TABEL IV: OPTIMAL PMUs PLACEMENT FOR PROPOSED HYPRID OBSERVABILITY

Test System	Critical Buses	NO. of PMUs	PMUs Placement
14-Bus	5, 9, 11	5	2, 6, 7, 10, 13
30-bus	10, 12, 13, 22, 26	12	2, 3, 6, 9, 10, 12, 13, 19, 24, 25, 26, 30
39-bus	5, 12, 14, 21, 33, 37	17	2, 4, 6, 9, 10, 11, 13, 16, 17, 19, 20, 22, 23, 25, 29, 33, 37
57-bus	7, 12, 13, 26, 33, 37, 50, 53, 56, 57	22	2, 6, 12, 13, 19, 22, 24, 26, 29, 31, 32, 33, 36, 39, 41, 44, 47, 49, 50, 52, 54, 56
118-bus	8, 12, 15, 26, 32, 35, 50, 52, 55, 57, 98, 100, 110	39	2, 5, 10, 12, 15, 17, 21, 23, 25, 28, 30, 36, 37, 42, 44, 46, 50, 51, 53, 55, 57, 59, 63, 67, 68, 71, 75, 77, 80, 84, 87, 89, 92, 94, 100, 103, 105, 110, 114

Table IV presents the selected critical busses and the OPP results for the test systems based on the proposed hybrid observability. It is observed that the number of needed PMUs to achieve hybrid observability less than the redundant observability, which are presented in Table III. As a comparison the number of needed PMUs for 57 bus system in single PMU outage scenario (redundant observability) was 33 PMUs. On the other hand, the number of needed PMUs was 22 PMUs for the proposed method. For more investigation Table V summarizes the minimum number of PMUs for single PMUs outage scenario and for hybrid observability.

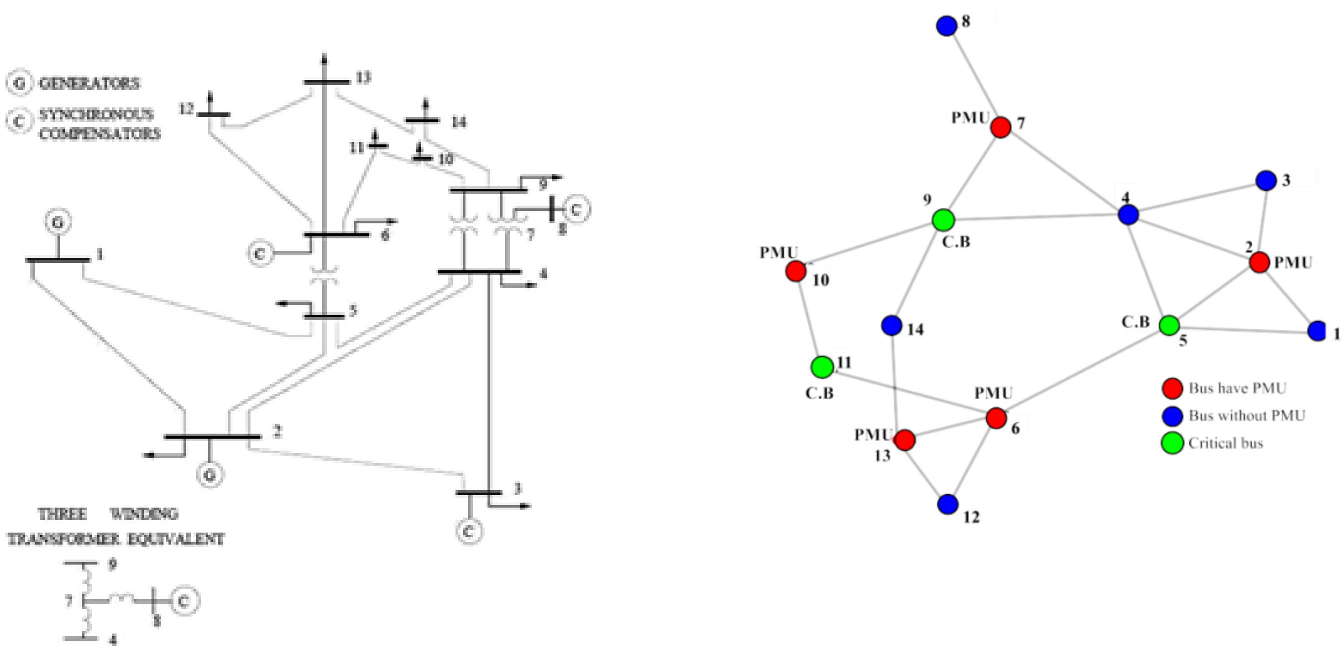

Fig. 1. Real IEEE 14 Bus system and visualization using graph theory with proposed hybrid observability.

TABEL V: COMPARISON BETWEEN REDUNDANT AND HYBRID OBSERVABILITY

System	Redundant Observability	Hybrid Observability	
	NO. of PMUs	List of Critical Buses C_i	NO. of PMUs
14-Bus	9	5, 9, 11	5
30-bus	21	10, 12, 13, 22, 26	12
39-bus	28	5, 12, 14, 21, 33, 37	17
57-bus	33	7, 12, 13, 26, 33, 37, 50, 53, 56, 57	22
118-bus	68	8, 12, 15, 26, 32, 35, 50, 52, 55, 57, 98, 100, 110	39

Fig. 1 presents graph theory visualization for IEEE 14 bus system. where the tables dose not present clear visualization of the OPP results. It observed when PMUs installed at bus 2, 6, 7, 10, and 13 the system is observable and the critical buses, which are 5, 9, and 11 have redundant observability. Fig.2 presents the graph theory visualization for IEEE 30 bus system It observed when PMUs installed at bus 2, 3, 6, 9, 10, 12, 13, 19, 24, 25, 26, and 30 the system is observable and the critical buses, which are 10, 12, 13, 22, and 26 have redundant observability. For example, bus number 10 is a critical bus in IEEE 30 bus system, as in Table IV, and it observable for two

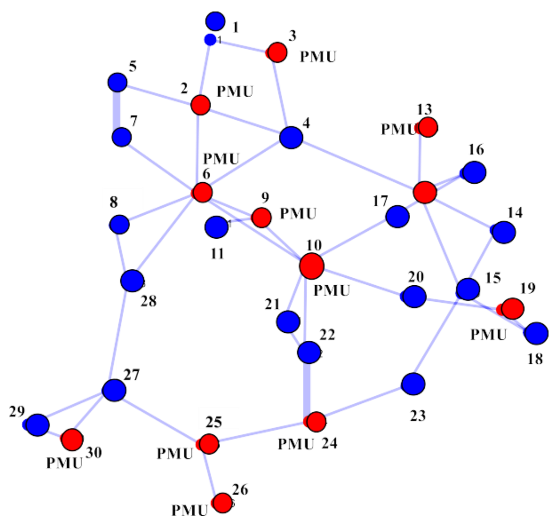

Fig. 2. IEEE 30 Bus system Visualization using graph theory with proposed hybrid observability.

PMUs located at bus 9 and 10. Similarly, for bus 12, 13 it is observable from two PMUs located at bus 12 and 13, bus 22 observable from two PMUs located at bus 24 and 10.

Fig.3 presents the graph theory visualization for IEEE 39 bus system It observed when PMUs installed at 2, 4, 6, 9, 10, 11, 13, 16, 17, 19, 20, 22, 23, 25, 29, 33, and 37 the system is observable and the critical buses, which are 5, 12, 14, 21, 33, and 37, have redundant observability.

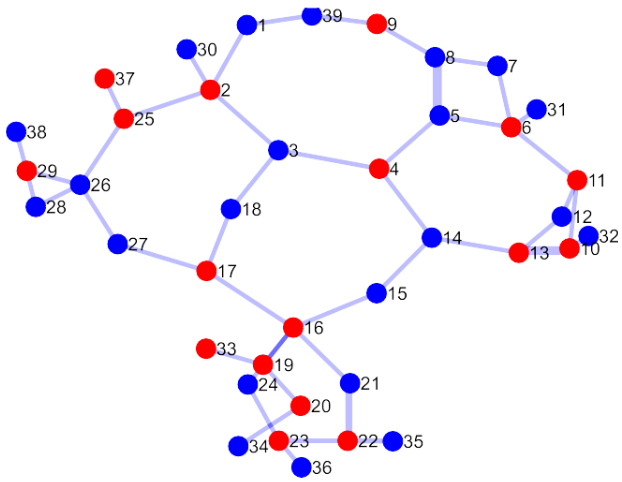

Fig. 3. IEEE 39 Bus system visualization using graph theory with proposed hybrid observability.

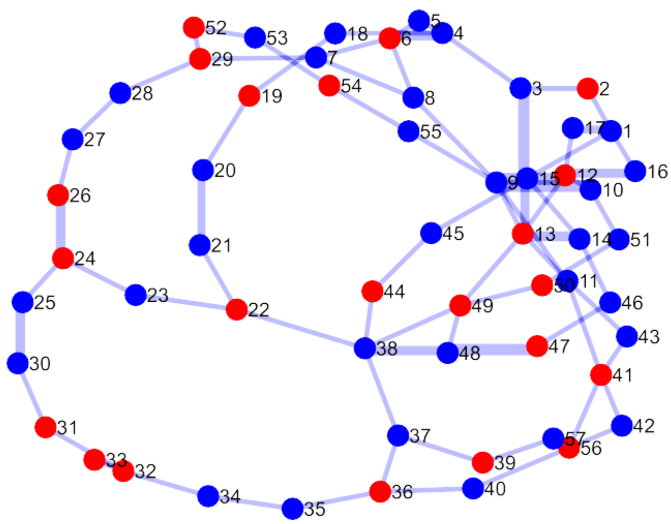

Fig. 4. IEEE 57 Bus system Visualization using graph theory with proposed hybrid observability.

Similarly, the graph theory presentation for 57, and 118 bus system are presented in Fig. 4, and 5, respectively. The IEEE 57 bus system has 22 PMUs and the critical buses, which are 7, 12, 13, 26, 33, 37, 50, 53, 56, and 57 have redundant observability. The IEEE 118 bus system has 39 PMUs and the critical buses, which are 8, 12, 15, 26, 32, 35, 50, 52, 55, 57, 98, 100, and 110 have redundant observability.

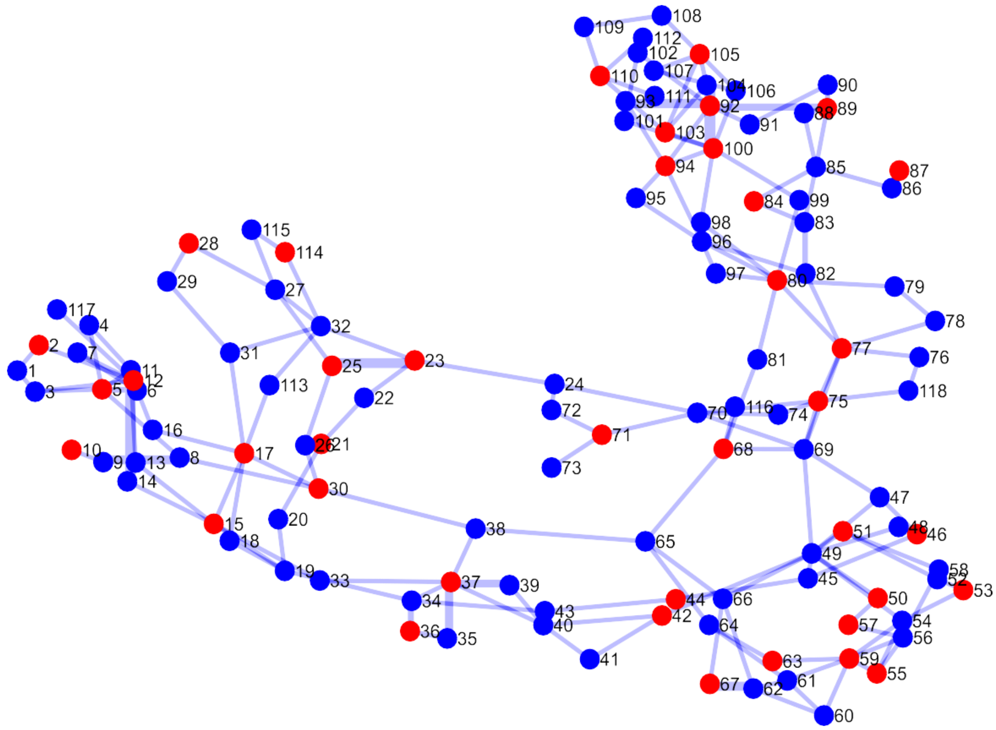

Fig. 5. IEEE 118 Bus system Visualization using graph theory with proposed hybrid observability.

TABEL VI: OPTIMAL PMUs PLACEMENT FOR ONE- AND TWO-LEVEL OPTIMIZATION WITH ZIB CONSEDERING

Test System	NO. of PMUs	One Level Optimization PMUs Placement	 M.R*	Two Level Optimization PMUs Placement
14-Bus	3	2, 6, 9	15	2, 6, 9
30-bus	7	2, 4, 10, 12, 18, 24, 27	35	2, 4, 10, 12, 18, 24, 27
39-bus	9	3, 8, 11, 13, 16, 20, 23, 25, 29	37	3, 6, 8, 13, 16, 20, 23, 25, 29
57-bus	13	1, 4, 9, 14, 19, 22, 25, 28, 32, 37, 50, 53, 56	53	1, 6, 9, 15, 20, 25, 27, 32, 37, 48, 50, 53, 56
118-bus	28	3, 8, 12, 15, 19, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 64, 70, 75, 77, 80, 85, 87, 90, 94, 102, 105, 110	145	3, 8, 12, 15, 17, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 59, 62, 70, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110

M.R*: Sum of measurement redundancy for all system, which is the objective function as in equation (10)

TABEL VII
OPTIMAL PMUs PLACEMENT FOR PROPOSED HYPRID OBSERVABILITY WITHOUT ZIB

Test System	Critical Buses	NO. of PMUs	M.R.C*	PMUs Placement
14-Bus	5, 9, 11	3	15	2, 6, 9
30-bus	10, 12, 13, 22, 26	7	13	2, 4, 10, 12, 19, 24, 30
39-bus	5, 12, 14, 21, 33, 37	9	4	3, 8, 11, 14, 16, 20, 23, 25, 29
57-bus	7, 12, 13, 26, 33, 37, 50, 53, 56, 57	13	19	1, 4, 13, 20, 25, 26, 29, 32, 37, 48, 51, 54, 56
118-bus	8, 12, 15, 26, 32, 35, 50, 52, 55, 57, 98, 100, 110	28	32	3, 8, 12, 15, 19, 21, 27, 28, 32, 34, 41, 45, 49, 52, 56, 62, 65, 70, 75, 77, 80, 85, 87, 91, 94, 101, 105, 110

M.R.C*: Sum of measurement redundancy for critical buses, which is the objective function as in equation (12)

4.4 Maximize The Measurement Redundancy

When the OPP result has more than one solution, the 2nd objective function can be used to maximize the measurement redundancy. Table VI shows the OPP results for one- and two- level optimizations. In two-level optimization, the first level minimizes the required number PMUs for full observability and the second level maximizes the measurement redundancy as in (10) – (11). In small test systems such 14 and 30 bus systems there are no reality different between the one-level and two-level OPP results.

In IEEE 39 bus test system the sum of measurement redundancy increased from 37 to 38. In IEEE 57 and 118 bus system the sum of measurement redundancy increased from 53 and 145 to 55 and 151, respectively. As stated early this method increases the measurement redundancy in global manner. In other word this method unable to increase the measurement redundancy for specific critical buses.

To maximize the measurement redundancy for specific critical buses the proposed objective function (12) is used. The critical buses, OPP result, and sum of measurement redundancy for critical buses are presented in Table VII. It is observed that for

14 bus test system the measurement redundancy for the selected critical buses, which are 5, 9, and 11, was 15. By using the proposed method, the measurement redundancy increased only for predefined critical buses. This helps the monitoring system fault tolerance since some of the buses in power system have characterized with high priority and importance.

4. Conclusions

This Paper proposed a hybrid observability method to enhance the fault tolerance of the monitoring system in electrical power system. the proposed hybrid observability combines the redundant observability and the full observability method, that to tradeoff between the measurement redundancy and the installation cost. Additionally, this paper proposed a new method to maximize the measurement redundancy for specific predefined critical buses in the entire system.

A new method to visualize the optimal PMUs placement results is proposed in this paper, which is based on graph theory Visualization. The proposed Visualization helps designers to choose the proper PMUs placement in graph or map manner.

The proposed methods are tested and investigated using IEEE 14, 30, 39, 57, and 118 bus test system. Integer Linear Programming ILP is used to solve the proposed optimization using optimization toolbox in MATLAB program.

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