An Absorbing Markov Chain Model for
Case – Based Reasoning

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Abstract: **-** Case-Based Reasoning (CBR) is the process of solving new problems based on the solutions of similar past problems. Here a Markov Chain model is constructed for a mathematical description of the CBR process by introducing an absorbing MC on its main steps. A method is also developed with the help of this model for evaluating the effectiveness of CBR systems, accompanied by suitable examples and hints are given for future research on the subject.

Keywords: **-** Case-Based Reasoning (CBR), Markov Chains (MCs), Absorbing MCs, CBR Systems, Artificial Intelligence (AI).

1. Introduction

Case-Based Reasoning (CBR) is a recent theory for problem-solving and learning in computers and people. Broadly construed it is the process of solving new problems based on the solutions of similar past problems.

In the paper at hands a *Markov Chain (MC)* model is developed for the mathematical description of the CBR process. The rest of the paper is organized as follows: Section II contains a brief description of the CBR process and its main steps. In Section III the MC model for CBR is constructed by introducing an absorbing MC on its steps and in Section IV a method is developed for measuring the effectiveness of CBR systems accompanied by suitable examples. Finally, Section V is devoted to conclusions a nd some hints for future research.

2. Case – based reasoning

In CBR the term *problem-solving* is used here in a wide sense, which means that it is not necessarily the finding of a concrete solution to an application problem, it may be any problem put forth by the user. For example to justify, or criticize a proposed solution, to interpret a problem situation, to generate a set of possible solutions, or generate explanations in observable data, are also problem solving situations. A lawyer, who advocates a particular outcome in a trial based on legal precedents, or an auto mechanic, who fixes an engine by recalling another car that exhibited similar symptoms, are using CBR; in other words CBR is a prominent kind of analogy making.

Its coupling to *learning* occurs as a natural by-product of problem solving. When a problem is successfully solved, the experience is retained in order to solve similar problems in future. When an attempt to solve a problem fails, the reason for the failure is identified and remembered in order to avoid the same mistake in future. Thus CBR is a cyclic and integrated process of solving a problem, learning from this experience, solving a new problem, etc.

The CBR systems' expertise is embodied in a collection (*library*) of past cases rather, than being encoded in classical

rules. Each case typically contains a description of the problem plus a solution and/or the outcomes. The knowledge and reasoning process used by an expert to solve the problem is not recorded, but is implicit in the solution.

CBR traces its roots in Artificial Intelligence to the work of Roger Schank and his students at Yale University – U.S.A. in the early 1980's. Scfhank's model of *dynamic memory* [1] was the basis of the earliest CBR systems that might be called case-based reasoners, Kolodner's CYRUS [2] and Lebowitz's IPP [3].

As an intelligent-systems method CBR has got a lot of attention over the last few years, because it enables the information managers to increase efficiency and reduce cost by substantially automating processes. CBR first appeared in commercial systems in early 1990's and since then has been sued to create numerous applications in a wide range of domains including *diagnosis, help-desk, assessment, decision support, design,* etc. Organizations as diverse as IBM, VISA International, Volkswagen, British Airways and NASA have already made use of CBR in fields like customer support, quality assurance, aircraft maintenance, process planning, and many more that are easily imaginable.

CBR has been formalized for purposes of computer and human reasoning as a four steps process. These steps involve:

- **R_1:** *Retrieve* the most similar to the new problem past case.
- **R_2:** *Reuse* the information and knowledge of the retrieved case for the solution of the new problem.
- **R_3:** *Revise* the proposed solution.
- **R_4:** *Retain* the part of this experience likely to be useful for future problem-solving.

More specifically, the retrieve task starts with the description of the new problem, and ends when a best matching previous case has been found. The reuse of the solution of the retrieved case in the context

of the new problem focuses on two aspects: The differences between the past and the current case, and what part of the retrieved case can be transferred to the new case. Usually in non trivial situations part of the solution of the retrieved case cannot be directly transferred to the new case, but requires an adaptation process that takes into account the above differences. Through the revision the solution generated by reuse is tested for success – e.g. by being applied to the real world environment, or to a simulation of it, or evaluated by a specialist – and repaired, if failed. When a failure is encountered, the system can then get a reminding of a previous similar failure and use the failure case in order to improve its understanding of the present failure, and correct it. In other words, there is a transfer from R_3 to R_1 in this case, and the same circle is repeated again. The revised task can then be retained directly (if the R_3 task assures its correctness), or it can be evaluated and repaired again. In the latter case the CBR process remains in fact in step R_3 for two successive phases. The final step R_4 involves selecting which information from the new case to retain, in what form to retain it, how to index the case for better retrieval in future for similar problems, and how to integrate the new case in the memory structure.

According to the above description the flow diagram of the CBR process can be represented as shown in Figure 1.
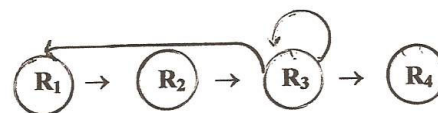

Figure 1: A flow-diagram of the CBR process

For general facts on the CBR process and methods we refer to [4, 5] and their relevant references.

3. The Markov Chain Model

Roughly speaking a MC is a stochastic process that moves in a sequence of phases through a set of states and has a *memory*

of only one state. This means that the probability of entering a certain state in a certain phase, although it is not necessarily independent of previous states, depends at most on the state occupied in the previous phase. This property is known as the *Markov property*.

When its set of states is a finite set, then we speak about a *finite MC.*. For general facts on finite MCs we refer to the book of Kemeny & Snell [6].

In this paper, assuming that the CBR process has the Markov property, we introduce a finite MC having as states the four steps of the CBR process described in the previous section. The above assumption is a simplification (not far away from the truth) made to the real system in order to transfer from it to the *assumed real system*. This is a standard technique of the mathematical modelling process of a real world problem, which enables the formulation of the problem in a form ready for mathematical treatment [7; Section 1).

Denote by p_{ij} the *transition probability* from state R_i to R_j, for i, j=1, 2, 3, 4, then the matrix A=[p_{ij}] is said to be the *transition matrix* of the MC.

With the help of the flow-diagram of Figure 1 one finds that

$$
A = \begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{array}{cccc} R_1 & R_2 & R_3 & R_4 \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p_{31} & 0 & p_{33} & p_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array},
$$

where we obviously have that

$$p_{31} + p_{33} + p_{34} = 1$$

(probability of the certain event).

Further let us denote by $\varphi_0, \varphi_1, \varphi_2, \ldots\ldots$ the successive phases of the above chain , and also denote by

$$P_i = [p_1^{(i)} \ p_2^{(i)} \ p_3^{(i)} \ p_4^{(i)}]$$

the row - matrix giving the probabilities $p_j^{(i)}$ for the MC to be in each of the states R_j, j = 1, 2, 3, 4, at phase φ_i, i = 1, 2, …. . We obviously have again that

$$\sum_{j=1}^{4} p_j^{(i)} = 1.$$

The above row-matrix is called the *probability vector* of the chain at phase φ_i .

From the transition matrix A and the flow diagram of Figure 1 one obtain the *tree of correspondence* among the phases of the MC and its states shown in Figure 2.
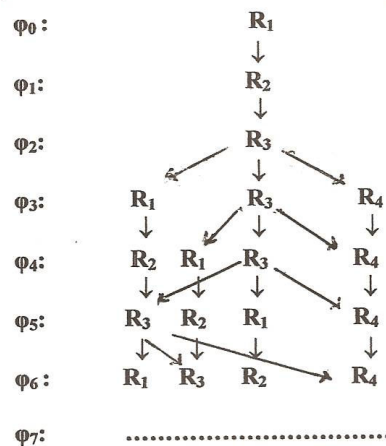

Figure 2: Tree of correspondence among phases and states of the MC

From the above tree it becomes evident that

$$P_0 = [1\ 0\ 0\ 0],\ P_1 = [0\ 1\ 0\ 0],$$

$$P_2 = [0\ 0\ 1\ 0]\ ,\ P_3 = [p_{31}\ 0\ p_{33}\ p_{34}].$$

Further it is well known that

$$P_{i+1} = P_iA,\quad i = 0, 1, 2, \ldots\ .$$

Therefore one finds that

$$P_4 = P_3A = [p_{33}p_{31}\ \ p_{31}\ \ p_{33}^2\ \ p_{34}(p_{33}+1)],$$

$$P_5 = P_4A = [p_{33}^2 p_{31}\ p_{33}p_{31}\ p_{31}+p_{33}^3 \\ p_{34}(p_{33}^2+p_{33}+1)],$$

etc.

Observe now that, when the chain reaches state R_4, it is impossible to leave it, because the solution of the new problem via the CBR approach finishes there. Therefore, we have an *absorbing* MC with R_4 its unique absorbing state.

Applying standard techniques from the theory of absorbing MCs we bring the transition matrix A to its *canonical* (or *standard*) *form* A* by listing the absorbing state first and then we make a partition of A* as follows:

$$A^* = \begin{array}{c} \\ R_4 \\ - \\ R_1 \\ R_2 \\ R_3 \end{array} \begin{array}{cccccc} R_4 & & R_1 & R_2 & R_3 \\ \left[\begin{array}{c} 1 \\ - \\ 0 \\ 0 \\ p_{34} \end{array}\right. & \begin{array}{c} | \\ - \\ | \\ | \\ | \end{array} & \begin{array}{ccc} 0 & 0 & 0 \\ - & - & - \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ p_{31} & 0 & p_{33} \end{array} & & \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right] \end{array}.$$

Symbolically we can write

$$A^* = \begin{bmatrix} I & | & 0 \\ - & | & - \\ R & | & Q \end{bmatrix},$$

where Q stands for the transition matrix of the non absorbing states. Then the *fundamental matrix* of the chain is given by

$$N = (I_3 - Q)^{-1} = \frac{adj(I_3 - Q)}{D(I_3 - Q)},$$

where I_3 denotes the 3X3 unitary matrix, adj $(I_3 - Q)$ denotes the adjoin matrix and $D(I_3 - Q)$ denotes the determinant of I_3-Q. It is recalled that the *adjoin* of a given matrix is the matrix of the algebraic complements of the transpose of the given matrix.

A straightforward calculation gives in our case that

$$N = \frac{1}{1 - p_{31} - p_{33}} \begin{bmatrix} 1 - p_{33} & 1 - p_{33} & 1 \\ -p_{31} & 1 - p_{33} & 1 \\ p_{31} & p_{31} & 1 \end{bmatrix} = [n_{ij}]$$

It is well known then that the entry n_{ij} of N gives the mean number of times at state R_j when the chain is started in state R_i. Therefore, since the present chain is always starting from R_1, the sum

$$t = n_{11} + n_{12} + n_{13} = \frac{3 - 2p_{33}}{1 - p_{31} - p_{33}}$$

gives the mean number of phases of the chain before the absorption. In other words, the mean number of steps for the completion of the CBR process is t+1. It becomes therefore evident that, the bigger is the value of t, the greater is the difficulty encountered for the solution of the given problem via the CBR process. Another indication of this difficulty is of course the total time spent for the completion of the CBR process.

The ideal case is when the CBR process is completed straightforwardly, i.e. without backwards from R_3 to R_1, or stays to R_3, as shown in Figure 1. In this case we have that $p_{31} = p_{33} = 0$ and $p_{34} = 1$, therefore t=3. Therefore, in general is $t \geq 3$.

The following simple example illustrates the above results:

Example: A physician takes into account the diagnosis and treatment of a previous patient having similar symptoms in order to determine the disease and suggest the analogous treatment of the patient in front of him.

If the initial treatment fails to improve the health of the patient, then the physician either revises the treatment (stay to R_3 for two successive phases), or, in more difficult cases, gets a reminding of a previous similar failure and uses the failure case to improve its understanding of the present failure and correct it (transfer from R_3 to R_1). The process is

completed, when the physician succeeds to cure the patient.

The recorded statistical data show that the probabilities of a straightforward cure of the patient as well as of each of the above two reactions of the physician in case of failure are equal to each other.

This means that $p_{13} = p_{33} = p_{34} = \frac{1}{3}$ and therefore t=7, i.e. the mean number of steps for the cure of the patient is 8.

Further, one finds that

$$P_3 = [\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}], P_4 = [\frac{1}{9} \quad \frac{1}{3} \quad \frac{1}{9} \quad \frac{4}{9}],$$

$$P_5 = [\frac{1}{27} \quad \frac{1}{9} \quad \frac{4}{9} \quad \frac{13}{27}]$$

and so on.

Observing, for example, the probability vector P_5 one finds that the probability for the CBR process to be at the step of revision (R_3) in the 6th phase from the beginning is $\frac{4}{9}$, or approximately equal to 44,44%, the corresponding probability to be at the step of retaining the acquired experience (R_4) is $\frac{13}{27}$, or approximately equal to 48,15%, etc. Note that in the last case it is possible that the CBR process has arrived to the absorbing state R_4 in earlier.

Remark: Knowing the exact movements of the physician during the CBR process one can calculate the number of steps needed for its completion directly from the flow-diagram of Figure 1. For example, assume that the initial treatment given to the patient failed to cure him/her and the physician got a reminding of a similar failure in the past in order to correct it. Assume further that the new treatment didn't give the expected results and the physician revised it again succeeding in this way to cure the patient. Under the above assumptions it is easy from Figure 1

to check that the number of steps needed for the absorption is exactly 8.

4. Evaluating the Effectiveness of CBR Systems

The challenge in CBR is to come up with methods that are suited for problem-solving and learning in particular subject domains and for particular application environments. Core problems addressed by CBR research can be grouped into five areas: Representation of cases, and methods for retrieval, reuse, revision and retaining the acquired experience. A CBR system should support the problems appearing in the above five areas. A good system should support a variety of retrieval mechanisms and allow them to be mixed when necessary. In addition, the system should be able to handle large case libraries with the retrieval time increasing linearly (at worst) with the number of cases.

Let us consider a CBR system including a library of n past cases and let t_i, as it has been calculated in the previous section, be the mean number of steps for the completion of the CBR process for case c_i, i=1,2,…,n. Each t_i could be stored in the system's library together with the corresponding case c_i. We define then the ***system's effectiveness***, say t, to be the mean value of the t_i's of its stored cases, i.e. we have that

$$t = \frac{\sum_{i=1}^{n} t_i}{n} \quad .$$

The more problems are solved in future applications through the given system, the bigger becomes the number n of the stored cases in the system's library and therefore the value of t is changing. As n increases it is normally expected that t will decrease, because the values of the t_i's of the new stored cases would be decreasing. In fact, the bigger is n, the better would be the chance of a new case to "fit" well (i.e. to have minor differences) with a known past case, and therefore the less would be the

difficulty of solving the corresponding problem via the CBR process. Thus we could say that a CBR system **behaves well** if, when n tends to infinity, then its efficiency t tends to 3.

Example: Consider a CBR system that has been designed in terms of the Schank's model of dynamic memory for the representation of cases [1]. The basic idea of this model is to organize specific cases which share similar properties under a more general structure called a **generalized episode (GE)**. During storing of a new case, when a feature of it matches a feature of an existing past case, a new GE is created. Hence the memory structure of the system is in fact dynamic, in the sense that similar parts of two case descriptions are dynamically generalized in to a new GE and the cases are indexed under this GE by their different features.

In order to calculate the effectiveness of a system of this type we need first to calculate the effectiveness of the GE's contained in it. For example, assume that the given system contains a GE including three cases, say c_1, c_2 and c_3. Assume further that c_1 corresponds to a straightforward successful application of the CBR process, that c_2 is the case described in the example of Section III, and that c_3 includes one "return" from R_3 to R_1 and two "stays" to R_3. Then $t_1=3$ and $t_2=7$, while for the calculation of t_3 observe that $p_{31}=p_{34}=\frac{1}{4}$ and $p_{33}=\frac{1}{2}$, therefore $t_3=8$. Thus the efficiency of this GE is equal to t $= \frac{3+7+8}{3} = 6$.

Note that a complex GE may contain some more complex GE's containing simpler ones [8; Figure 3]. In this case we need to calculate the effectiveness of the complex GE by considering all its cases only once, regardless if they belong or not to more than one of the simpler GE's contained in it. Finally, the system's effectiveness is the mean value of the effectiveness of all its GE's.

Remark: An alternative approach for the representation of cases in a CBR system is the **category and exemplar model** applied first to the PROTOS system by Porter and Bareiss [9].. In this model the case memory is embedded in a network of categories, cases and index pointers. Each case is associated with a category. Finding a case in the case library that matches an input description is done by combining the features of the new problem case into a pointer to the category that shares most of these features. A new case is stored in a category by searching for a matching case and by establishing the appropriate feature indices. The process of calculating the effectiveness of a system of such type is analogous to the process described in the above example, the only difference being that one has to work with categories instead of GE's. In a similar way one may calculate the effectiveness of systems corresponding to other case memory models including Rissland and Ashley's HYPO system in which cases are grouped under a set of domain-specific dimensions [10], the MBR model of Stanfill and Waltz [11], designed for parallel computation rather than knowledge-based matching, etc.

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