$$det(M) = \omega^2 \cdot (\omega^2 - \boldsymbol{k}^2)^2.$$

6.2 Solution of the Equations

Solution of the system is as follows:

$$\beta_1 = ik_2A_3 - ik_3A_2, \quad \beta_2 = ik_3A_1 - ik_1A_3,$$

$$\beta_3 = ik_1A_2 - ik_2A_1, \quad \varepsilon_1 = i\omega A_1 - ik_1\Phi,$$

$$\varepsilon_2 = i\omega A_2 - ik_2\Phi, \quad \varepsilon_3 = i\omega A_3 - ik_3\Phi,$$

where are:

$$A_i = -\frac{\gamma_i}{\omega^2 - {k_1}^2 - {k_2}^2 - {k_3}^2},$$

$$\Phi = (k_1A_1 + k_2A_2 + k_3A_3)/\omega.$$

The solution is written in the variables $\boldsymbol{B}, \boldsymbol{E}, \boldsymbol{j}$ as

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}, \qquad \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{A} - \boldsymbol{\nabla}\,\Phi,$$

where $\boldsymbol{A}$ satisfies the wave equation:

$$\left(\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2\right)\boldsymbol{A} = \boldsymbol{j},$$

and Φ satisfies the Lorentz condition:

$$\frac{\partial}{\partial t}\,\Phi + \boldsymbol{\nabla} \cdot \boldsymbol{A} = 0.$$

Solution of the wave equation may be done using for example the Green lagging function. The Lorentz condition can be also reduced to the wave equation computing the divergence of the vector $\boldsymbol{E}$:

$$div\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{\nabla} \cdot \boldsymbol{A} - \boldsymbol{\nabla} \cdot \boldsymbol{\nabla}\,\Phi = \left(\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2\right)\Phi.$$

Thus, for Φ we get also the wave equation:

$$\left(\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2\right)\Phi = \rho,$$

Where $\rho = div\boldsymbol{E}$ is the charge density satisfying the continuity equation

$$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0,$$

which is assumed to be a known function.

7 Conclusion

Derivation of the Doppler Effect from the modified Maxwell equations with total derivatives by time was considered and analyzed in this paper. It was shown that the modified Maxwell equations contain a description of the Doppler Effect in the form of O.E. Akimov [2]. The Doppler Effect takes place when waves of any nature (not only electromagnetic) propagate in a homogeneous and isotropic continuous medium. The problem raised the new attention of scientists during the last time, e.g. [4].

Researchers in the field and the students including the application of the Computerized Educational Platform (CompEdu) [5-7] may use the presented materials by the spreading of the electromagnetic waves and analysis of the Maxwell equations. It can also be useful for studying the Doppler effect of electromagnetic wave propagation.

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