Synchronization of SA and AV Node Oscillators Using PSO Optimized RBF-based Controllers and Comparison with Adaptive Control

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Abstract - This paper studies the synchronization of SA and AV Node Oscillators using PSO optimized RBF-based controllers systems. High levels of control activities may excite unmodeled dynamics of a system. The objective is to reach a trade-off between tracking performance and parametric uncertainty. Two methods are proposed to synchronize general forms of Van Der Pol (VDP) Model and their performance. These methods use the radial basis function (RBF)- based neural controllers for this purpose. The first method uses a standard RBF neural controller. Particle swarm optimization (PSO) algorithm is used to derive and optimize RBF controller parameters. In the second method, an error integral term is added to the equations of RBF neural network. The coefficients of error integral component and parameters of RBF neural network are also derived and optimized via PSO algorithm. Simulation results show the effectiveness and superiority of proposed methods in both performances in comparison with adaptive controller.

Key-Words- Synchronization, Van der Pol Model, SA and AV Node Oscillators, RBF, PSO Algorithm, Adaptive Conrol, Optimization Algorithm, system Dynamics, Simulation Results, Controller Parameters, impulses.

1 Introduction

The present paper examines synchronizations of Van der Pol oscillatory systems. Synchronization problem has found many applications in laser, chemical reactors, secure communications, and biology. This paper deals with one such application in cardiac synchronization. This seems to be particularly important as cardiovascular diseases are among the major causes of death worldwide. Disruption in the heart electrical function is a type of such diseases generally referred to as “cardiac arrhythmia”[1],[2]. Thus, electrical conduction system of the heart can be modeled and used in preventing serious heart diseases. One practical way to investigate how a member of an organism works is to develop a model which accurately reflects the function of this part. Such a model may serve as a hypothesis for some physiological observation. For simulating how stimulation propagates over the heart tissue, it seems necessary to develop an accurate model of action potential of cells[3]. For this purpose, Van der Pol model was used in the present study to examine synchronization of heart oscillators. The main goal of this study is to synchronize atrio-ventricular (AV) oscillator with sino-atrial oscillator based on a particular model by the use of different methods. We will also discuss how pacemakers including SA node and AV node can be resynchronized in cases where one is out of synch with the other (which is a major cause of arrhythmia)[4].

1.1 An Overview of Cardiovascular Physiology

The heart will not be able to pump unless it receives an electrical excitation which originates from pumping. Generation and transmission of electrical impulses depend on automaticity, excitability, conductivity, and contractibility of cardiac cells. Transmission of cardiac impulses creates depolarization-repolarization cycles in cardiac cells. When at rest, the cardiac cells are polarized, *i.e.* they show no sign of electrical activity[5]. The cell membranes separate different concentrations of such ions as K^+ and Na^+, and create larger negative charges inside the cell. The phenomenon is known as resting membrane potential. As soon as an electrical excitation arrives, the ions are transported at either side of the cell membrane leading to action potential or depolarization. Once a cell is completely depolarized, it tries to return to its initial conditions or resting state. This process is referred to as repolarization[6],[7]. The electrical charges are reversed and return to the normal state. A typical depolarization-repolarization cycle consists of five phases (0 to 4) (Fig. 1 presents action potential curve and voltage variations in these five phases):

Phase 0: A cell receives an impulse from its adjacent cell and becomes depolarized.

Phase 1: An initial immediate repolarization takes place.

Phase 2: This slow repolarization step is also known as plateau phase. In Phase 1, Phase 2, and early in Phase 3, cardiac cells are at total inexcitability state. In this phase, not every stimulus with any intensity can result in cellular response[8],[9].

Phase 3: This phase is known as rapid repolarization. At this point of time, the cell returns to its initial state. At the last one-third of this phase, when the cell enters the relative excitability state, very strong excitations can depolarize it.

Phase 4: This step is the resting state for action potential. By the end of the fourth phase, the cell is ready for next excitations. All these activities can be recorded on electrocardiogram (ECG)[10].

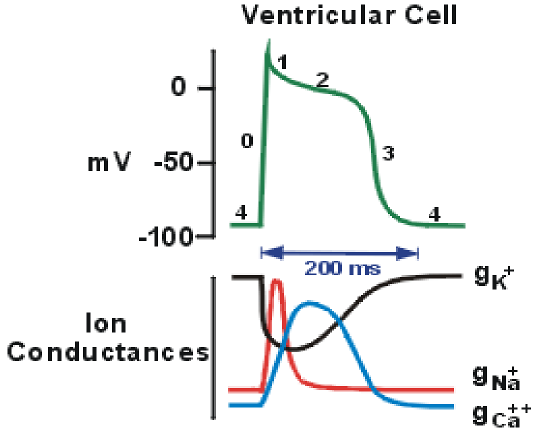

Fig. 1- Action Potential Curve

1.2 Electromechanical Conduction Mechanism of the Heart

Immediately after depolarization and repolarization, electrical impulses are propagated along a pathway known as conduction system (Fig. 2). These impulses start traveling out of the SA node, through the atrium and Bachmann's bundle and into the AV node[11]. The impulses, then travel through the bundle of His, left and right branches, and eventually into the Purkinje fibers. This conduction system is an electromechanical one. The electrical section orders the contraction of all cells, and the mechanical section (muscles) implements these orders. Some diseases are caused by failure in these mechanical functions while most diseases are the result of the malfunction of electrical system. Electrical conduction system of the heart can be thought of as a self-exciting pacemaker. This system is responsible for proper and synchronized contraction of cardiac muscles[12].

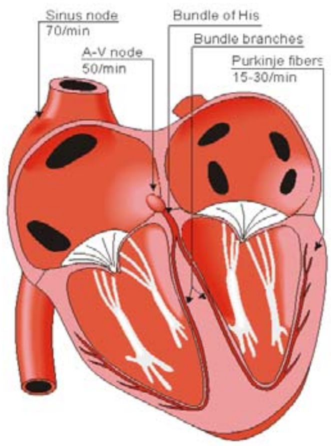

Fig. 2. Pacemakers and Impulses Routes

1.3 Introduction to Synchronization

The word "synchronous" has its origin in a Greek word meaning "sharing the same time period", and since its origin, the word has been used in everyday applications to denote agreement or dependency of the different processes in terms of time. Historically, analysis of synchronization of dynamic systems has received considerable attention as a very important subject in physics. The phenomenon dates back to the 17th century when Hyugens patented two synchronized pendulum clocks with very weakly coupled oscillations[13],[14].

In synchronization of oscillatory systems, two identical systems oscillate simultaneously. If one system is designated as the master and another identical system is assigned as the slave when a proper control input is applied to the slave, the dynamic behavior or the two systems will become identical after a period of time. The slave which often has to become synchronized with the master is usually referred to as the response system or received while the master is sometimes called the drive or sender[15],[16],[17].As mentioned earlier, the objective here is to synchronize the slave with the master. For this purpose, a nonlinear control system must be designed to receive the control signals from the master and control the slave. Here, behavior of the slave is clearly controlled by the master. In addition, the slave may have conditions different from those of the master[18],[19].

2 Materials And Methods

2.1 Van Der Pol (VDP) Model

The first attempts to explain the heart cells oscillatory behavior was made in 1926 by Van der Pol [20]. Balthasar van der Pol was a German physicist and an electronic engineer. He achieved discovering stable oscillations, now called 'limit cycle' . Van der Pol was the first person who examined relaxation oscillations by studying an electrical circuit which had self-entertained oscillations with the amplitude independent of initial conditions. The schematic of this circuit is shown in Figure 3.

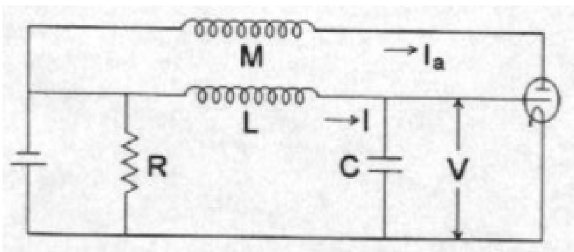

Fig. 3- Van der Pol's Circuit

The equations of the voltages and currents in this circuit are

$$I_a = CV'(\theta) \quad I_a = V - \frac{1}{3}V^3, \quad , \quad L\frac{dI}{d\theta} + RI + V = M\frac{dI_a}{d\theta} \qquad (1)$$

where

$$c = \frac{M}{\sqrt{LC}} - R\sqrt{\frac{C}{L}}, \quad \theta = t\sqrt{LC} \quad V = x\sqrt{1 - R\frac{C}{M}} \qquad (2)$$

By substitution x, t, and c from (2) into the current-voltage equations in (1), the following differential equation, known as Van der Pol's equation is obtained[21] :

$$\frac{d^2x}{dt^2} + c(x^2 - 1)\frac{dx}{dt} + x = 0$$

This circuit serves as an essential model for self-entertained oscillations in physics, electronic engineering, biology, neurology and many other sciences. Since c is the control parameter in this equation, different periodic responses can be initiated by changing the value of c with large values of c resulting in relaxation oscillations[22].

For some important properties, Van der Pol nonlinear equations are used to model the oscillations of the heart. First, Van der Pol oscillator adjusts its natural frequency to the frequency of the input signal without changing the oscillation amplitude. This is critically important as the low-frequency slave oscillator has to adjust itself to the dominant high-frequency pacemaker of the heart[23].

Therefore, Van der Pol model was used in this project to model the oscillators at SA and AV nodes. Each oscillator at SA node and AV node is modeled using Van der Pol differential equations. The interaction between the oscillators of the heart is modeled by the following Van der Pol equations[24],[25]. The coupling between these interacting oscillators is modeled as follows:

$$SA : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -w_1{}^2 x_1 + c_1(\mu - x_1^2)x_2 + R_1(x_4 - x_2) \end{cases}$$

$$AV : \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -w_2{}^2 x_3 + c_2(\mu - x_3^2)x_4 + R_2(x_2 - x_4) - \end{cases}$$

The first equation which models SA oscillations is the drive in the present problem while the second equation for modeling AV oscillations represents the response system[38].

2.2 Generating Action Potential by the Model

Heart rhythm is determined by a series of electric impulses (action potential) which travels throughout the heart. Action potentials for SA and AV cells obtained through are confirmed[25]. The validity of the Van der Pol model used here in terms of how these waveforms match the actual forms generated in the heart is clarified[26],[27].

2.3 Synchronization Using Adaptive Control

Adaptive control is used to design a controller which can create a desirable response in the face of smooth changes in the system and modeling errors. The difference between adaptive control and robust control is that in adaptive control, no information is required about the range within which the system operates or the error is involved in the system parameters. In other words, a design based on robust approach results in a controller which leads to stability within a certain range without any requirements for changing the control laws while in adaptive control, control laws may be changed depending on the conditions in order to make the system stable[28],[29].

2.3.1 Designing a Synchronizer Using Adaptive Control

Many techniques rely on complete knowledge of the system structure and parameters. However, some parameters may not be available for designing a synchronization mechanism. In such cases, adaptive synchronization can be helpful in solving the problem. In many cases, the parameters of the master and slave systems are unknown[30]. Therefore, adaptive techniques should be used to synchronize two systems with unknown parameters. In adaptive control, parameter estimation forms a basis for designing the controller. Using the Lyapunov method for estimating the parameters, these parameters approach the values of the corresponding parameters in the actual system. These estimated values are employed in the controller while closed-loop stability of the system is maintained by utilizing Lyapunov's theorem for stability[31].

In cases where the parameters of the drive system and/or the response system are unknown, adaptive control is a useful and simple technique for synchronizing the two systems[32]. The objective here is to find a controller and a rule to update the parameters so that the states of the drive and the response systems become globally and asymptotically synchronized[33]. In this technique, control inputs with the same number as the states of the system are applied to the slave or response system, and the controller is selected such that the nonlinear portion of the drive-response error dynamic is eliminated[34].

Lyapunov's theorem for stability is utilized to obtain a rule for updating the estimated parameters used in the controller and to demonstrate closed-loop stability of the system. More details are provided in the following section on adaptive synchronization of the heart oscillators[9].

In this section, adaptive control is used to synchronize non-identical SA and AV oscillators with one totally unknown parameter. In both cases, the initial conditions of the master are different from those of the slave[35].

The descriptive equations for the two nodes are:

$$SA: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -w_1^2 x_1 + c_1(1 - x_1^2)x_2 \end{cases}$$

$$AV: \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -w_2^2 x_3 + c_2(1 - x_3^2)x_4 + R(x_2 - x_4) + u \end{cases}$$

(5)

(6)

Up to this point, we assumed that all parameters are known. Now, we assume that one parameter (c_2) is unknown. The parameter error is defined as:

$$\tilde{c}_2 = c_2 - \hat{c}_2$$

(7)

The control signal is determined using the estimation found for c_2:

$$\dot{e}_1 = e_2$$

$$\dot{e}_2 = -w_1^2 e_1 + (c_1 - R)e_2 - c_1 x_1^2 x_2 + c_2 x_3^2 x_4 + (c_1 - c_2)x_4 + (w_2^2$$

(8)

Lyapunov method is used to obtain the rules for updating $\hat{c}_2$. Consider the following Lyapunov candidate function:

$$u = (c_1 - \hat{c}_2)x_4 - c_1 x_1^2 x_2 + \hat{c}_2 x_3^2 x_4 + ke_2 + (w_2^2 - w_1^2)x_3 \quad (9)$$

By differentiating both sides of (10) and substituting the error dynamic from (8), we have:

$$V(e_2, \tilde{c}_2) = (e_2^2 + \tilde{c}_2^{\,2})/2 \quad (10)$$

Now, (5-69) can be used to obtain the updating rule:

$$\dot{v} = \dot{e}_2 e_2 + \tilde{c}_2 \dot{\tilde{c}}_2 = \left[(c_1 - R - k)e_2 - \tilde{c}_2 x_4 + \tilde{c}_2 x_3^2 x_4 + w_1^{\,2} e_1\right] e_2 + \dot{\tilde{c}}_2 \tilde{c}_2 \quad (11)$$

The parameter $\hat{c}_2$ is determined by the signals of the system; the nonlinear nature of the adaptive control system can be easily seen here.

$$\dot{\hat{c}}_2 = (-x_4 + x_3^2 x_4)e_2 \quad (12)$$

By selecting $k > c_1 - R$, $\dot{V}$ is obtained as:

$$\dot{V} = -e_1^2 - e_2^2 < 0 \quad (13)$$

Now, Lyapunov's theorem for stability and Barbalat's Lemma can be used to show that the control function (9) and the parameter adjustment rules (12) asymptotically synchronize the systems described by (5) and (6) with one unknown parameter. It can be seen that, even with one unknown parameter, synchronization error asymptotically converges to zero[36].

3 Discussions And Results

3.1 Simulation Results

The initial value for the estimated parameter is selected as $\hat{c}_2(0) = 3$. Based on the states of the two systems synchronized using adaptive control [37],[38], it can be seen that, even with one unknown parameter, synchronization error vanishes symptotically.

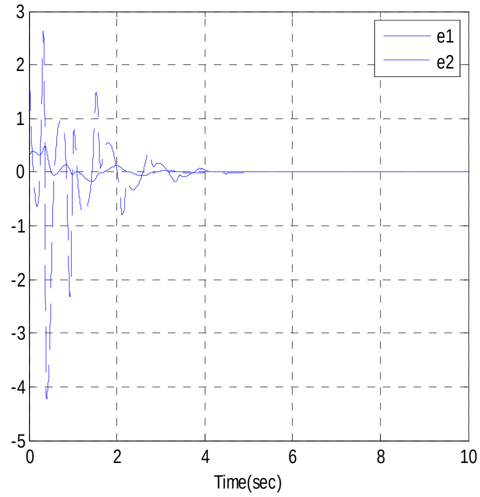

Fig. 4. Error

Figure 4 Shows how the estimated values for the unknown parameter converge to the actual values.

3.2 PSO Algorithm

PSO is a population-based stochastic optimization technique which does not use the gradient of the problem which was optimized, so it does not require to be differentiable for the optimization problem as is necessary in classic optimization algorithms. Therefore, it can also be used in optimization problems that are partially irregular, time variable, and noisy. In PSO algorithm, each bird referred to as a 'particle' represents a possible solution for the problem.[39] Each particle moves through the D-dimensional problem space by updating its velocities with the best solution found by itself (cognitive behavior) and the best solution found by any particle in its neighborhood (social behavior). Particles move in a multidimensional search space, and each particle has a velocity and a position as follows:

$$v_i(k+1) = v_i(k) + \gamma_{1i}\left(P_i - x_i(k)\right) + \gamma_{2i}\left(G - x_i(k)\right)$$

$$x_i(k+1) = x_i(k) + v_i(k+1)$$

where i is the particle index, k is the discrete-time index, vi is the velocity of ith particle, xi is the position of ith particle, Pi is the best position found by i^{th} particle (personal best), G is the best position found by swarm (global best), and γ1,2 are random numbers in the interval [0, 1] applied to ith particle. In our simulations, the following equation is used for velocity:

$$v_i(k+1) = \phi(k)v_i(k) + \beta_1[\gamma_{1i}(P_i - x_i(k))] + \beta_2[\gamma_{2i}(G_i - x_i(k))] \quad (16)$$

in which ϕ is inertia function and β1, β2 are acceleration constants[40],[41].

3.3 Proposed Synchronization Schemes

As mentioned before, the synchronization scheme consists of two systems: the master and the slave. In this scheme, an RBF- or "RBF + error integral"-based controller is used to make the states of the slave system follow the states of the master system, in the presence of uncertainties and external disturbances[42]. It should be noted that h(·) can be considered as any continuous function. In this section, two proposed methods of system synchronization are described: (14) RBF-based nonlinear controller, (15) "RBF + error integral" model in which an integral term is added to RBF model to improve the robustness of the proposed controller. To optimize the parameters of these controllers, PSO is also used as a continuous evolutionary algorithm[43],[44]. The mathematical formulation:

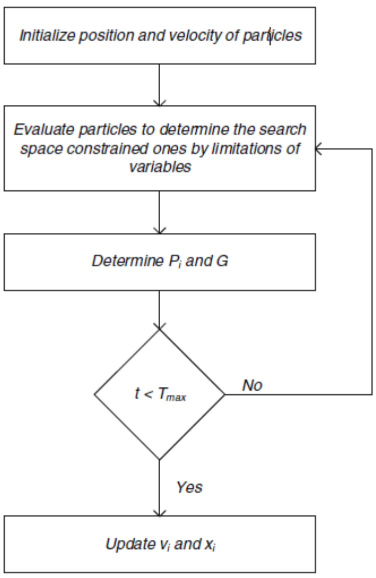

Fig 5. Flowchart of PSO Algorithm

The proposed methods are stated in the following subsections:

3.4 Control by RBF Model

Consider the control system in (15), for this system the following RBF-based controller is proposed:

$$u(t) = W^T \xi(e)$$

in which u(t) is the control signal, and $e = [e_1, e_2, \ldots, e_n]^T, e_i = x_{im} - x_{is}, i = 1,2,\ldots,n, x_{im}$ and $x_{is}, i = 1,2,\ldots n$ are the states of master and slave systems, respectively.[45],[46] $W = [w_1, w_2, \ldots, w_M]^T \in \Re^M$ is the weight matrix, and $\xi(e) = [\xi_1(e), \xi_2(e), \ldots, \xi_M(e)]^T \in \Re^M$ is a set of basis functions of the corresponding RBF model. The basis function $\xi_i(e)^M$ of i[th] node in the hidden layer is considered a Gaussian function as:

$$\xi_i(e) = \exp\left(-\frac{\|e - c_i\|^2}{\delta_i^2}\right)$$

in which ci and δi are the center and the width, respectively. Considering $c = [c_1, c_2, \ldots, c_n]^T \in \Re^n$ and $\delta = [\delta_1, \delta_2, \ldots, \delta_n]^T \in R^n$ the goal is to find the optimized $W = W^*, c = c^*$ and $\delta = \delta^*$ such that the following cost functional is minimized:

$$IAE = \int_0^T \|e(t)\| \, dt$$

Where $\|.\|$ is the Euclidean norm of a vector. To find the optimized parameters $W = W^*, c = c^*$ and $\delta = \delta^*$, the PSO algorithm is used[47],[48].

3.5 "RBF + error Integral" Model

As mentioned before, there may be modeling uncertainties and external disturbances in the control problem. Therefore, the controller should be robust enough such that it can cope with these uncertainties. Now, as a modification of the method proposed, the integral components are added to the basis function vector to increase the robustness of the system. Therefore, the following controller is proposed:

$$u(t) = W_I^T \xi_1(e)$$

Where

$$W_I = [w_1, w_2, ..., w_M, w_{M+1}, w_{m+2}, ..., w_{M+n}]^T \in \Re^{M+n}, \xi_I(e) = [\xi_{1I}(e), \xi_{2I}(e), ..., \xi_{MI}(e), \int edt]^T \in \Re^{M+n} \quad (21)$$

$\xi_{iI}(e) = \xi_i(e), i = 1,2,...,M$ and $\int edt = [\int e_1 dt, \int e_2 dt, ..., \int e_n dt] \in \Re$ .The goal in this scheme is to find the optimized $W = W^*, c = c^*$ and $\delta = \delta^*$ such that the cost functional (8) is minimized.

3.6 Simulation and Experimental Results

As mentioned before, the system consists a master and a slave. Considering $h(.)$ to be any continuous function, in system masking scheme, the message signal *m(t)* is added to the output of the master system, $h(x_m)$. The controller is designed such that the master and the slave systems are synchronized [49]. Thus by subtracting the output of the slave system, $h(x_s)$, from the resulted signal, the message signal can be thoroughly recovered[50]. It should be noted that the controller should be designed such that it can cope with uncertainties and external disturbances.

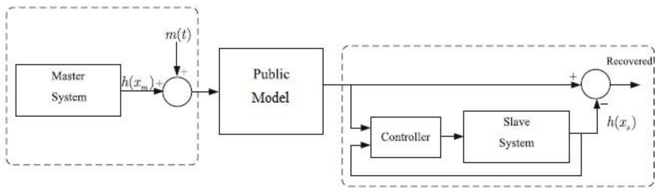

Fig.6. Block Diagram of System Masking Scheme

The descriptive equations for the SA and AV oscillators are:

$$SA: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -w_1^{\,2} x_1 + c_1(1 - x_1^2) x_2 \end{cases}$$

$$AV: \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -w_2^{\,2} x_3 + c_2(1 - x_3^2) x_4 + R(x_2 - x_4) + u \end{cases}$$

The initial conditions for the master and slave are (1,4) and (0.7, 2), respectively. Based on the physiological facts, a one-way coupling is considered here. The frequency is 60 pulses per minute for the first oscillator and 40 pulses per minute for the second oscillator. It is assumed that c2 contains uncertainty and $\hat{c}_2 = c_2 + \Delta c_2$ where Δc_2 represents uncertainty[47].Eventually the system output converges to the desirable output. The SA and AV oscillators have become synchronized once a time period is passed. There is a synchronization error. This error vanishes over time indicating that the two oscillators have become synchronized.

3.6.1 Two-way Coupling

If two-way coupling is used for the two oscillators - which in its physiological sense it means that AV oscillator impacts SA oscillator as well, sometimes in a relatively weak manner – the equations become:

$$SA: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -w_1^{\,2} x_1 + c_1(1 - x_1^2) x_2 + R_1(x_4 - x_2) \end{cases}$$

$$AV: \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -w_2^{\,2} x_3 + c_2(1 - x_3^2) x_4 + R_2(x_2 - x_4) + u \end{cases}$$

First, we select a value for R_1 which was about one tenth of R_2. The computation results indicated that two-way coupling has no effect on synchronization time. The value of R_1 was then increased, but no impact was observed on synchronization time. This is in line with physiology of heart as AV oscillator has negligible effect on SA oscillator[18],[28].

4. Conclusion

The table below shows the results of the two methods. As seen on the table, PSO optimized RBF-based controllers outperforms in terms of synchronization time and variance of error.

Table 1. Results of Two Methods

SA and AV Node Oscillators	Synchronization Time(sec)	Error Variance	Control Effort(max)	Control Effort (min)
PSO optimized RBF-based Controllers	0.2	0.001152	149+	149-
Adaptive Control	4.8	0.0026186	149+	149-

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