The Box Fractal Sets Based on Bilinear Transformation Iterated Function System

HAI-LONG HU, HUI-LAN FANG
Department of Application Mathematics,
Zhejiang A&F University, Hangzhou city,
P.R.China
E-mail: huhailonga@163.com

Abstract: - For solving the problem which typical Iterated Function System(IFS for short) attractor is very "regular",we attempt to use bilinear transformation IFS through geometric approach, and use it to generate a kinds of fractal set: box fractal set. The result shows that bilinear transformation IFS can generate attractors with a higher degree of flexibility、reality, and a higher modeling capability. It's important to research nonlinear interactive fractal modeling algorithm.

Key-Words: - fractal, bilinear transformation, iterated function system, Attractor

1 Introduction

The books fractal geometry of nature[1] by Mandelbrot and Fractal everywhere [2] by Barnsley, and a great deal of later work show that the mathematical language of fractal geometry is well-suited for describing natural phenomena.In molecular biology field,[3-8] have used fractal theory to research nucleus,DNA sequences and complete genomes,characterisation of structure of macromolecules like carbohydrate and so on.Michel L. Lapidus etc. Gained pointwise tube formulas for fractal sprays by “tubular zeta function”[9],and used it to generate some tilings.To generate fractal attractor fast,Tomasz Martyn presented a novel approach for realistic real-time rendering scenes consisting of many affine IFS fractals[10]; M.R.Browna presented a highly efficient algorithm for the generation of random fractal aggregates[11]. fractal interpolation was studied in [12-14].

Iterated Function System(shorted as IFS) is an important topic in fractal, and has become a powerful tool for generating attractors. Especially it is useful for image compression and natural scene simulation.Typical IFS consists of contractive transformations which are often linear transformations.Linear transformation only can map line to line, parallelogram to another parallelogram,so typical IFS attractor is not natural and inflexible.For solving this problem, Eduard groller and CHEN Lian studied nonlinear IFS respectively[15-17], LIU Shu-qun et al. studied IFS based on polynomial transformation, and generated fractal attractors[18], LUO yan et al. improved L-system and used Bezier curve to simulated bamboo trunk bending by gravity[19];ZHOU wen-li studied plant modeling based on B-spline curve[20]; their results are all natural and elaborate.

In this paper, we studied a kinds of fractal sets based on bilinear transformation iterated function system. The experimental result shows that,this method not only can solve the difficulty of getting IFS code ,but also can generate much more realistic and natural fractal sets.

2 Iterated Function System Theory [21]

If (X, d) is a metric space, then $(H(X), h(d))$ is the Hausdorff metric space.

Theorem1: Suppose that $\omega: X \to X$ is a continuous mapping in (X, d) space,then $\omega: H(X) \to H(X)$.

Theorem2: Suppose that $\omega: X \to X$ is a contractive mapping in (X, d) space, with contractive factor $s\,(0 \le s < 1)$, then $\omega: H(X) \to H(X)$,

$$\omega(B) = \{\omega(x): x \in B\},\ \forall B \in H(X)$$

is a contractive mapping with contractive factor s in $(H(X), h(d))$.

Theorem3: Suppose (X, d) is a metric space, $\{\omega_n : n = 1, 2, 3, \cdots, n\}$ is a collection of contractive mapping in $(H(X), h(d))$, the contractive factor of ω_n is s_n, define mapping W:

$$W(B) = \omega_1(B) \bigcup \omega_2(B) \bigcup \cdots \bigcup \omega_N(B) = \bigcup_{n=1}^{N} \omega_n(B),\ \forall B \in H(X)$$

then $W: H(X) \to H(X)$ and W is a contractive mapping with contractive factor

$$s = \max\{s_n : n = 1, 2, 3, \cdots, N\}.$$

Definition 1: (X, d) denotes a complete metric space with metric d. Let $\omega_n : X \to X, n = 1, 2, \cdots, N$, be a collection of continuous functions. The pair $\{X, \omega_n, n = 1, 2, \cdots, N\}$ is called an iterated function system(IFS).If,in addition, there exists a constant $0 \le s < 1$, such that

$$\max_{1 \le n \le N} \mathrm{d}(\omega_n(x), \omega_n(y)) \le s \cdot d(x, y)$$

then $\{X, \omega_n, n = 1, 2, \cdots, N\}$ is called a hyperbolic iterated function system.The constant s is referred to as the contractivity of the IFS $\{X, \omega_n, n = 1, 2, \cdots, N\}$.

Associated with the collection of functions $\omega_n : X \to X, n = 1, 2, \cdots, N$ is a set-valued mapping W from the hyperspace $H(X)$ of nonempty compact subsets of (X, d) into itself. More precisely, $W(B) = \bigcup_{n=1}^{N} \omega_n(B), \forall B \in H(X)$.

There exists a natural metric on $H(X)$, called the Hausdorff metric, which completes $H(X)$.

Theorem 4: When $\{X, \omega_n, n = 1, 2, \cdots, N\}$ is a hyperbolic IFS with contractivity s, it is well-known that ω_n is a contraction on the complete metric space $(H(X), h(d))$ with the same contractivity s:

$$h(W(B), W(C)) \le s \cdot h(B, C), \forall B, C \in H(X)$$

then there exists a unique set $A \in H(X)$ called the attractor of the hyperbolic IFS, such that $A = \lim_{n \to \infty} W^n(B)$, and $W(A) = A$.

3 IFS Based on Bilinear Transform-ation

3.1 Bilinear Transformation

Bilinear transformation is

$$p = (1-\xi)(1-\eta)p_0' + \xi(1-\eta)p_1' + \xi\eta p_2' + (1-\xi)\eta p_3' \qquad (1)$$

It can map parallelogram to arbitrary quadrilateral [22].

If $\xi = 0$,equation(1) is $p = p_0' + (p_3' - p_0')\eta$; (2)

If $\eta = 0$,equation(1) is $p = p_0' + (p_1' - p_0')\xi$. (3)

Equations (2) and (3) are all linear transformation.Of course, linear transformation is the special case of bilinear transformation.

3.2 IFS Based on Bilinear Transformation

Definition 2.In IFS $\{X, \omega_i, i = 1, 2, \cdots, n\}$, if ω_i $(i = 1, 2, 3 \cdots, n)$ is bilinear transformation, then this IFS is called IFS based on Bilinear transformation, denoted as BIFS $\{X, \omega_i, i = 1, 2, \cdots, n\}$.

This paper did not discuss the convergence of BIFS.That is if exist attractor or not, and what conditions must satisfy, BIFS exists an attractor, will be discussed in next work.

3.3 Interactive Designing of Bilinear Transformation

We use four-point transformation represent bilinear transformation, which can map standard square to arbitrary quadrilateral ,shown in Fig.1,of course, "L, R, B, T"are the corresponding four borders of the standard square.

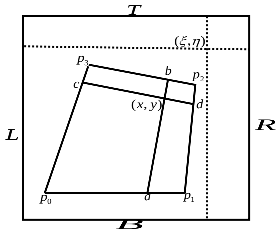

Fig.1 Bilinear Transformation

Written $W = R - L$, $H = T - B$,so point (x, y) location proportion are

$$\alpha = \frac{\xi - L}{W}, \qquad \beta = \frac{\eta - B}{H},$$

Corresponding fix-proportion points in border side are

$$a = p_0 + \alpha(p_1 - p_0), \qquad b = p_3 + \alpha(p_2 - p_3),$$

$$c = p_0 + \beta(p_3 - p_0), \qquad d = p_1 + \beta(p_2 - p_1).$$

Line $\overline{ab}$ equation is

$$y = a_y + \frac{b_y - a_y}{b_x - a_x}(x - a_x)$$,noted as $y = Ax + B$,

Line $\overline{cd}$ equation is $y = c_y + \frac{d_y - c_y}{d_x - c_x}(x - c_x)$,noted as $y = Cx + D$.

From $\begin{cases} y = Ax + B \\ y = Cx + D \end{cases}$, get $X = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{B - D}{C - A} \\ \frac{BC - AD}{C - A} \end{pmatrix}$.

We must use division to get X ,so we first get a , b or c , d points,and then determine X according to ratio. So (for example first getting a , b):

$$X = a + \beta(b - a) = p_0 + \alpha(p_1 - p_0) + \beta[p_3 + \alpha(p_2 - p_3) - p_0 - \alpha(p_1 - p_0)]$$
$$= p_0 + \alpha(p_1 - p_0) + \beta(p_3 - p_0) + \alpha\beta(p_0 - p_1 + p_2 - p_3)$$
$$= (1-\alpha)(1-\beta) \cdot p_0 + \alpha(1-\beta) \cdot p_1 + \alpha\beta \cdot p_2 + (1-\alpha)\beta \cdot p_3$$

So

$$X = \frac{(R-\xi)(T-\eta)}{W \cdot H} p_0 + \frac{(\xi - L)(T-\eta)}{W \cdot H} p_1 + \frac{(\xi - L)(\eta - B)}{W \cdot H} p_2 + \frac{(R-\xi)(\eta - B)}{W \cdot H} p_3.$$

If $L=B=-1, R=T=1$,above equation is bilinear transformation,substitute (x,y) for (ξ,η) ,

substitute $X^{'}(x^{'},y^{'})$ for X , above equation is

$$\begin{pmatrix} x^{'} \\ y^{'} \end{pmatrix} = \frac{1}{4}\begin{pmatrix} (1-x)(1-y)x_0+(1+x)(1-y)x_1+(1+x)(1+y)x_2+(1-x)(1+y)x_3 \\ (1-x)(1-y)y_0+(1+x)(1-y)y_1+(1+x)(1+y)y_2+(1-x)(1+y)y_3 \end{pmatrix}$$

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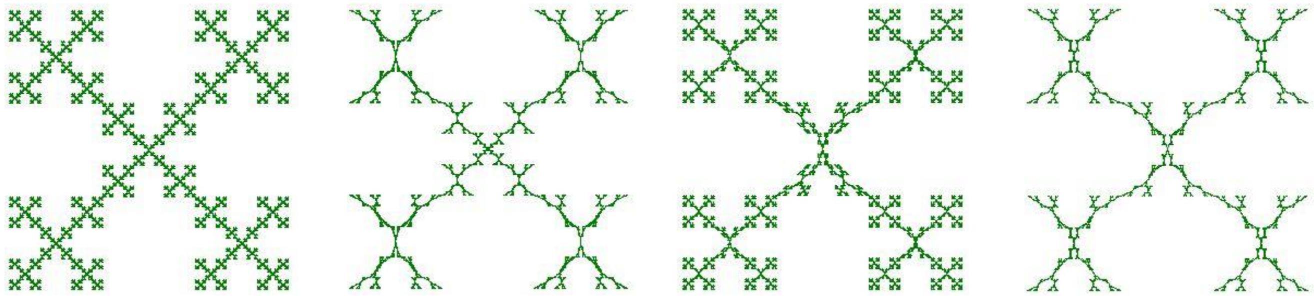
4 Generating Box Fractal Sets Examples

4.1 3×3 Box Fractal Sets

We chose four point coordinates $(-1,-1), (1,-1), (1,1), (-1,1)$ are the four vertexes of standard square , i.e. $[-1,1]\times[-1,1]$ is the first set. According to section 3, we realized above algorithm, and then generated a lot of fractal sets.

In BIFS1, $\omega_i, i=1,2,\cdots,5$ are all linear transformations, so the fractal set is very regular and rigid self-similar.BIFS2 is composed of 4 bilinear transformations (in 4 corner) and 1 linear transformation(in center).BIFS3 is composed of 4 linear transformations (in 4 corner) and 1 bilinear transformation(in center).BIFS4 is composed of 5 bilinear transformations. These BIFSs are in table 1 and Their attractors are shown in Fig. 2.



attr1 attr2 attr3 attr4

Fig. 2 3×3 box fractal sets

Table 1 3×3 BIFSs transformations

	ω_i	transformations		ω_i	transformations
BIFS1	ω_1	$x'=\frac{1}{3}x-\frac{2}{3}, y'=\frac{1}{3}y-\frac{2}{3}$	BIFS2	ω_1	$x'=-\frac{1}{3}xy-\frac{2}{3}, y'=\frac{1}{3}y-\frac{2}{3}$
	ω_2	$x'=\frac{1}{3}x+\frac{2}{3}, y'=\frac{1}{3}y-\frac{2}{3}$		ω_2	$x'=-\frac{1}{3}xy+\frac{2}{3}, y'=\frac{1}{3}y-\frac{2}{3}$
	ω_3	$x'=\frac{1}{3}x+\frac{2}{3}, y'=\frac{1}{3}y+\frac{2}{3}$		ω_3	$x'=-\frac{1}{3}xy+\frac{2}{3}, y'=\frac{1}{3}y+\frac{2}{3}$
	ω_4	$x'=\frac{1}{3}x-\frac{2}{3}, y'=\frac{1}{3}y+\frac{2}{3}$		ω_4	$x'=-\frac{1}{3}xy-\frac{2}{3}, y'=\frac{1}{3}y+\frac{2}{3}$
	ω_5	$x'=\frac{1}{3}x, y'=\frac{1}{3}y$		ω_5	$x'=\frac{1}{3}x, y'=\frac{1}{3}y$
	ω_i	transformations		ω_i	transformations
BIFS3	ω_1	$x'=\frac{1}{3}x-\frac{2}{3}, y'=\frac{1}{3}y-\frac{2}{3}$	BIFS4	ω_1	$x'=-\frac{1}{3}xy-\frac{2}{3}, y'=\frac{1}{3}y-\frac{2}{3}$
	ω_2	$x'=\frac{1}{3}x+\frac{2}{3}, y'=\frac{1}{3}y-\frac{2}{3}$		ω_2	$x'=-\frac{1}{3}xy+\frac{2}{3}, y'=\frac{1}{3}y-\frac{2}{3}$
	ω_3	$x'=\frac{1}{3}x+\frac{2}{3}, y'=\frac{1}{3}y+\frac{2}{3}$		ω_3	$x'=-\frac{1}{3}xy+\frac{2}{3}, y'=\frac{1}{3}y+\frac{2}{3}$
	ω_4	$x'=\frac{1}{3}x-\frac{2}{3}, y'=\frac{1}{3}y+\frac{2}{3}$		ω_4	$x'=-\frac{1}{3}xy-\frac{2}{3}, y'=\frac{1}{3}y+\frac{2}{3}$
	ω_5	$x'=-\frac{1}{3}xy, y'=\frac{1}{3}y$		ω_5	$x'=-\frac{1}{3}xy, y'=-\frac{1}{3}y$

4.2 4×4 Box Fractal Sets

In table 2,there are 8 BIFSs and their fractal sets are shown in Fig.3. These BIFS transformations are shown in table 2 and the bilinear transformations denoted by the red boxes in every fractal set are shown in Fig.3.

The attr5 is generated by BIFS5 which composed of 8 linear transformations,so it is very regular and rigid self-similarity; the attr6、attr7、attr8、attr9、attr10 and attr11 are generated by BIFS which composed of1、 2、3 or 4 bilinear transformations, so they are flexible and natural;the attr12 is generated by BIFS12 which composed of 8 bilinear transformations, the set is much more flexible and natural.

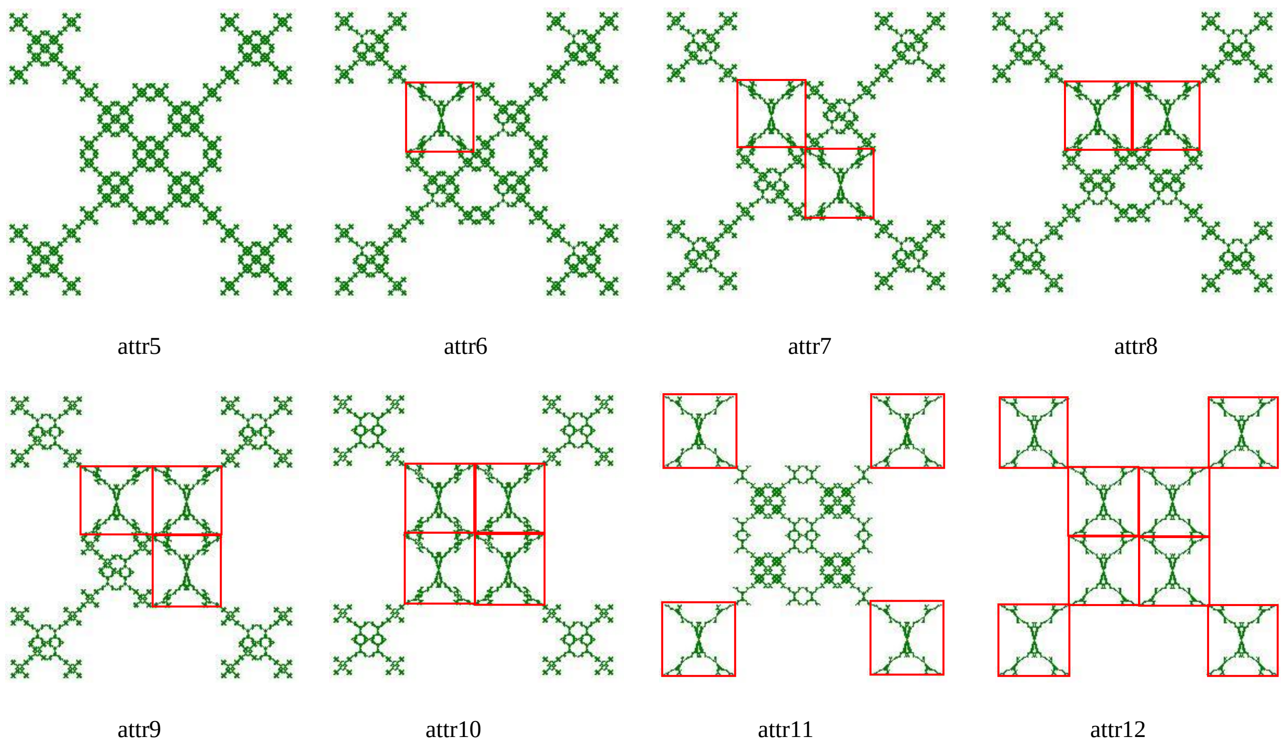


Fig. 3 4×4 box fractal sets

Table 2 4×4 BIFSs transformations

	ω_i	transformations		ω_i	transformations
BIFS5	ω_1	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$	BIFS6	ω_1	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$
	ω_2	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$		ω_2	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$
	ω_3	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$		ω_3	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$
	ω_4	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$		ω_4	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$
	ω_5	$x'=\frac{1}{4}x-\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$		ω_5	$x'=\frac{1}{4}x-\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$
	ω_6	$x'=\frac{1}{4}x+\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$		ω_6	$x'=\frac{1}{4}x+\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$
	ω_7	$x'=\frac{1}{4}x+\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$		ω_7	$x'=\frac{1}{4}x+\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$
	ω_8	$x'=\frac{1}{4}x-\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$		ω_8	$x'=\frac{1}{4}xy-\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$
BIFS7	ω_i	transformations	BIFS8	ω_i	transformations
	ω_1	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$		ω_1	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$
	ω_2	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$		ω_2	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$
	ω_3	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$		ω_3	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$
	ω_4	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$		ω_4	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$
	ω_5	$x'=\frac{1}{4}x-\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$		ω_5	$x'=\frac{1}{4}x-\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$
	ω_6	$x'=\frac{1}{4}xy+\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$		ω_6	$x'=\frac{1}{4}x+\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$
	ω_7	$x'=\frac{1}{4}x+\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$		ω_7	$x'=\frac{1}{4}xy+\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$
	ω_8	$x'=\frac{1}{4}xy-\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$		ω_8	$x'=\frac{1}{4}xy-\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$
BIFS9	ω_i	transformations	BIFS10	ω_i	transformations

	ω_1	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$		ω_1	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$
	ω_2	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$		ω_2	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$
	ω_3	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$		ω_3	$x'=\frac{1}{4}x+\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$
	ω_4	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$		ω_4	$x'=\frac{1}{4}x-\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$
	ω_5	$x'=\frac{1}{4}x-\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$		ω_5	$x'=\frac{1}{4}xy-\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$
	ω_6	$x'=\frac{1}{4}xy+\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$		ω_6	$x'=\frac{1}{4}xy+\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$
	ω_7	$x'=\frac{1}{4}xy+\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$		ω_7	$x'=\frac{1}{4}xy+\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$
	ω_8	$x'=\frac{1}{4}xy-\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$		ω_8	$x'=\frac{1}{4}xy-\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$
BIFS11	ω_i	transformations	BIFS12	ω_i	transformations
	ω_1	$x'=-\frac{1}{4}xy-\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$		ω_1	$x'=-\frac{1}{4}xy-\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$
	ω_2	$x'=-\frac{1}{4}xy+\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$		ω_2	$x'=-\frac{1}{4}xy+\frac{3}{4}, y'=\frac{1}{4}y-\frac{3}{4}$
	ω_3	$x'=-\frac{1}{4}xy+\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$		ω_3	$x'=-\frac{1}{4}xy+\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$
	ω_4	$x'=-\frac{1}{4}xy-\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$		ω_4	$x'=-\frac{1}{4}xy-\frac{3}{4}, y'=\frac{1}{4}y+\frac{3}{4}$
	ω_5	$x'=\frac{1}{4}x-\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$		ω_5	$x'=\frac{1}{4}xy-\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$
	ω_6	$x'=\frac{1}{4}x+\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$		ω_6	$x'=\frac{1}{4}xy+\frac{1}{4}, y'=-\frac{1}{4}y-\frac{1}{4}$
	ω_7	$x'=\frac{1}{4}x+\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$		ω_7	$x'=\frac{1}{4}xy+\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$
	ω_8	$x'=\frac{1}{4}x-\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$		ω_8	$x'=\frac{1}{4}xy-\frac{1}{4}, y'=-\frac{1}{4}y+\frac{1}{4}$

5 The Fractal Dimension of Box Fractal Sets

Fractal dimension can describe the complex of fractal set. The fractal dimension has been employed as a useful parameter in the diagnosis of retinal disease[24], of course, it can also be applied to plant identification[25,26], phonem classification and domain of words[27,28]. There exists many fractal dimension definition[29],but Hausdorff dimension (denoted as $\dim_H$) and Box dimension(denoted as $\dim_B$)are often used.

Theorem 5[21,23]: Let $\{X, \omega_n, n=1,2,\cdots,N\}$ be an IFS on R^n with contraction constants $\{s_1, s_2, \cdots, s_N\}$ respectively.

The IFS is said to satisfy the Open Set Condition,provided there is bounded non-empty open set O in R^n which satisfies the following conditions.For each $n, \omega_i(O)\subset O$ and for all $1\le i\ne j\le N$, $\omega_i(O)\cap\omega_j(O)=\Phi$.

If A is the attractor of the IFS,that is $A=\bigcup_{n=1}^{N}\omega_n(A)$, then $\dim_H A=\dim_B A=d$,where d is given by $\sum_{n=1}^{N}s_n^d=1$. Moreover, for this value of d , $0<H^{(d)}(A)<\infty$,where $H^{(d)}(X)$ is the Hausdorff $d-measure$ of X .

We used Theorem 5,in Fig.2 ,the dimension d of attr1 is computed by $\sum_{n=1}^{5}(\frac{1}{3})^d=1$,

So $d=\frac{\log 5}{\log 3}\approx 1.46498$.

We also used Theorem 5 to compute the dimension for attr12 which is generated by 8 bilinear transformations, the dimension d is computed by

$\sum_{n=1}^{8}(\frac{1}{4})^d=1$,So $d=\frac{\log 8}{\log 4}=1.5$.

Although we can compute the dimension of BIFS fractal attractors, we do not know it is right or wrong.Next,we will study the dimension of BIFS fractal attractors.

6 Conclusions and Next Work

Compared BIFS with affine map and L-system,affine map can map one line to another line, parallelogram to another parallelogram, so IFS which consists of affine maps, its attractor is not natural and inflexible,bilinear transformations attractor is very natural and flexible;L-system is very useful for bifurcation structure,especially it has absolute advantage to simulate plant. But L-system which depends on formal language and characteristic string rewriting,lacks of visual effect, and is not easy to realize.BIFS is composed of transformations, so it can be represented by geometry graph,and easy to realize.

This paper presented the IFS based on bilinear transformation,and generated a kind of fractal set: box fractal set,it can solve the problem that typical IFS attractor is inflexible and not natural.The result shows that this algorithm can generate more natural,realistic and complicated fractal graph.Bilinear transformation is extended by affine map,so all typical IFS can be represented by bilinear transformation IFS.

Next,we will study fractal morphing and recursive IFS by using this algorithm;will use B-spline and bicubic transformation to make more powerful modeling by IFS;will study the fractal dimension of the attractor of the BIFS .

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References:

[1] Mandelbrot BB. *Fractal geometry of nature*. New York: W.H. Freeman, 1983

[2] Barnsley MF. *Fractals everywhere*, 2nd ed. Boston Academic Press;,1993

[3] Ping Wang,Lei Li,et al.,Effects of fractal surface on C6 glioma cell morphogenesis and differentiation in vitro. *Biomaterials* 31(2010)6201-6206

[4] James G McNally, Davide Mazza, Fractal geometry in the nucleus.*The EMBO Journal* (2010) 29, 2–3

[5] Daniele Mancardi, Gianfranco Varetto, et al., Fractal parameters and vascular networks: facts & artifacts. *Theoretical Biology and Medical Modelling* 2008, 5:12

[6] Bai-lin Hao, H.C. Lee, et al., Fractals related to long DNA sequences and complete genomes.Chaos, *Solitons and Fractals* 11 (2000) 825-836

[7] Jianbo Gao,Yan Qi,et al.,Protein coding sequence identification by simultaneously characterizing the periodic and random features of DNA sequences.*Journal of Biomedicine and Biotechnology* 2 (2005) 139–146

[8] Nazneen Akhter, Yusuf Talib, et al.,Fractal application for characterisation of structure of macromolecules like carbohydrate. *J. Microbiol. Biotech. Res.*, 2012, 2 (1):108-114

[9] Michel L. Lapidus, et al.,Pointwise tube formulas for fractal sprays and self-similar tilings with arbitrary generators. *Advances in Mathematics* 227 (2011) 1349–1398

[10] Tomasz Martyn,Realistic rendering 3D IFS fractals in real-time with graphics accelerators.*Computers & Graphics* 34 (2010) 167–175

[11] M.R. Browna,R.,Errington P. Rees,et al.,A highly efficient algorithm for the generation of random fractal aggregates. Physica D 239 (2010) 1061-1066

[12] Enrique de Amo, Manuel Díaz Carrillo,et al.,PCF self-similar sets and fractal interpolation. *Mathematics and Computers in Simulation* 92 (2013) 28–39

[13] Zhigang Fenga,Yizhuo Feng,et al.,Fractal interpolation surfaces with function vertical scaling factors.*Applied Mathematics Letters* 25 (2012) 1896－1900

[14] Song-Gyong Ri,Huo-Jun Ruan,Some properties of fractal interpolation functions on Sierpinski gasket..*J. Math. Anal. Appl.* 380 (2011) 313－322

[15] Eduard Groller,Modeling and rendering of nonlinear iterated function systems. *Comput. & graphics*, 1994, 18(6):739- 748

[16] Eduard Groller, Rainer Wegenkittl,Interactive design of nonlinear functions for iterated function systems. 93-102. http://wscg.zcu.cz/WSCG1996/papers96/Groller_96.pdf

[17] CHEN Lian,Nonlinear models and applications of IFS[J]. *Journal of Computer Applications*, 2011, 21(8): 130-131

[18] LIU Shu-qun, LIU Luo-Sheng,Iterated function system based on polynomial transformation. *Journal of Lanzhou University of Technology*, 2011,37(1):81-85

[19] LUO Yan,WU Zhong-fu,et al.,"Bamboo" simulation based on improvement fractal algorithm and displacement and texture mapping.*Computer Science*, 2009,36(12):285-289

[20] ZHOU Wen-Li,Basic spline based reasoning for plant modeling.*Computer Science*, 2007,34(6):245-247

[21] Kenneth Falconer,*Fractal Geometry: Mathematical foundations and applications(Second Edition)*.Wiley & Sons, New York, 2003

[22] Heckbert Paul S,Fundamentals of texture mapping and image Warping.Computer Science,University of California, Berkeley ,June 1989:14-17. http://www.cs.cmu.Edu/~ph

[23] Louis Block, James Keesling,Iterated function systems and the code space.*Topology and its Applications*,2002,122: 65-75

[24] A.C.B. Kunicki,A.J. Oliveira,et al.,Can the fractal dimension be applied for the early diagnosis of non-proliferative diabetic retinopathy?.Braz J Med Biol Res, 2009,10: 42(10) 930-934

[25] YANG Hui-jun,CHEN Li-wei, Plants recognition based on fractal feature.*Computer Engineering and Design*. 2010, 31(24): 5321-5324

[26] Odemir Martinez Bruno, Rodrigo de Oliveira Plotze, et al., Fractal dimension applied to plant identification.*Information Sciences* 178 (2008) 2722－2733

[27] Rubina J Khan,Madki M R,Fractal Dimension for Vowel Extraction.*World Journal of Science and Technology* 2011, 1(12): 59-62

[28] M.Fernandez-Martinez,M.A.Sanchez-Granero,et al.,Fractal dimension for fractal structures: Applications to the domain of

words.*Applied Mathematics and Computation* 219 (2012) 1193 – 1199

[29] M.Fernández-Martínez,M.A.Sánchez-Granero, Fractal dimension for fractal structures: A Hausdorff approach.*Topology and its Applications* 159 (2012) 1825 – 1837