On the control of a muscular force model

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Abstract: Athletes and Physiotherapists may need electromyostimulation to reinforce muscle or to treat deficient muscles in order to speed up their recovery. Generally, the electromyostimulator does not take into account the physiological parameters necessary to adapt automatically the stimulation parameters of the system in order to reach a desired force value. To remedy at this problem and to optimize the rehabilitation sessions, we investigate the feasibility of controlling the muscular force by using an experimentally-based model.

Key–Words: Electrostimulation, Force muscular, Control

1 Introduction

Electromyostimulation is a process which arouses a lot of interest. As a consequence, this technique is often used for reinforcement and for reconditioning. Electromyostimulation (EMS) is carried out by placing electrodes on the area to be stimulated and electrical pulses are sent into the muscle to create involuntary contractions. The effects of these stimulations on the generated force are difficult to model. The modeling should be realized by mathematical models which take into account the physiological parameters, varying according to the subject state but also from one person to another, which renders the modeling difficult as explained in [1].

The model used in [2, 3] is obtained from experimental data and therefore satisfies specification requirements. Furthermore, it is based on a specific protocol which proves its reliability to solve the modeling problem of the relationship between force and stimulation accurately. Thus, this model could be used to maximize the force, which could be realized for instance by the prediction of the necessary number of muscular contractions. The predicted contraction frequency could also be used to analyze the correlation between the physiological aspects and the mathematical model [4]. The relationship between the force and the stimulation frequency can also be estimated on the same model, showing a correlation between the stimulation frequency and the effects on the force [5,6]. This model was tested in the case of children with cerebral paralysis (CP) in order to predict the obtained force level [7]. In this study, we based our work on the model proposed in [2, 3], where the control variable acts on the impulse amplitude while the frequency is fixed. Section 2 details the equations of the model. Then in section 3, the nonlinear control strategy is proposed. In section 4, simulation results are presented, leading to quantify the control method efficiency. Finally, section 5 is devoted to the conclusion.

2 Model

The model used is defined by a set of differential equations, as follows:

$$\frac{dC_n}{dt} = \frac{1}{\tau_c}\sum_{i=1}^{n} R_i e^{\frac{-(t-t_i)}{\tau_c}} - \frac{C_n}{\tau_c}, \qquad (1)$$

$$\frac{dF}{dt} = A\frac{C_n}{K_m + C_n} - \frac{F}{\tau_1 + \tau_2(\frac{C_n}{K_m+C_n})}, \qquad (2)$$

with

$$R_i = 1 + (R_0 - 1)e^{-(\frac{t_i - t_{i-1}}{\tau_c})}. \qquad (3)$$

Equation (1) represents the C_n derivative depending on C_n the normalized amount of $C_a^{2+} - troponin$ complex obtained at each stimulation, the time constant τ_c and the sum of successive pulses that are gathered in the term:

$$\sum_{i=1}^{n} R_i e^{\frac{-(t-t_i)}{\tau_c}}, \qquad (4)$$

where t_i is the time of i^{th} pulse and R_i the mathematical term defining the magnitude of enhancement

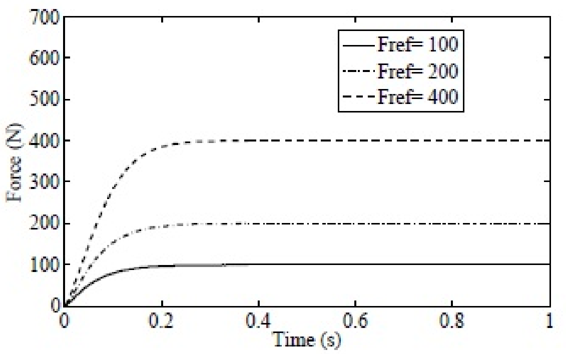

Figure 1: Evolution of the force value of control for F_{ref}= 100, 200 and 400 N with dt= 10 ms.

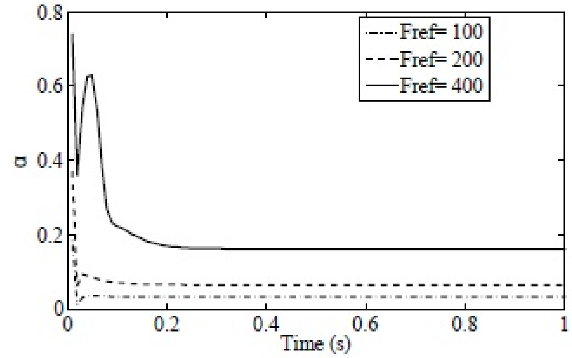

Figure 2: Evolution of the obtained impulse amplitude of control of the force for F_{ref}= 100, 200 and 400 N with dt= 10 ms.

in C_n from the next stimuli. R_i is initialized at R_0. The stimulation generates a muscular force F according to (2). The parameters A (the scaling factor of the force and the muscle shortening velocity), R_0 and τ_1 (a force time constant) are for this model at their resting values: A= 5.1 N/ms, R_0=2 and τ_1= 43.8 ms and the parameters τ_2= 124.4 ms, K_m= 0.3 and τ_c= 20 ms, as defined in [2] with τ_2 the second force time constant and K_m the sensitivity of a strong bound crossbridges to C_n.

3 Control Method

The control method presented in this section is a nonlinear control applied to the force model. The variable u represents the control acting on the variation of impulse amplitudes. These amplitudes are found in the sum of exponential functions between two successive stimulations $dt_i = t_{i+1} - t_i$. The control is defined by the expression:

$$u = \frac{1}{\tau_c} \sum_{i=1}^{n} \alpha_i R_i e^{\frac{-(t-t_i)}{\tau_c}}, \tag{5}$$

so that (1) is modified as:

$$\frac{dC_n}{dt} = u - \frac{C_n}{\tau_c}. \tag{6}$$

Equation (5) includes a new parameter α_i which is determined by the control u. Therefore, controlling the force corresponds to finding α_i values. The control method is based on the prediction of the amplitude of the next pulse with dt_i constant. In the first part, we consider the case of a continuous control, named $u_{NL}(t)$ then we deduct from it the discrete control $u(t)$, leading to determine α_i.

In a general way, a nonlinear affine system function of $u_{NL}(t)$ is described by the system (7), that is:

$$\dot{x}(t) = f(x(t)) + g(x(t))u_{NL}(t), \tag{7}$$

and

$$y(t) = h(x(t)). \tag{8}$$

According to the force model, one can write:

$$x(t) = \begin{bmatrix} C_n(t) \\ F(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \tag{9}$$

$$y(t) = x_2(t), \tag{10}$$

where f and g represent vector fields:

$$f = \begin{bmatrix} \dfrac{-x_1}{\tau_c} \\ \dfrac{Ax_1}{x_1 + K_m} - \dfrac{x_2}{\tau_1 + \tau_2 \frac{x_1}{x_1+K_m}} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}, \tag{11}$$

$$g^T = \begin{bmatrix} 1 \ 0 \end{bmatrix}^T. \tag{12}$$

Equation (10) expresses the fact that only the force $F(t)$ is measured. Let us assume that the system is controllable, so that a control by output feedback would be possible. To perform this, we compute the first derivative of $y(t)$. From (7) and (8), it comes:

$$\dot{y}(t) = L_f h(x) + L_g h(x) u_{NL}(t), \tag{13}$$

where

$$L_f h(x) = \frac{\partial h}{\partial x} f, \tag{14}$$

$$L_g h(x) = \frac{\partial h}{\partial x} g. \tag{15}$$

It leads to:

$$L_f h(x) = [0 \ 1] f = f_2, \tag{16}$$

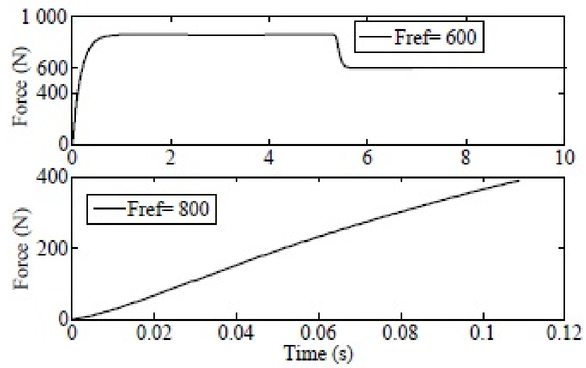

Figure 3: Representation of the generated forces for the control of the force for F_{ref}= 600 and 800 N with dt= 10 ms.

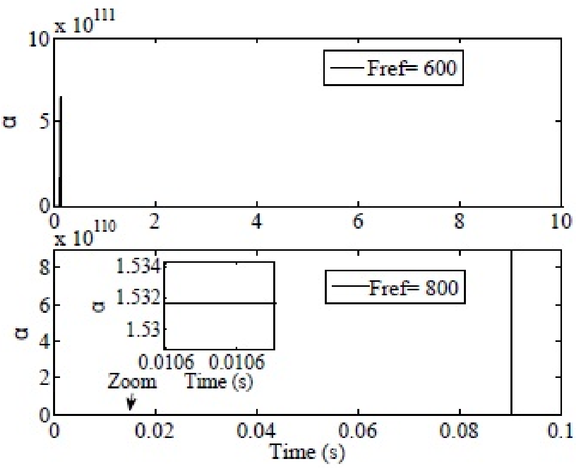

Figure 4: Evolution of the obtained impulse amplitude for the control of the force for F_{ref}= 600 and 800 N with dt= 10 ms.

$$L_g h(x) = [0\ 1] g = 0. \qquad (17)$$

As $L_g h(x)$ is null, we compute the second derivative of $y(t)$. From (13), one can write:

$$\ddot{y}(t) = L_f^2 h(x) + L_g L_f h(x(t)) u_{NL}(t), \qquad (18)$$

where

$$L_f^2 h(x) = L_f(L_f h(x)) = \frac{\partial L_f h(x)}{\partial x(t)} f, \qquad (19)$$

and

$$L_g L_f h(x) = \frac{\partial L_f h(x)}{\partial x(t)} g = [\frac{\partial L_f h(x)}{\partial x_1} \frac{\partial L_f h(x)}{\partial x_2}] g. \qquad (20)$$

So,

$$L_g L_f h(x) = \frac{A K_m}{(K_m + x_1)^2} + \frac{x_2 K_m \tau_2}{(\tau_1(K_m + x_1) + \tau_2 x_1)^2}. \qquad (21)$$

From (21), one can verify that $L_g L_f h(x)$ is $\neq 0$. Therefore, the nonlinear control $u_{NL}(t)$ is defined such as:

$$u_{NL}(t) = \frac{-L_f^2 h(x) + v(t)}{L_g L_f h(x)}, \qquad (22)$$

where $\ddot{y}(t)$ has to be equal to $v(t)$, which is defined to stabilize the system to $y_{ref}(t)$, the target force. Let us assume that:

$$v(t) = \ddot{y}_{ref} - C_1(\dot{y}(t) - \dot{y}_{ref}(t)) - C_0(y(t) - y_{ref}(t)), \qquad (23)$$

where C_1 and C_0 are constant parameters to be calculated in order to ensure stability system. Then,

$$\ddot{y}(t) = \ddot{y}_{ref}(t) - C_1(\dot{y}(t) - \dot{y}_{ref}(t)) - C_0(y(t) - y_{ref}(t)), \qquad (24)$$

giving:

$$\ddot{y} - \ddot{y}_{ref} + C_1(\dot{y} - \dot{y}_{ref}) + C_0(y(t) - y_{ref}(t)) = 0. \qquad (25)$$

Let us now define the error $e(t)$ between the force $y(t)$ and the target force $y_{ref}(t)$, that is :

$$e(t) = y(t) - y_{ref}(t). \qquad (26)$$

Equation (25) becomes:

$$\ddot{e}(t) + C_1 \dot{e}(t) + C_0 e(t) = 0, \qquad (27)$$

which corresponds to:

$$s^2 E(s) + C_1 s E(s) + C_0 E(s) = 0, \qquad (28)$$

where $E(s)$ is the Laplace transformation of $e(t)$ ans s the Laplace variable. $e(t)$ tends to zero if $s^2 + C_1 s + C_0 = 0$ is stable. The unknown parameters C_0 and C_1 are then chosen such that the roots of (28) have negative real parts (stable eigenvalues). A set of test has been performed using different couples (C_1, C_0) in order to choose acceptable tendency (overshoot, convergence speed) of the force evolution. For all the following, we used (C_1, C_0)=(-0.006,0.05). The nonlinear control ($u_{NL}(t = ndt)$) being computed, it is used to compute the impulse amplitude to be applied at t (the real control u is a sum of successive impulses):

$$u = \frac{1}{\tau_c} \sum_{i=1}^{n} \alpha_i R_i e^{\frac{-(t-t_i)}{\tau_c}}. \qquad (29)$$

To do that, we suppose that $u_{NL}(t)$ is constant during the interval $[t, t + dt[$ and chosen to be equal to the mean value of $u(t)$ during the above interval. The effect of a control u will be approximately the same as

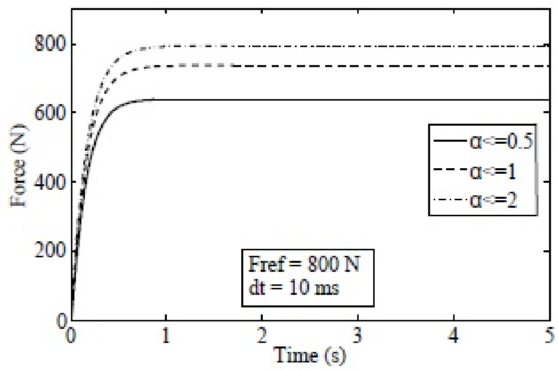

Figure 5: Different limits of impulse amplitudes for a F_{ref} of 800 N (dt= 10 ms).

$u_{NL}(t)$ which leads F to F_{ref},

$$u_{NL}(t) = \frac{1}{H+1}\sum_{k=0}^{H} u(t+\frac{k}{H}dt) = \frac{1}{H+1}\sum_{k=0}^{H}\frac{1}{\tau_c}\sum_{i=1}^{n} R_i\alpha_i e^{\frac{-(t+\frac{k}{H}dt-t_i)}{\tau_c}}, \quad (30)$$

where H is the number of simulation values of u during $[t, t+dt[$ equal to a choosen value fixed to 10 with the integration step $\Delta T = \frac{dt}{H+1}$. Therefore:

$$u_{NL}(t) = \frac{1}{H+1}[\sum_{k=0}^{H}\frac{1}{\tau_c}[\sum_{i=1}^{n-1} R_i\alpha_i e^{\frac{-(t+\frac{k}{H}dt-t_i)}{\tau_c}} + R_n\alpha_n e^{\frac{-(t+\frac{k}{H}dt-t_n)}{\tau_c}}]], \quad (31)$$

α_i $(i = 1, ..., n-1)$ being the previous pulses amplitudes, which were computed using the previous stimulation steps. At time $t = ndt$, only the value of α_n is unknown. From (31), it comes:

$$\alpha_n = \frac{(H+1)u_{NL}(t) - \sum_{k=0}^{H}\frac{1}{\tau_c}[\sum_{i=1}^{n-1} R_i\alpha_i e^{\frac{-(t+\frac{k}{H}dt-t_i)}{\tau_c}}]}{\frac{R_n}{\tau_c}\sum_{k=0}^{H} e^{\frac{-(t+\frac{k}{H}dt-t_n)}{\tau_c}}}. \quad (32)$$

Until this step, α is considered as a free parameter. However, it is obvious that the muscle will not be stimulated by whatever amplitude. In our case, we suppose that α_i can range between 0 and 2, which corresponds to 2 times the stimulation constraint imposed in [2] (the pulse amplitude is still in an acceptable range).

4 Simulations Results

Differential equations of the force model (1) to (3) are solved by numerical methods. The control method is applied on the force model for the force references F_{ref}= 100, 200, 400, 600 and 800 N with stimulation times dt= 10, 20, 40, 60, 80, 100 ms.

In the Figures 1 and 3, the generated force for F_{ref}=100, 200, 400, 600 and 800 N is represented with the corresponding computed α (Figures 2 and 4) (case of unconstrained α). The developed force staying constant at its final value during the simulation, we represent just the obtained force for a 1 s and 10 s duration. Contrary to the cases of small reference values (100, 200 and 400 N) where the impulse amplitude is at most equal to 0.2 (Figure 2), the α values for F_{ref}= 600 and 800 N reach very high values (Figure 4). As discussed above, the applied pulse to the muscle must have a reasonable amplitude. The effects of this constraint is showed in Figure 5 where α is limited to 0.5, 1 and 2 for F_{ref}= 800 N. It can be observed that α's limit influences the final value of F. In fact, the smaller α is, the farther the final value of F is from F_{ref}.

In Figure 6, we treat the effect of dt (dt varies for 10, 20, 40, 60, 80 and 100 ms) on the final value of the force for F_{ref}= 100, 200, 400, 600 and 800 N. It is clear that the difference between F and F_{ref} increases with dt.

To get closer to experimental results, a white noise of 5% seems adequate to mimic realistic conditions, that's why a 5% noise is added to the force measurements in all the control simulations. On the Figure 7, the obtained force is presented. At high reference force, the noise is not negligible and we must take into account the disturbance on the force and on the stimulation. With a low reference force, the developed force is not disturbed by noise.

5 Conclusion

In this work, we applied a control method to control the force value of a muscle during a stimulation session. The computed control method acts on the electrical impulse amplitude during an EMS. The simulation results showed a good efficiency of this control by maintaining the force at the reference force. Stimulation time and impulse amplitude effects were also explored for different cases of reference forces. A small stimulation time with a constrained impulse amplitude seems to be the best strategy to compute an efficient control. In a next study, it would be interesting to include muscular fatigue effects in order to check if this control could be still efficient then to test experimentally.

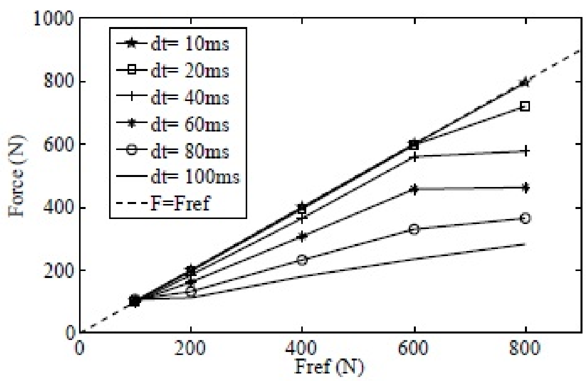

Figure 6: Application of the nonlinear control for the reference forces F_{ref}= 100, 200, 400, 600 and 800 N, the obtained forces for dt= 10, 20, 40, 60, 80, 100 ms.

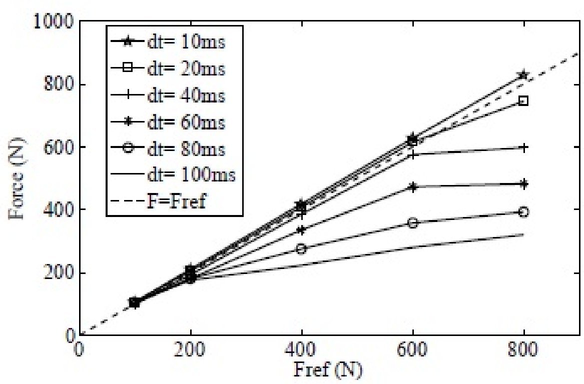

Figure 7: Application of the white noise of 5% for the F_{ref}= 100, 200, 400, 600 and 800 N and dt= 10, 20, 40, 60, 80, 100 ms.

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