{\mathrm{Re}}{\sqrt{\mathrm{Re}^2 + 16k^2}}$ it is available to come at

$$\left(32k^2 - \mathrm{Re}^2\right)\left[2\sqrt{2}k + \mathrm{Re}\left(\frac{1}{Fr^2} + \frac{8k^2}{We}\right)\right] + $$

$$+6\sqrt{2}k^3\,\mathrm{Re}\sqrt{\frac{k}{\pi}}\frac{32k^2 - \mathrm{Re}^2 + 6k\,\mathrm{Re}^2}{\sqrt[4]{16k^2 + \mathrm{Re}^2}} > 0 \qquad (40)$$

based on condition $re(\omega_*) > 0$. The condition (40) answers increase of Eigen fluctuations of system in time (development of film instability). At big enough k condition (40) is not satisfied, i.e., starting from some value of k all subsequent fluctuations of smaller length become fading in time. At $\mathrm{Re} \gg k$ is possible to get the following approximate formula:

$$re(\omega_*) = \frac{2k\,\mathrm{Re}^2\left[\sqrt{2}\left(Ga^2 + 8k^2/Oh^2\right) - 8k\,\mathrm{Re}\right]}{2\left(Ga^2 + 8k^2/Oh^2\right)^2 + 8\sqrt{2}k\,\mathrm{Re}\left(Ga^2 + 8k^2/Oh^2\right) + \mathrm{Re}^4}, \qquad (41)$$

where from follows that decrease of perturbations in a film is possible only on condition that is extremely seldom in practice. Film is unstable, but, apparently from (41), the increase rate of perturbations is small at $\mathrm{Re} \gg 1$ that practically means uselessness of suppression of Eigen modes by parametric excitation of film flow disintegration on the set wavelength.

4 Conclusions by the results obtained

The results obtained may be used for analysis of parametric excitation and suppression of films. Horizontal progressive electromagnetic wave excides on a film surface the wave with amplitude $\sim Al \cdot \mathrm{Re}$; parametric resonance in a film flow by action of constant magnetic field is impossible in contrast to the alternating field; viscous forces are important: application of inviscid approach lead to inaccuracy in calculation of a wave amplitude 3-4 decimal orders; parametric resonance is impossible by means of horizontal electromagnetic wave. The perturbation amplitude in case of $\omega \gg k$, $\omega \gg k^2/\mathrm{Re}$ does not depend on the wave number and is determined only by frequency of a field.

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