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About new conditions of wave-wave interaction

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Abstract: - New conditions for wave interaction are considered. It is shown that using these conditions allows us to discover and describe new features of wave interaction. Specifically, it is shown that an electromagnetic wave in a stationary periodic medium can excite a wave with a different frequency. It is shown that waves with different frequencies can effectively interact in such a medium. It is shown that these conditions can reveal new features of three-wave interaction in nonlinear media.

Key-Words: - wave-wave interaction, three-wave

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1 Introduction

In plasma physics, two fundamental physical processes are distinguished: wave-particle interactions and wave-wave interactions. The main results of the theory of wave interactions are based on local interaction conditions. This means that phase-matching conditions are imposed along each of the four axes of four-dimensional spacetime $(t, \vec{r})$:

$$\Delta\vec{k} = \sum_i \vec{k}_i = 0,$$

$$\Delta\omega = \sum_i \alpha_i \omega_i = 0; \ \alpha_i = \pm 1 \qquad (1)$$

Here ω_i are the frequencies of the interacting waves, and $\vec{k}_i$ are the wave vectors of these waves. These conditions coincide with the conditions for resonant particle interaction. These conditions are often referred to as the law of conservation of momentum and the law of conservation of energy. Indeed, if we multiply the left-hand and right-hand sides of these conditions by Planck's constant, they truly describe the conservation laws of elementary processes during particle interactions.

Moreover, these are interactions in quantum mechanics. They are not applicable to conservation laws within classical electrodynamics. They only indicate the necessary phase relationships between interacting waves. Moreover, such a beautiful analogy hinders attempts to find other conditions for effective wave interaction. Of course, conditions (1) correctly describe both particle interactions and wave interactions. They underlie all wave-wave interaction processes not only in plasma physics. See, for example, in monographs and reviews [1], [2], [3], [4], [5], [6]).

It should be kept in mind, however, that the processes of particle and wave interaction differ significantly. Particle interaction is indeed local, occurring within a small spatiotemporal region. Wave interaction most often occurs in a region whose spatiotemporal characteristics are significantly greater than the wavelength and wave period. It may turn out that a phase mismatch, for example along the time axis ($\Delta\omega \neq 0$), can be compensated for by a mismatch along one of the spatial axes ($\Delta k \neq 0$). Analysis of these features showed that such compensation is indeed possible. As a result, the wave interaction conditions took the form $\Delta\omega = (\Delta k \cdot V)$. In [7] described some consequences of the new interaction conditions. It should be noted that there are some other results on generalizing the conditions of wave interaction under various conditions (see, for example, [8-11]). These generalized conditions apply to specific cases, for example, those where geometric optics methods can be used [8-10]. However, in all cases, these generalizations are based on considerations related to conditions (1). Paper [11] discusses the problem of wave-wave resonances, as the most fundamental nonlinear phenomena that can occur between waves.

In this paper, we continue to explore some of the consequences of using the new wave interaction conditions [7]. In the next (second) section, the problem and all the constraints used will be formulated. The basic system of truncated partial differential equations will be written out. In the third section, it will be shown that the use of the new conditions allows us to prove the fact that a wave propagating in a stationary, periodically inhomogeneous medium can excite another wave. Moreover, the frequency of the excited wave can be differ from the frequency of the excited wave. This result undermines the well-known notion (the well-known paradigm) that a wave in a stationary medium cannot excite waves with different frequencies. In other words, only elastic scattering processes can occur in a stationary medium. It is further shown that such waves (waves with different frequencies) can effectively exchange energy among themselves.

2 Problem Statement and Basic Equations

Let's consider the simplest model of wave interaction, in which new conditions for wave interaction can be realized. This is a three-wave interaction model, in which one of the interacting waves is a fixed wave. The characteristics of this wave do not change. The permittivity of the medium can be considered as such a wave. For example, consider a medium whose permittivity can be represented as:

$$\varepsilon = \varepsilon_0 + \tilde{\varepsilon},\ \tilde{\varepsilon} = q\cos(\vec{\kappa}\vec{r} - \Omega t),\ q << 1 \quad (2)$$

Let's assume that two electromagnetic waves with different frequencies propagate in such a medium. We will be interested in the conditions for the effective interaction of these waves in such a medium. The equations for each of these waves are Maxwell's equations. From Maxwell's equations, it is easy to find an equation for the electric field vectors of the waves in such a medium:

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2(\varepsilon\vec{E})}{\partial t^2} = -\vec{\nabla}\left(\frac{1}{\varepsilon}\vec{E}\cdot\vec{\nabla}\varepsilon\right) \quad (3)$$

Let all three vectors $\left(\vec{k}_0, \vec{k}_1, \vec{\kappa}\right)$ lie in the plane of the page. Then equation (3) will take on a simpler form:

$$\Delta\vec{E} - \frac{1}{c^2}\frac{\partial^2(\varepsilon\vec{E})}{\partial t^2} = 0 \quad (4)$$

By assumption, there are two waves, therefore we will seek the solution to equation (4) as the sum of two terms:

$$\begin{aligned}\vec{E} &= \vec{A}_0(\vec{r},t)\exp(-i\omega_0 t + i\vec{k}_0\vec{r}) + \\ &+ \vec{A}_1(\vec{r},t)\exp(-i\omega_1 t + i\vec{k}_1\vec{r})\end{aligned} \quad (5)$$

Here $k_0^2 = \omega_0^2\varepsilon_0 / c^2$, $k_1^2 = \omega_1^2\varepsilon_0 / c^2$.

Each term in (5) describes a plane electromagnetic wave, the amplitudes of which are functions of time and coordinates. Since the spatiotemporal inhomogeneity of the medium is assumed to be small, we will assume these amplitudes to be slowly varying functions of space and time.

$$\frac{\partial^2 A_j}{\partial z^2} \sim \frac{\partial^2 A_j}{\partial t^2} \sim q^2 << 1 \quad (6)$$

Let also $\vec{A}_0 = A_{0y} \equiv A_0$, and $\vec{A}_1 = A_{1y} \equiv A_1$;

$\vec{k}_0 = \{k_{0x}, 0, k_{0z}\}$; $\vec{k}_1 = \{k_{1x}, 0, k_{1z}\}$; $\vec{\kappa} = \{\kappa_x, 0, \kappa_z\}$

$k_0^2 = \omega_0^2\varepsilon_0 / c^2$, $k_1^2 = \omega_1^2\varepsilon_0 / c^2$ $V_j \equiv v_{phj} = \dfrac{\omega_j}{k_j}$

Considering all these features and limitations, we can write the following system of equations to find slowly changing functions A_j:

$$\begin{aligned}\hat{L}_0 A_0 &\equiv \left[k_{0z}\frac{\partial}{\partial z} + k_{0x}\frac{\partial}{\partial x} + \frac{\varepsilon_0\cdot\omega_0}{c^2}\frac{\partial}{\partial t}\right]A_0 = \\ &= -\frac{q}{4i}\frac{(\omega_1 \pm \Omega)^2}{c^2}A_1\cdot\exp\left[+i\cdot\delta(z,t)\right]\end{aligned} \quad (7)$$

$$\begin{aligned}\hat{L}_1 A_1 &\equiv \left[k_{1z}\frac{\partial}{\partial z} + k_{1x}\frac{\partial}{\partial x} + \frac{\varepsilon_0\cdot\omega_1}{c^2}\frac{\partial}{\partial t}\right]A_1 = \\ &= -\frac{q}{4i}\frac{(\omega_0 \pm \Omega)^2}{c^2}A_0\cdot\exp\left[-i\cdot\delta(z,t)\right]\end{aligned} \quad (8)$$

Here $\delta(\vec{r},t) \equiv \Delta\vec{k}\cdot\vec{r} - \Delta\omega\cdot t$, $\Delta\vec{k} \equiv \left(\vec{k}_1 - \vec{k}_0 \pm \vec{\kappa}\right)$, $\Delta\omega \equiv \omega_1 - \omega_0 \pm \Omega$

3 Excitation of a wave with frequency ω_1 by a wave with frequency ω_0 in a stationary medium ($\Omega = 0$).

Let $\hat{L}_0\exp(i\delta) = 0$. This means:

$$\Delta\omega = \left(\Delta\vec{k}\cdot\vec{V}_0\right) \quad (9)$$

Let's use this property of the operator $\hat{L}_0$. We'll apply it to equation (8). Then equation (8) for determining the amplitude A_1 will take the form

$$\hat{L}_0\hat{L}_1 A_1 = -\frac{q^2}{16\cdot c^4}\left[(\omega_0 \pm \Omega)(\omega_1 \pm \Omega)\right]^2 A_1 \quad (10)$$

First, let's consider the case of a stationary medium $(\Omega = 0)$. Furthermore, we will simplify the equations (where possible) in the future. For our purposes, it is sufficient to consider spatially one-dimensional dynamics. Equation (10) can then be rewritten as

$$\begin{aligned}&\left[\frac{\partial^2}{\partial z^2} + \frac{1}{V_0 V_1}\frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial z}\frac{\partial}{\partial t}\left(\frac{1}{V_0} + \frac{1}{V_1}\right)\right]A_1 = \\ &= -\frac{q^2 k_0 k_1}{16}A_1\end{aligned} \quad (11)$$

To evaluate the characteristic spatial and temporal quantities that determine the process of excitation of a wave with frequency, we determine the temporal dynamics at fixed coordinates ($z = const$):

$$\frac{\partial^2 A_1}{\partial t^2}+\frac{q^2\omega_0\omega_1}{16}A_1=0 \qquad (12)$$

The amplitude of the excited wave periodically varies with a characteristic frequency $\Omega_{ch}=(q/4)\sqrt{\omega_0\omega_1}$. The dispersion diagram of these waves is shown in Figure 1. Similarly, the characteristic scale of spatial variation of this amplitude ($t=const$) can be estimated:

$$\frac{\partial^2 A_1}{\partial z^2}+\frac{q^2 k_0 k_1}{16}A_1=0 \qquad (13)$$

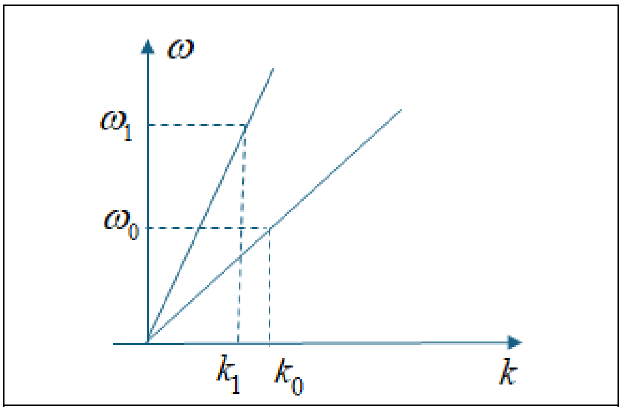

Fig. 1: $\omega_1 \neq \omega_0$, $k_0 k_1 > 0$. A possible scheme for exciting a wave with a frequency (ω_1) by another wave with a frequency (ω_0)

Spatial "dynamics" are richer. If $k_0 k_1 > 0$, then the amplitude of the excited wave periodically changes with a spatial period equal to $K_{ch}=(q/4)\sqrt{k_0 k_1}$. If $k_0 k_1 < 0$, then the spatial dynamics resemble Bragg reflection, with the only difference being $\omega_1 \neq \omega_0$ (see Figure 2). The change in amplitude A_0 can be found by substituting the solution of equation (11) into equation (7). However, restricting the process under consideration to the excitation of a wave by the field of another wave is sufficiently limited by the fact that the average value A_0 does not change ($\langle A_0\rangle = const$).

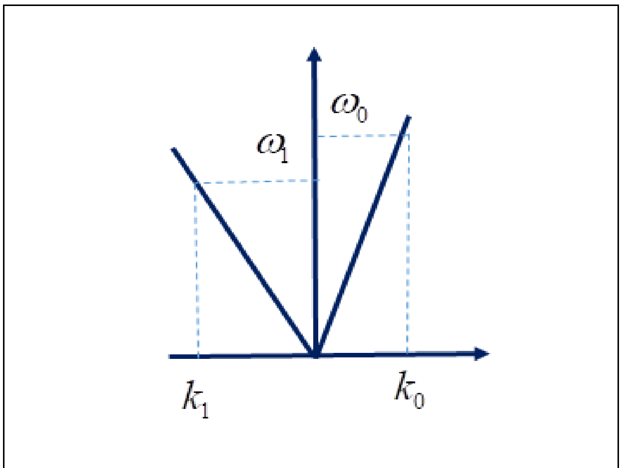

Fig. 2: $\omega_1 \neq \omega_0$, $k_0 k_1 < 0$. Dispersion lines of the exciting (ω_0) and excited (ω_1) waves

Using (9), we obtain the following expression for the frequency of the excited wave

$$\omega_1 = \pm\frac{\kappa V_0 V_1}{(V_1-V_0)}\ , \quad V_1 \neq V_0 \qquad (14)$$

4 Interaction of Waves with Different Frequencies

Above, we examined the process of waves generating new waves in stationary media. In this section, we will demonstrate that it is possible to study the interaction of such waves as well.

To do this, we will rewrite the system of equations (7), (8):

$$\left[\frac{c}{\sqrt{\varepsilon_0}}\frac{\partial A_0}{\partial z}+\frac{\partial A_0}{\partial t}\right]=-a_{10}A_1\cdot\exp\left[+i\cdot\delta_+(z,t)\right] \quad (15)$$

$$\left[\frac{c}{\sqrt{\varepsilon_1}}\frac{\partial A_1}{\partial z}+\frac{\partial A_1}{\partial t}\right]=-a_{01}A_0\cdot\exp\left[-i\cdot\delta_-(z,t)\right] \quad (16)$$

Here $\alpha_{10}=\frac{q}{4i}\frac{(\omega_1+\Omega)^2}{\varepsilon_0\omega_0}$, $\alpha_{01}=\frac{q}{4i}\frac{(\omega_0-\Omega)^2}{\varepsilon_1\omega_1}$,

$\delta_\pm(\vec r,t)\equiv\Delta\vec k_\pm\cdot\vec r-\Delta\omega_\pm\cdot t$, $\Delta\vec k_\pm\equiv\left(\vec k_1-\vec k_0\pm\vec\kappa\right)$,

$\Delta\omega_\pm\equiv\omega_1-\omega_0\pm\Omega$

In deriving this system of equations, we limited ourselves to a one-dimensional inhomogeneity. Furthermore, we considered that one wave can interact with one component of the inhomogeneity $\delta_+(\vec r,t)$ and another wave can interact with another ($\delta_-(\vec r,t)$). Let us use the characteristics of equations (15) and (16):

$$\frac{dt}{1}=\frac{dz}{V_0}=-\frac{dA_0}{\alpha_{10}A_1 exp(i\delta_+)} \ ;$$

$$\frac{dt}{1}=\frac{dz}{V_1}=-\frac{dA_1}{\alpha_{01}A_0 exp(-i\delta_-)} \qquad (17)$$

These characteristics allow us to replace the two partial differential equations (15) and (16) with four ordinary differential equations:

$$\frac{dA_0}{dt}=-\alpha_{10}A_1\exp(i\delta_+)\ , \quad \frac{dz}{dt}=V_0\ ;$$

$$\frac{dA_1}{dt}=-\alpha_{01}A_0\exp(-i\delta_-)\ , \quad \frac{dz}{dt}=V_1\ ; \qquad (18)$$

Using the solutions of the second and fourth equations ($z=V_0 t$, $z=V_1 t$) we obtain the following expressions for $\delta_\pm(\vec r,t)$:

$$\delta_+=(V_0\cdot\Delta k_+-\Delta\omega_+)\cdot t\ , \ \delta_-=(V_1\cdot\Delta k_--\Delta\omega_-)\cdot t \qquad (19)$$

We can impose the following conditions on these quantities

$$\delta_+ = \delta_- = 0\ ;\ \Delta\omega_+ = V_0 \Delta k_+\ ,\ \Delta\omega_- = V_1 \Delta k_- \quad (20)$$

We are considering in the most interesting case: $\Omega = 0$. Then formula (20) gives the following expression for the frequency of the excited wave

$$\omega_1 = \omega_0 \frac{(V_0 + V_1)}{(3V_0 - V_1)} \quad (21)$$

Using the system of equations (18) and conditions (20), the following useful relationships can be obtained

$$\frac{d^2 A_j}{dt^2} = -\Omega_q^2 A_j\ ,\ \ j = \{0,1\}\ ,$$

$$A_0^2 - \mu A_1^2 = const \quad (22)$$

Where $\Omega_q^2 = \frac{q^2 (\omega_1 + \Omega)^2 (\omega_0 - \Omega)^2}{16 \cdot \varepsilon_1 \cdot \varepsilon_0 \cdot \omega_1 \cdot \omega_0}$,

$$\mu = \frac{(\varepsilon_1 \cdot \omega_1)}{(\varepsilon_0 \cdot \omega_0)} \left(\frac{\omega_1 + \Omega}{\omega_0 - \Omega} \right)^2$$

From the first equation, we can immediately determine the characteristic time of energy exchange between interacting waves ($T_q \sim (25/q\omega)$). The second relation indicates that the exchange process is bounded by a certain integral. Note that, as follows from (18), the complex amplitudes A_0 *and* A_1 are shifted relative to each other by $\pi/2$.

5 Wave Interaction in Nonlinear Media.

The above-described conditions for wave synchronism in inhomogeneous media are apparently the easiest to implement and use for experimental verification of the formulated interaction conditions. However, the formulated conditions can also be implemented in many other physical processes. Such processes can include wave interactions in nonlinear media. In this section, we consider three-wave interactions in nonlinear media. The main purpose of this consideration is to demonstrate the fact that new conditions of interaction can reveal new features of many other processes.

The equations that describe the interaction of three waves ($\omega_0, \omega_1, \omega_2$; $\vec{k}_0, \vec{k}_1, \vec{k}_2$) in a nonlinear medium can be represented as (see, for example, [1]):

$$\frac{\partial a_0}{\partial l_0} \equiv \dot{a}_0 + (\vec{V}_0 \vec{\nabla}) a_0 = -\sigma_0 a_1 a_2 \exp(-i\delta) \equiv f_0$$

$$\frac{\partial a_1}{\partial l_1} \equiv \dot{a}_1 + (\vec{V}_1 \vec{\nabla}) a_1 = \sigma_1 a_0 a_2^* \exp(i\delta) \equiv f_1 \quad (23)$$

$$\frac{\partial a_2}{\partial l_2} \equiv \dot{a}_2 + (\vec{V}_2 \vec{\nabla}) a_2 = \sigma_2 a_0 a_1^* \exp(i\delta) \equiv f_2$$

Here, the dots denote partial derivatives with respect to time. $\delta(\vec{r},t) \equiv \Delta\vec{k} \cdot \vec{r} - \Delta\omega \cdot t$ is the detuning, which we do not consider a slow function of coordinates and time; $\Delta\omega = \omega_0 - \omega_1 - \omega_2$; $\Delta\vec{k} = (\vec{k}_0 - \vec{k}_1 - \vec{k}_2)$; $\vec{V}_i$ are the group velocities of the waves; σ_i are the matrix elements of the nonlinear interaction.

The left-hand side of each equation in system (23) is represented as derivatives along the characteristic directions. For clarity, we will consider the interaction of waves with positive energy ($\sigma_i > 0$), and we will also focus on decay processes. The equations for the characteristics of each equation in system (23) can be written as:

$$\frac{dt}{1} = \frac{dx}{V_{ix}} = \frac{dy}{V_{iy}} = \frac{dz}{V_{iz}} = \frac{da_i}{f_i} \quad (24)$$

Unit vectors directed along these characteristics will have the form: $\vec{l}_i = \{V_{ix}, V_{iy}, V_{iz}, 1\} / N_i$.

Here $N_i = \sqrt{V_{ix}^2 + V_{iy}^2 + V_{iz}^2 + 1}$. The condition for synchronism of the interacting waves will be the condition of parallelism of the characteristic lines of the hyperplane $\delta(\vec{r},t) \equiv \Delta\vec{k} \cdot \vec{r} - \Delta\omega \cdot t = const$, i.e., the condition:

$$\Delta\omega - \Delta\vec{k} \cdot \vec{V}_i = 0 \quad (25)$$

The linear stage of the decay process proceeds as instability. At this stage, the wave with the maximum frequency can be considered fixed ($a_0 = const$). In this case, system (23) can be conveniently rewritten as:

$$\frac{\partial a_1}{\partial l_1} = \sigma_1 a_0 a_2^* \exp(i\delta) \quad (26)$$

$$\frac{\partial a_2^*}{\partial l_2} = \sigma_2 a_1 a_0^* \exp(-i\delta)$$

It is convenient to rewrite system (26) as a single second-order partial differential equation with constant coefficients:

$$\frac{\partial^2 a_1}{\partial l_1 \partial l_2} = \sigma_1 \sigma_2 |a_0|^2 a_1 \quad (27)$$

In deriving (27), we used the fact that the derivatives along the characteristic directions of the function $\delta(\vec{r},t)$ are equal to zero. Substituting the solution in the form $\sim \exp(i\Omega t - i\vec{\kappa}\vec{r})$ into (27), we obtain the following dispersion equation, which determines the relationship between the frequency Ω and the vector $\vec{\kappa}$:

$$\Omega^2 - \Omega \cdot \vec{k}(\vec{V}_1 + \vec{V}_2) + \left[k^2 (\vec{V}_1 \vec{V}_2) + \sigma_1 \sigma_2 |a_0|^2 \right] = 0 \quad (28)$$

From the synchronism conditions (25) it follows that synchronism will be possible and under the condition of equality of group velocities ($\vec{V}_1 = \vec{V}_2$). Solving equation (28) for Ω , in this case we obtain:

$$\Omega = \vec{k}\vec{V} \pm i|a_0|\sqrt{\sigma_1\sigma_2} \qquad (29)$$

The imaginary part of the frequency $\left(\operatorname{Im}\Omega = |a_0|\sqrt{\sigma_1\sigma_2}\right)$ determines the increment of decay instability. It is of interest to discover new possibilities for decays that do not fit within the known decay conditions. Let us consider the simplest. Let us assume that a transverse wave decays into a transverse wave and one of the eigenmodes of a magnetized plasma waveguide ($t_0 \to t_1 + l$). We will assume that all three waves fit into the linear dispersion region (see Fig.3). In this case, the group velocities of all three waves coincide ($\vec{V}_0 = \vec{V}_1 = \vec{V}_2 = \vec{V}$). Moreover, the group velocities in this case coincide with the phase velocities. The synchronism conditions (25) in this case will be satisfied for any triplet of waves. Indeed, conditions (25) in this case take the form of an identity:

$$\omega_0 - \omega_1 - \omega_2 = \left(\frac{\omega_0}{V} - \frac{\omega_1}{V} - \frac{\omega_2}{V}\right)V \qquad (30)$$

Thus, in the case under consideration, the decay process can involve a large number of wave triplets. Moreover, these waves may differ little in their characteristics. In this case, the decay process becomes chaotic. Let us demonstrate this. System of equations (26) has the following integral: $\sigma_2|a_1|^2 - \sigma_1|a_2|^2 = C = const$. (31)

It is convenient to make the following substitution of dependent complex variables: $a_j = |a_j|\exp(i\Phi_j)$. Next, we take into account that at the initial stage of the decay process, the amplitudes $|a_1|$ and $|a_2|$ are small ($C \to 0$), and also that

$$l_0 = l_1 = l_2 \equiv l, \ \left(\partial\delta/\partial l\right) = 0 .$$

In this case, it is easy to see that dynamics of the phase $\Phi = 2(\Phi_0 - \Phi_1 - \Phi_2 + \delta)$ obey the equation of a mathematical pendulum:

$$\frac{\partial^2\Phi}{\partial l^2} + (2|a_0|\sqrt{\sigma_1\sigma_2})^2 \sin\Phi = 0 . \qquad (32)$$

Simultaneously with the decay of a transverse wave into a plasma wave and a transverse wave, this transverse wave decays into another plasma wave and another transverse wave, both lying on the dispersion line (Figure 3). It is obvious that the interaction process of this new trio of waves will be described by a system of equations similar to (23). The magnitude of the phase $\Psi = 2(\Phi_0 - \Phi_3 - \Phi_2 + \delta_1)$ will then obey the equation of a mathematical pendulum, similar to (32). The distance between nonlinear resonances can be arbitrarily small. Nonlinear resonances overlap. The decay regime becomes chaotic (Chirikov criterion).

It should be noted that the case under consideration of the coincidence of all velocities can be solved analytically not only at the linear stage but also at the nonlinear stage. For this, it is sufficient to use the effective potential method (see, for example, [1]). Such a solution is beyond the scope of our interests. Moreover, because, as we noted above, the decay process will be chaotic, this solution loses its meaning.

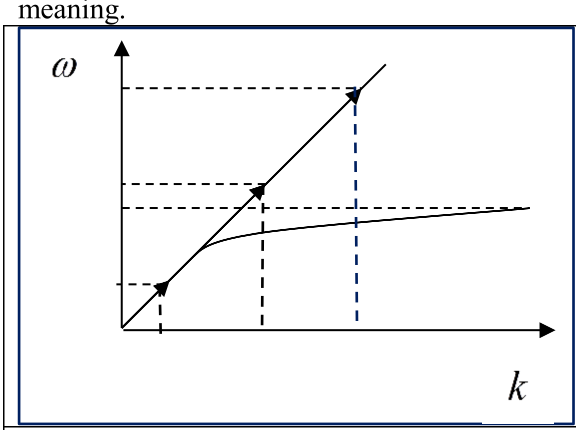

Fig. 3: Schematic diagram of the decay of a transverse wave into a transverse wave and a wave of a magnetized plasma waveguide

6 Conclusion

Thus, the results obtained above demonstrate that the new wave interaction conditions are a useful tool for discovering new features of such an important process as wave-wave interaction. Let us briefly highlight the most important of these.

First, it's worth noting the result that a wave propagating in a periodic stationary medium can excite another wave. Importantly, the frequency of the excited wave can be differed from the frequency of the wave that excited it. This result challenges our notion (our paradigm) that only elastic processes can occur in a stationary medium.

It should be noted that a similar result was previously described in the paper [6]. However, the experimental conditions for observation were difficult. The result presented in section 3 can be easily observed. However, it should be noted that a system containing a wave is, strictly speaking, not stationary. It is important that such waves can interact (section 4). Clearly, these results are primarily important for diagnostic purposes.

An important result is described in the fifth section. This result exemplifies the successful use of new conditions in studying wave interactions in nonlinear media. However, it is not only an example. The considered scheme for the stochastic decay of a transverse wave into a transverse wave and a Langmuir wave can be useful for plasma heating. This is especially true if the Langmuir wave is an Ion Langmuir wave (this wave interacts most effectively with the ion component of the plasma). In this case, the considered scheme offers the most direct way to deliver energy for plasma heating.

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