$$Y_z = f(X_z; \beta_z) + \varepsilon_z$$

Where z = 1, 2, 3... n; Y_z represent the observed output level of the z^{th} sample farmer; f (X_z; β_z) is the convenient frontier production function (e.g. Cobb-Douglas or Trans log); X_z denotes the actual input vector by the z^{th} farmer; β_z stand for the vector of unknown parameters to be estimated; ε_z is a composed disturbance term made up of two error elements (V_z and U_z) and n represents the number of farmers involved in the survey.

Thus, Cobb-Douglas frontier function was specified as follows:

$$Y_z = AX_1{}^{\beta 1} X_2{}^{\beta 2} \ldots X_n^{\beta n} \quad (2)$$

The linear form of Cobb-Douglas production functions for this study is defined as:

$$\ln(Y_z) = \beta o + \sum_{j=1}^{4} \beta_j \, lnX_{jz} + \varepsilon_z \quad (3)$$

$$\ln(Y_z) = \beta o + \beta 1 \ln(SEED) + \beta 2 \ln(LAB) + \beta 3 \ln(OXEN) + \beta 4 \ln(LAND) + \varepsilon_z$$

$$\varepsilon_z = V_z - U_z$$

Where, ln denotes the natural logarithm (i.e., base e); j represents the number of inputs used; z represents the z^{th} farm in the sample; Y_z represent the observed sorghum output of the z^{th} sample farmer; X_{jz} denotes z^{th} farm input variables will be used in sorghum production of the z^{th} farmer; β_0 represent intercept; $\beta_1 - \beta_4$ stands for the vector of unknown parameters to be estimated and represent elasticity of production; ε_z is a composed disturbance term made up of two error elements (V_z and U_z); the symmetric component (V_z) is assumed to be independently and identically distributed as random errors with zero mean and variance N (0, σ^2v), which captures inefficiency as a result of factors beyond control of farmers and U_z proposed to capture inefficiency effects in the production of sorghum.

We can define the farm-specific TE in terms of observed output (Y_z) to the corresponding frontier output (Y^*) using the existing technology.

$$TE_z = \frac{Y_z}{Y^*} = \frac{f(X_z;\beta)\exp(V_z - U_z)}{f(X_z;\beta)\exp(V_z)} = \exp(-U_z) \quad (4)$$

Determinants of Technical Efficiency

In this study, two-limit Tobit regression model was used to know factors affecting TE of farmers, which was specified as:

$$y_z^* = \beta_0 + \sum_{k=1}^{12} \beta_k X_{kz} + U_z \quad (5)$$

Where: y_z^*, latent variable representing the TE scores of farm z ; β_o intercept; β_k unknown parameter; X_{kz} are demographic, institutional, soci-economic and farm-related variables which are expected to affect TE; k is a number of explanatory (independent) variables that affect technical efficiency, and U_z is an error term that is independently and normally distributed with mean zero and variance σ^2.

Denoting y_z as the observed variables:

$$y_z = \begin{cases} 1 \text{ if } y_z^* \geq 1 \\ y_z^* \text{ if } 0 < y_z^* < 1 \\ 0 \text{ if } y_z^* \leq 0 \end{cases}$$

Likelihood ratio statistic

According to Bravo and Pinheiro (1997) gamma (γ) can beformulated as:

$$\frac{\lambda^2}{1+\lambda^2} \quad (7)$$

In this study, the likelihood ratio was conducted to select the appropriate functional form that best fits the data. The value of the generalized likelihood ratio (LR) statistic to test the hypotheses that all interaction terms, including the square specification is equal to zero ($H_0 = 0 = \beta_{jz} = 0$) calculated as follows.

Following Greene (2003), the hypothesis tests conducted using the LR statistics.

$$LR(\lambda) = -2\ln\left[\frac{L(H_0)}{L(H_1)}\right] = -2\left[lnL(H_0) - lnL(H_1)\right] \quad (8)$$

Where: LR= Generalized log-likelihood ratio

L (H_o) = Denotes the likelihood function value under the null (H_o)

L (H_1) = Denotes the likelihood function value under alternative hypothesis (H_1)

This value compared with the upper 5% point for the χ^2distribution and the decision made based up on the model result. If the calculated χ^2 value is less than the tabulated upper 5% point of the critical value, we accept the specified null hypothesis at 5% level of significance.

Yield gap measurement

Yield gap is the difference between yield potential and actual farmers' yields over a given spatial or temporal scale (Ittersum *et al*., 2013). In this study, yield gap analysis was applied to determine how much sorghum yield is lost because of efficiency variation among farmers in the study area. From the stochastic model defined in TE of z[th] farmer was estimated as follows:

$$TE_z = \frac{Y_z}{Y_z{}^*} = \frac{f(Xz;\beta)\exp(Vz-Uz)}{f(Xz;\beta)\exp(Vz)} = \exp(-U_z) \quad (9)$$

Then solving for Y_z^*, the potential yield (qt/ha) of each sample household was represented as:

$$Y_z^* = \frac{Y_z}{TE_z}$$

$$= f(X_z;\beta)\exp(V_z)$$

TE_z= technical efficiency of the z[th] sample household in sorghum production.

Y_z^*= the frontier or potential output of the z[th] sample household in sorghum production in qt/ha.

Y_z=the actual or observed output of the z[th] sample household farmer in sorghum production in qt/ha.

Hence, yield gap (qt/ha) =potential yield (qt/ha)-actual yield (qt/ha).

Thus, Yield gap $= Y_z^* - Y_z$

3. Results and Discussion

Descriptive Statistical Results

Inputs used for sorghum production

The production function for this study was estimated using four input variables. On average, sample households produced 15.72 qt of sorghum, which is the dependent variable in the production function. The land allocated for sorghum production, by sample households during the survey, ranged from 0.1 ha to 4.75 ha with an average of 1.14 ha. On average, the amount of seed, human labor and oxen power the households used was 22.33kg, 43.97 and 9.17 respectively (Table 1)

Table 1: Summary statistics of variables used to estimate the production

Variable	Unit	Mean	Std. Dev.	Minimum	Maximum
Output	Qt	15.72	8.23	3	41
Seed	Kg	22.33	14.45	2	98
Labor	MD	43.98	28.51	7.8	166.10
Oxen power	Pair of oxen day	9.17	5.47	1	30
Land	Ha	1.14	0.72	0.1	4.75

Source: Own computation (2019)

Table 2: Generalized likelihood ratio tests of hypothesis for the parameters of the SFPF

Null hypothesis	df	LR	χ^2 value at 5%	Decision
$H_0 = \beta_{zj} = 0$	10	15.46	18.31	Accept H_0
$H_0 = \gamma = 0$	1	10.04	3.84	Reject H_0
$H_0: \delta_0 = \delta_{1=}\delta_{2=}..\delta_{12} = 0$	12	149.38	21.03	Reject H_0

Econometric Result

Test of Hypothesis

The attractive feature of SPF model is that, it is possible to test various hypotheses using maximum likelihood ratio test, which were not possible in non-parametric models. Therefore, before discussing about parameter estimates of production frontier function and the inefficiency effects, it is advisable to run the several hypotheses tests in order to choose an appropriate model for further analysis and interpretation (Greene, 2003). Accordingly three hypotheses were tested, to select the correct functional form for the given data set, the existence of inefficiency and variables that explain the difference in efficiency respectively using the likelihood ratio test specified in equation 8. The result of the first hypothesis tested indicates that Cobb-Douglas production function best fit the data. The second and third hypothesis tested also implies that there was inefficiency among smallholder sorghum producers in the study area (Table 2).

Parameter estimates of the SFPF model

The maximum likelihood estimates of the parameters of the SFPF for sorghum farm in Hidabu Abote district was presented in Table 3. The results of the model showed that the input elasticity for each input in the SFPF. Among four input variables analyzed in the stochastic frontier model, the parameter for labor and oxen power were found to be significant at 1%, as hypothesized implying that increasing the amount labor and oxen power by one percent would result in 0.444% and 0.345% increase in sorghum output. The result is similar with the finding of Kusse *et al*. (2018).

Efficiency Scores and their Distribution

The model output presented in Table 4 indicates that the mean values of TE of the sample households were about 65.2%. This result is similar with Ali *et al*. (2012) studied efficiency of faba bean in Northern State Sudan and found the mean TE of 65%. The mean TE of sample farmers was about 0.652 with a minimum level of 0.24 and the maximum level of 0.903. This means that if the average farmer in the sample was to achieve the technical efficient level of its most efficient counterpart, then the average farmer could realize 27.8% derived from (1-0.652/0.903)*100 increase sorghum output by improving TE with existing inputs and technology, using the resource at their disposal in an efficient manner without introducing other improved or external inputs and practice.

Yield gap due to technical efficiency variation in the study area

Using the values of the actual output obtained the predicted TE indices; the potential output was estimated for each household in sorghum production on the hectare bases. Hence, the mean level of both the actual and the potential sorghum yield in the cropping season was thus 17.19 qt/ha and 25.77 qt/ha respectively. Using the t-test method, the mean difference of the actual and the potential yield was found to be statistically significant at 1% level of significance. Therefore, the average yield gap that lost due technical inefficiency, which was the mean difference between actual (17.19 qt/ha) and the potential output (25.77 qt/ha) was, 8.58 qt/ha. This indicates that there is a room to increase the production level on average by 8.58 qt/ha with the existing level of input use. On average, the money value of sorghum output that lost due to technical inefficiency (yield gap) was 10725 birr/ha.

Determinants of technical efficiency

The result of a two-limit Tobit regression model showed that among 12 variables used in the analysis, education, soil fertility, striga weed, and frequency of extension contact were found to be statistically significant in affecting the level of TE of the farmers (Table 5). The marginal effects of change in explanatory variables were presented in Table 6 below.

Table 3: MLE for the parameters of the SFPF

Variables	Parameter	Coef.	Std. Err.
Intercept	β_0	0.796	0.537
Lnseed	β_1	-0.003	0.145
lnLAB	β_2	0.444^{***}	0.064
lnOXEN	β_3	0.345^{***}	0.084
Lnland	β_4	0.002	0.146
Variance parameter:			
$\sigma^2 = \sigma_V^2 + \sigma_U^2$		0.425	0.1028
$\lambda = \sigma_{u/}\sigma_v$		2.438	0.173
Gamma (γ)		0.856	
Log likelihood function	-87.31		

Note: *** refers to 1% significance level, respectively

Table 4: Estimated TE scores

Types of efficiency	Mean	Std. Dev.	Min	Max
TE	0.652	0.167	0.240	0.903

Table 5: A two- limit Tobit regression results of determinants of TE

		TE	
Variables	Parameters	Coef.	Std. Err.
Const	δ_0	0.5013^{***}	0.0479
SEX	δ_1	0.0109	0.0210
EXPRNCE	δ_2	-0.0008	0.0009
EDUC	δ_3	0.0092^{**}	0.0046
TRCNDTN	δ_4	-0.0292	0.0250
FAMSZE	δ_5	0.0007	0.0042
SOILFERT	δ_6	0.0620^{**}	0.0247
STRIGA	δ_7	-0.0864^{***}	0.0233
DISTNCE	δ_8	0.0001	0.0002
FEXTN	δ_9	0.0504^{***}	0.0064
NONFI	δ_{10}	-0.0112	0.0162
TCLAND	δ_{11}	0.0101	0.0080
LIVSTOK	δ_{12}	0.0036	0.0031

Note: ***, ** and *sign represents significance at 1%, 5% and 10% levels, respectively.

Table 6: Marginal effects of change in explanatory variables
Note: $\frac{\partial E(y)}{\partial X_j}$ (total change), $\frac{\partial E(y^*)}{\partial X_j}$(expected change) and $\frac{\partial[\varphi(Z_U)-\varphi(Z_L)]}{\partial X_j}$ (change in probability).

	Marginal effect of TE		
Variables	$\partial E(y)$	$\partial E(y^*)$	$\partial[\varphi(Z_U)-\varphi(Z_L)]$
SEX	0.0109	0.0107	0.0008
EXPRNCE	-0.0008	-0.0008	-0.0001
EDUC	0.0092**	0.0090**	0.0008**
TRCNDTN	-0.0291	-0.0286	-0.0022
FAMSZE	0.0007	0.0007	0.0001
SOILFERT	0.0618**	0.0605**	0.0060**
STRIGA	-0.0863***	-0.0850***	-0.0056***
DISTNCE	0.0001	0.0001	0.0000
FEXTN	0.0503***	0.0494***	0.0042***
NONFA	-0.0112	-0.011	-0.0009
TCLAND	0.0101	0.0099	0.0008
LIVSTOK	0.0036	0.0035	0.0003

Accordingly, the result of two- limit Tobit model (Table 5) for each significant variable and its marginal effects of change in explanatory variables (Table 6) on TE were discussed as follows.

Education level of household (EDUC): The finding of the study shows that education affected TE of sorghum producers significantly and positively at 5% significance level. The positive sign implies that farmers that are more educated tend to be more efficient in agricultural production than the less educated in the study area. This is due to the fact that better educated household head can understand agricultural instructions easily, have higher tendency to adopt improved agricultural technologies, have better access to information, and are able to apply technical skills imparted to them than less educated ones. Additionally, the calculated marginal effect revealed that, a one year increase in educational level of the household head increases the probability of a farmer being technically efficient by 0.08% and the mean values of technical by about 0.9% with an overall increase in the probability and levels of technical efficiencies by 0.92%. In line with this study, research done by Solomon (2012), Chepng'etich **(**2013), Sisay *et al.* (2015) and Nigusu (2018) explains that the more educated the farmer, the more technically efficient s/he becomes.

Frequency of extension contact (FEXTN): The result showed that the variable had positive sign and significant effect on TE at 1% level as expected. The reason is that farmers who had more frequency of extension; could lead them to improvements in resource allocation, facilitates practical use of modern techniques and use inputs in appropriate way. In addition, the computed marginal effect revealed that, a one times increase in frequency of extension of household head increases the probability of a farmer being technically efficient by 0.42% and the mean values of technical efficiencies by about 4.94% with an overall increase in the probability and levels of technical efficiencies by 5.03%. This is in line with previous studies by Haileselassie (2005), Wudineh and Endrias (2016) and Getachew *et al*. (2018).

Perception of farmers on fertility status of soil (SOILFERT): The result indicated that soil fertility status was positive and significant effect on TE at 5% level of significance as expected. This implies that, fertility of land is an important factor in influencing the level of TE in the production of sorghum or positively contributes to TE of sorghum in the study area. This may be associated with those fertile lands have high nutrients which leads to increase to increase in the production and efficiency of farmers. This implies that households who allocated land which was relatively fertile were better in TE. Therefore, decline in soil fertility could be taken as cause for significant productivity difference. Moreover, a change in the dummy variable, fertility status of the soil from (0 to 1), would increase the probability of the farmer being TE by about 0.6% and the expected values of TE by about 6.05% with an overall increase in the probability and levels of TE by 6.18% respectively. This result is similar with the empirical findings of Awol (2014) and Getachew *et al.* (2018)

Striga weed (STRIGA): It was found to have a significant and negative effect on TE at 1% significance level. The result indicates that the sample farmer whose sorghum exposed to striga infestation was less efficient than others. This is due the fact that striga weed caused the output loss of sorghum in terms of quantity and quality by competing minerals and water with sorghum plant. Even if there is no quantitative information on yield loss assessment specific to the area, striga weed in general is becoming a big threat in Ethiopia more than any other parasitic weeds (Fassil, 2002). The finding of Birhane (2016) shows that striga attributed 25% annual sorghum losses in Ethiopia. There is also additional evidence by Gebisa (2007) that there was an estimated yield reduction of 65-70 % in major sorghum growing areas where in heavy striga infestation losses often reach 100 %. Additionally, the computed marginal effect result also shows that, a change in the dummy variable, striga weed infestation from (0 to 1), would decrease the probability of the farmer being technically efficient by about 0.56% and the expected values of TE by about 8.5% with an overall decrease in the probability and levels of TE by 8.63%.

4. Conclusion and Recommendation

Conclusion

The estimated stochastic production frontier model indicated that labour and oxen power were significant determinants of production level. The coefficients related with the inputs measure the elasticity of output with respect to inputs. The results showed that the input variables specified in the model had positive effect on the output of sorghum production except quantity of seed used. The stochastic frontier model results revealed that there was inefficiency in smallholder sorghum production in the district. The study found that the average TE was 65.2%. This implies that farmers can increase sorghum production by 34.8% without increasing inputs if they were technically efficient. The results of the two- limit Tobit regression model revealed that, education level of household head, soil fertility and frequency of extension contact were significantly and positively affected TE, while striga weed had significant and negative effect on TE as expected. The positive and significant variables implies that they play great role in enhancing efficiency and productivity of sorghum while the negative and significant variables indicates that it decreases the efficiency and production of sorghum producers in the study area. An important conclusion coming from the analysis is that, sorghum producers in the study area are not operating at full TE level which implied that there is an opportunity for sorghum producers to increase output at existing levels of inputs without compromising yield with present technologies.

Recommendation

The result of the study provides information and got some policy recommendations to policymakers as follows:

- Giving education for farmers on practical training on different agronomic practices by *woreda* agriculture and natural resource office.
- Appropriate capacity building program for farmers and development agents should be provided by *woreda*, zonal and regional agriculture and natural resource offices.
- Improvement of soil fertility status through practicing different soil conservation techniques should have to done *woreda* agriculture and natural resource offices.
- Research center with integration of government should supply striga resistance sorghum variety at a time and also developmental program should give attention on this issue.

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