

# Location Problems in Wireless Sensor Network for Improving Its Reliability and Performance

DENIS MIGOV

Institute of Computational Mathematics and Mathematical Geophysics of SB RAS  
Laboratory of Dynamical Processes Modeling in Information Networks  
Lavrentieva 6, Novosibirsk  
RUSSIA  
mdinka@rav.sccc.ru

*Abstract:* The wireless sensor networks (WSNs) are considered in this paper. Approach of sink nodes placement in WSNs which maximizes its reliability is proposed. For WSNs with wireless chargers, both mobile or static, we propose technique for searching the optimal points for a simultaneous omnidirectional wireless charging of a group of sensors. Corresponding problem is stated as P-center location problem.

*Key-Words:* Wireless Sensor Network, Network Reliability, Wireless Charging, Charger Deployment, P-median, P-center, Location Problem.

## 1 Introduction

The wireless sensor networks gain a lot of attention due to many applications. WSN usually consists of a considerable amount of micro sensors, which monitor required index and can establish a wireless communication with each other for delivering gathered data. These data may be temperature, pressure, speed, etc. Obtained data is collected by sink nodes.

As a rule, used sensors are cheap enough, so they are subject to failures. Random graph models have been extensively studied for reliability problems of networks, including the WSNs. In our previous paper [1] we've offered the new reliability index for WSNs. It is assumed, that WSN has perfectly reliable links and imperfect nodes, which fail statistically independently. An operational probability is associated with every node. A special feature of our model is that the network contains initially excessive amount of nodes to provide a proper functioning of the network. For example, wireless sensor network may work acceptably even if some amount of nodes fails. In other words, it works until there is a sufficient number of workable nodes which are connected to any sink node. Another requirement for the network operation is the connectivity of sink nodes through workable nodes. We define the reliability of such a network as probability of the proper functioning in the above meaning. In the present paper we propose the heuristic method for sinks placement in WSN for optimizing its reliability.

At the same time, WSNs experience difficulties due to finite electrical batteries in sensors. One ap-

proach to overcome energy problems of WSNs is the wireless energy transfer [2, 3], which gives an alternative opportunity to prolong the wireless sensor network lifetime [4–7].

In this study, we offer some optimization techniques for WSNs with wireless chargers which can be both mobile or static. More specifically, we state problem of the search for the points which are most suitable for the simultaneous omnidirectional wireless charging of a group of sensors. This problem is quite similar to the problem of  $p$ -center location in a graph and may be solved by appropriate methods.

For the WSNs with mobile chargers, these optimal points should be visited by chargers. This approach may significantly reduce the path of a mobile charger when traversing places for recharging sensors. Further, the optimal path planning for mobile chargers in order to reduce the time of bypass can be performed by one of the known methods [8–10]. For WSNs with static wireless chargers these optimal points are useful for the chargers deployment.

## 2 Reliability of WSNs

### 2.1 Definitions and Notations

We model the WSN by an undirected probabilistic graph  $G = (V, E)$  whose vertices represent the nodes and whose edges represent the links. We assume that each node succeeds or fails independently with an associated probability. Further on we refer to this probability as node reliability. We suppose that the links are perfectly reliable. We use following notations for the

number of network elements:  $|V| = N$ ,  $|E| = M$ .  $K$  — the specific set of nodes that correspond to the sink nodes of ad hoc network. Elements of this set we call terminals. It is assumed that  $K$  contains at least one element. As a rule, sink nodes are perfectly reliable. We have also an integer  $T$  such that  $1 \leq T \leq N - |K|$ . It is assumed that the ad hoc network is functioning properly if sink nodes are connected with each other and at least  $T$  nodes are workable and connected to any sink node.

Let us introduce some definitions.

An elementary event  $Q$  is a special realization of the graph defined by existence or absence of each node. By  $V_Q$  we denote the set of all existing nodes in  $Q$  that are not sink nodes.

The probability of an elementary event equals to the product of probabilities of existence of operational nodes times the product of probabilities of absence of faulty nodes.

An elementary event  $Q$  is called successful if:

- all sink nodes are connected with each other by nodes from  $V_Q$ ;
- at least  $T$  nodes from  $V_Q$  are connected to any sink node.

Otherwise, it is called unsuccessful.

An arbitrary event (an event is a union of elementary events) is called successful if it consists only of the successful elementary events.

An event is called unsuccessful if it consists only of the unsuccessful elementary events.

We define the reliability of the ad hoc network as the probability of the event consisting of all successful events and of them only. We denote it by  $R_{K,T}(G)$ . Further on, under the network reliability will be assumed this index, unless stated otherwise. In other words, the introduced reliability index is the probability that sink nodes are connected with each other and at least  $T$  nodes are workable and connected to any sink node.

It is obvious that problem of proposed reliability measure calculation is NP-hard since it includes the problem of  $k$ -terminal reliability calculation, which is known to be NP-hard [11].

## 2.2 Method for Reliability Calculation and Optimal Sink Nodes Placement

Calculation of  $R_{K,T}(G)$  may be done by the well-known factoring method [12] which has been modified for this purpose. This technique partitions the probability space into two sets, based on the success or failure of one network's particular element (node or link). The chosen element is called factored element.

So we obtain two subgraphs, in one of them factored element is absolutely reliable (branch of contraction) and in second one factored element is absolutely unreliable that is, absence (branch of removal). The probability of the first event is equal to the reliability of factored element; the probability of the second event is equal to the failure probability of factored element. Thereafter obtained subgraphs are subjected to the same procedure. The law of total probability gives expression for the network reliability, in the general case for system  $S$  with unreliable elements it takes the following form:

$$R(S) = r_e R(S|e \text{ works}) + (1 - r_e) R(S|e \text{ fails}), \quad (1)$$

where  $R(S)$  is the reliability of the  $S$  and  $R(S|e \text{ works})$  is the reliability of the system  $S$  when the element  $e$  is in operation,  $R(S|e \text{ fails})$  is the reliability of the system  $S$  when the element  $e$  is not in operation,  $r_e$  is reliability of element  $e$ .

Calculation of  $R_{K,T}(G)$  may be performed in the same way, but it is more complicated because of the need to fulfill two conditions: connectivity of terminals and the availability of a sufficient number of other nodes attached to them. It is convenient to choose as a factoring element one of the nodes adjacent to any terminal or the node adjacent to any already passed with reliability 1. Thus, it can be accumulating the number of nodes connected to chosen terminal. We keep the name "branch of contraction" and "branch of removal" despite the fact the process of contraction in graph is optional, as well as the process of removal. Let us consider separately the branch of contraction and the branch of removal of such process.

**Branch of contraction.** In this branch the number of nodes attached to the terminal is increasing. If its number reached  $T$  it is necessary to check connectivity of all terminals via absolutely reliable nodes. If the check is successful then a final subgraph is received which corresponds to a successful event. If the check is unsuccessful then further factoring procedure continues only in order to ensure connectivity of the terminals. In other words, we calculate the probability of terminals connectivity in graph with unreliable nodes. It is convenient to use the method [11] for this purpose.

**Branch of removal.** In this branch the number of nodes which in the process of further factoring could potentially be absolutely reliable is decreasing. Therefore, the event corresponding to the graph obtained by this branch can be authentically unsuccessful due to the disconnectedness of terminals or impossibility of reaching the required number of nodes attached to the terminals. It is suitable initially to check the first con-

dition, that is, check whether the terminals are connected via non-zero reliability nodes. If the check is successful then we should check the second condition: the number of pending nodes (with the reliability of 0 up to 1) should be sufficient to achieve the required number of nodes attached to the terminals, that is, the number  $T$ . If the pending nodes are just enough to ensure this condition, all they become absolutely reliable. Thus successful subevent fully stands out from the considering event. It remains only to take into account that in order to obtain probability of this event it is required to multiply the value of reliability of pending nodes.

The pseudocode of the proposed algorithm is described in detail in [1]. The optimization problem for improving WSN reliability can be formulated as optimal sink nodes placement problem: for given  $G$  graph and integer  $k, T$  find  $K \subseteq V$  such that  $|K| = k$  and  $R_{K,T}(G)$  takes the maximum value.

For solving of the stated problem we offer the greedy algorithm, which is well-known heuristic technique. However, calculation of WSN reliability still demands enormous computational effort, since this problem is NP-hard. Nevertheless, the proposed approach can operate with medium scale WSNs, as it is shown in the next section. For a large scale ones it is possible to adapt the technique proposed in [13, 14]. This technique can handle with various reliability measures, including the proposed one. It allows to decide the feasibility of a given WSN without performing exhaustive calculation of reliability.

### 2.3 Case Studies

Let us show how the proposed algorithm works. The test problem was the problem of optimal sink nodes placement in ad hoc network with unreliable nodes in order to obtain the most reliable version. We consider  $5 \times 5$  grid network topology, supposing that all sink nodes are perfectly reliable and reliabilities of the other nodes equal to each other. Our goal is to place in the nodes of this grid three sink nodes to maximize the probability of access to sinks for at least  $T$  of other nodes in the network with the condition of sinks connected to each other. Three values for the indicator  $T$  were considered: 10, 15, 20. For each value of  $T$ , the solution was searched for three values of node reliability: 0, 1, 0, 5, 0, 9. We have solved this problem by exhaustive search of all combination of nodes; for each combination its reliability was calculated. The calculation time was about three hours; PC with Intel Core Duo 2,93 GHz was used for testing. On the other hand, we obtain the same optimal results by greedy search algorithm within a several minutes.

If  $T = 15$ , we have obtained different optimal

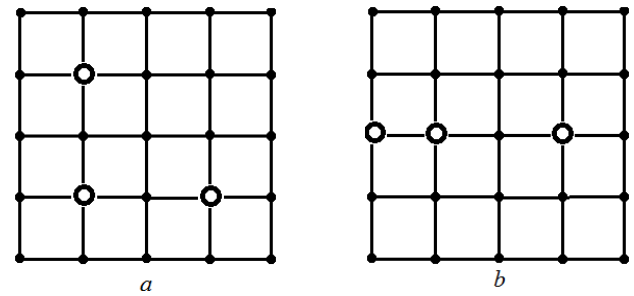


Figure 1: Optimal sink nodes placements for  $T = 15$  for  $p = 0, 1, 0, 5$  (a) and for  $p = 0, 9$  (b)

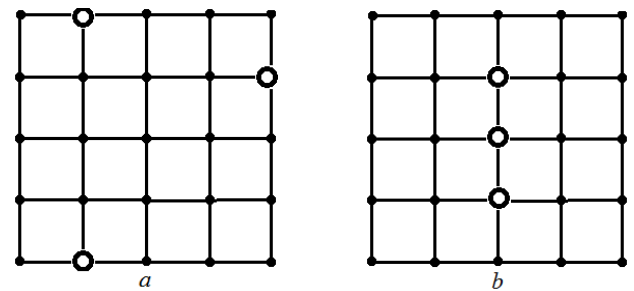


Figure 2: Optimal sink nodes placements for  $T = 20$  (a) and for  $T = 10$  (b)

sink nodes placements for different values of node reliability (Fig. 1).

If  $T = 10, 20$ , optimal sink nodes placements are not different for different values of node reliability (Fig. 2).

It is obvious that any sink nodes placement, which is isomorphic to optimal, is optimal, too. For both placements shown in Fig. 1 there are 4 isomorphic combinations of nodes. Placement shown in Fig. 2 (a) has 32 isomorphic combinations and placement shown in Fig. 2 (b) has 2 isomorphic combinations.

## 3 Location Problems for WSNs with Wireless Chargers

### 3.1 Wireless Energy Transfer and Its Application to WSNs

As shown in [5], there are three ways of wireless energy transfer: inductive coupling, electromagnetic radiation and magnetic resonant coupling. Among them, the technology of magnetic resonant coupling is most useful for applications in WSNs. This technology, developed by Kurs et al. [2], is based on the principle of resonant coupling. In accord with this technology, energy can be transferred from a source coil to a receiver coil which are operating at the same resonance frequency. The efficiency of energy trans-

fer depends on the size of effective area of the receiver coil in the magnetic field of the transmitter coil.

Further, Kurs et al. [3] developed an enhanced magnetic resonant coupling technology that allows charging multiple devices simultaneously. The efficiency is high within several-meter range. Moreover, there are no requirements of a line of sight and appropriate weather conditions.

The above mentioned technologies may be applied to solving the energy problems in WSNs with the use of wireless energy chargers (WC). One approach is using mobile energy chargers (MWC) for charging sensors [4, 8–10]. There are two ways of using this technology. The first one is the so-called offline scenarios [4, 9, 10] when nodes are charged in a periodic and deterministic manner. The second way of utilization of mobile energy chargers is the on-demand energy replenishment in WSNs [8].

The static wireless energy chargers (SWC) utilization in WSNs has been subject to research, too [15]. The authors [15] study the WSNs with wireless static chargers equipped with 3D-beamforming directional antennas. It is assumed that chargers are deployed at grid points at a fixed height, and the charging space of a charger equipped with a directional antenna is simulated by a cone. The authors solve the optimization problem of how to deploy as few as possible chargers in a WSN to cover all sensor nodes to make the network sustainable. Two greedy algorithms for solving this optimization problem were proposed.

### 3.2 Definitions and Notation

Let us assume that a wireless sensor network is distributed on some two-dimensional bounded plane area  $\Omega$ . Also, we suppose that due to the terrain features, certain subareas are prohibited for visiting by mobile wireless chargers and for placing a static wireless charger on them. We denote the union of these subareas as  $P$ .

We denote a set of sensors as  $S$ . A geometrical distance between the two objects  $x$  and  $y$  on  $\Omega$  is denoted as  $d(x, y)$ . As such objects, we consider sensors and wireless chargers both mobile and static.

We denote as  $R_{WCD}$  a maximum distance at which the wireless power transfer can be performed. That is, the wireless charger WC can recharge a sensor  $s$  within a reasonable time iff  $d(WC, s) \leq R_{WCD}$ . Otherwise, the wireless charging is completely ineffective.

### 3.3 Optimal Points for a Wireless Charging of a Group of Sensors

One of the primary problems for the WSNs with MWC is the optimal path planning for mobile chargers for reducing the bypass time. With the use of the enhanced magnetic resonant coupling technology [3] it is possible to wireless charge multiple devices at the same time. Thus, MWC may travel inside WSN and visit the points which are most suitable for the simultaneous wireless charging of a group of sensors. In case of static chargers, SWCs may be placed at these points. So, we arrive at the problem of finding a sufficient amount of these points depending on the effective wireless charging distance and their locations. Having these optimal points, we can further solve the path planning problem by one of the existing methods. For example, we may use methods based on finding the shortest Hamiltonian cycle [4, 5], the optimal offline path planning [9, 10], the on-demand charging method [8], and others methods.

The approach proposed may significantly reduce the mobile charger path when traversing places for recharging sensors. For instance, when using the methods from [4, 5], the Hamiltonian cycle is sought for in a graph of a smaller dimension than the initial one. Although the on-demand charging method [8] originally operates with direct-contact – based energy transferring technologies, it may also be used for the on-demand wireless charging with minor changes. Instead of visiting each sensor for the direct-contact energy transfer, MEC may visit the above-mentioned points which are most suitable for the simultaneous wireless charging of a group of sensors. Thus, a few sensors may be charged per one request therefore requests the total number of requests is reduces. For the offline path planning methods [9, 10], the application of the approach proposed is exactly the same: visiting locations which are suitable for the simultaneous wireless charging of a few sensors instead of visiting each sensor separately.

We formulate the location problem of points for visiting by WC as follows: find the subset  $L \subseteq \Omega \setminus P$  with a minimum amount of elements such that

$$\forall s \in S \quad \exists l \in L : d(s, l) \leq R_{WCD}. \quad (2)$$

We propose the discrete approach for solving the stated problem. In other words, it is the search among variants from a certain discrete subset  $V \subseteq \Omega \setminus P$ . The subset  $V$  may be formed by dividing  $\Omega$  into squares of the same size  $h$ , so we obtain a discrete coordinate grid 1. Points of grid lines intersections may be taken as elements of the auxiliary set  $W$ . The set  $V$  is obtained from the set  $W$  by elimination of elements from the set  $P$ :  $V = W \setminus P$ .

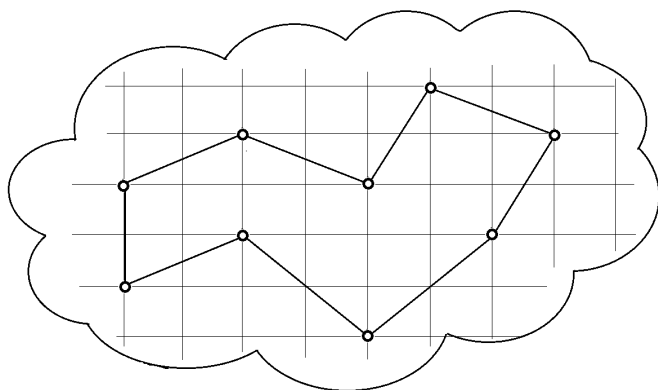


Figure 3: Example of area's discretization, points location for wireless charging, and path for a mobile charger

Let us define the graph  $G = (S \cup V, E)$ , where  $S \cup V$  is a set of vertices and  $E$  is a set of edges. An edge between two vertices exist iff one of vertices is from  $V$ , another is from  $S$ , and the distance between them is shorter or equal to  $R_{WCD}$ .

Thus, the formulated problem is quite similar to that of  $p$ -center location in the graph  $G$ . This well-known graph problem is how to locate  $p$  centers (service nodes) at graph vertices (served nodes) in order to reduce a distance from the most remote vertex to the nearest center. However, there are certain differences between the  $p$ -center location problem and problem (2). First, we initially do not know the exact value of  $p$ . Second, centers should be placed at the nodes from  $V$ , while the nodes from  $S$  are considered as the ones being served. Third, the distance between any sensor and the nearest point for a visit by WC should be shorter or equal to  $R_{WCD}$ .

However, despite these differences, known methods for solving the  $p$ -center problem [16, 17] with minor changes can be successfully applied for solving the stated problem (2). Although the exact value of points for visiting by MEC is initially unknown, problem (2) in the discrete case has obviously a decision since condition (2) is satisfied if we assume that  $h = R_{WCD}$  and  $p = |S|$ . So, we start the solution process by solving  $p$ -median problem by greedy algorithm for mentioned above  $h, p$  values. In the second step we reduce  $p$  and solve the obtaining  $p$ -median problem, and so on. The process stops when  $p$ -median problem for the next  $p$  has no solution. We may notify it if the greedy algorithm in the preliminary step can't generate any initial solution for long enough period of time.

## 4 Conclusion

This study presents some optimization technique for improving reliability and performance of WSNs. For WSNs whose nodes are subject to random faults, we offer the method of sink nodes placement in order to maximize WSN reliability. This method is based on our previous results in WSNs reliability analysis and greedy method.

For the WSNs with wireless chargers we have present methods which improves efficiency of utilization of wireless chargers. We have formulated the problem of the search for the points which are most suitable for the simultaneous omnidirectional wireless charging of a group of sensors. The problem is quite similar to the problem of  $p$ -center location in a graph and may be solved by appropriate methods. This approach may significantly reduce the path of a mobile charger when traversing the places for recharging sensors. For the WSNs with static wireless chargers these optimal points are useful for the chargers deployment.

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