f) and keep only terms to zeroth order in ε, we find linear system for u_0, v_0 and P_0, whose expressions were find by integration in radial direction and apply boundary conditions after assigning and using a prescribed volume flow rate Q_0 at this order in the form

$$Q_0 = 2\pi\int_0^R r\, u_0\, dr. \qquad (3b)$$

Next, we apply similar procedure at the first order in ε and find some rather lengthy expressions for u_1, v_1 and P_1.

3. Results and Discussion

We carried out numerical calculations of several expressions for several different values of λ, γ, t_s, z and r for fixed values of Q_0 =1, b=0.5, ε=0.001 and ε_1 =0.1. Our first calculated results are for the case of aspect ratio γ<<1, where the results are qualitatively the same for γ≤0.01. For given value of z=0.7 inside the stenosis zone and at a given instant in time, we find that dP/dz (axial rate change of the blood pressure in the artery) is oscillatory in time with magnitude of oscillation increases with the hematocrit effect (Figure 3). The blood speed also oscillates with the same frequency, but its magnitude of oscillation decreases with increasing the hematocrit parameter. This result is in contrast to the case where the pulse frequency has moderate value [2]. The wall shear stress (negative of the radial derivative of the axial velocity at the artery wall) was found to have similar behavior to that of the blood pressure force and agree with the results for moderate pulse frequency case [1]. Comparing these results with the corresponding results for order one or large values of γ, we find that both blood pressure force, wall shear stress and the blood speed have smaller magnitudes of the oscillations in the case of small value of the aspect ratio. Next we consider the case of moderate values of the aspect ratio, such as a typical value was found to be γ=1. Similar to the case of small aspect ratio, the magnitudes of oscillations of the blood pressure force and the wall shear stress decrease with increasing the hematocrit effect, while the magnitude of the oscillations of the blood speed decreases with increasing the hematocrit effect. Comparing these results for the moderate value of the aspect to those for either small or large values of γ, we find that the magnitudes of the oscillations of the blood flow quantities increase with γ.

We also calculated the values of the blood pressure force, blood speed and the wall shear stress for order one values of γ such as γ=1 and for different values of the hematocrit parameters, axial and radial variables and given instant in time. We find that in contrast to the case of small γ and moderate frequency of oscillations [2], the values of these blood flow quantities are lower in the stenosis zone as compare to the corresponding values of in the artery flow regions outside the stenosis zone.

4. Conclusion

We investigated the unsteady blood flow in an elastic artery with stenosis to calculate blood flow quantities such as blood pressure force, blood speed and the wall shear stress for the case of very low frequency of the oscillation and for arbitrary values of the ratio γ of the artery radius to the axial extent of the stenosis. We found, in particular, that the blood pressure force, blood speed and the wall shear stress are oscillatory in time. The magnitudes of the blood pressure force, blood speed and the wall shear stress are smaller for γ<<1. The magnitudes of oscillations of blood pressure force and the wall shear stress increase with the hematocrit effect, while the magnitude of the oscillation of the blood speed decreases with increasing the hematocrit parameter regardless of the imposed value of the aspect ratio. For moderate values of the aspect ratio, the magnitudes of the flow oscillations for pressure force, velocity and wall shear stress are lower in the stenosis zone than the corresponding ones outside the stenosis region.

The present paper and the corresponding results are consider the first part of our ongoing research work, and we plan to investigate other flow quantities in the elastic artery, extend to two-phase blood flow, and the more general cases to include the nonlinear effects for both steady and unsteady cases.

Another important extension of the present study can be for the unsteady stenosis evolution and for medically realistic finite systems with cases conforming more to the medically generated data in order to identify the components of elastic arterial blood flow diseases which can improve the health conditions of the corresponding patients.

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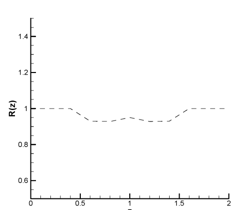

Figure 1. Shape of the stenosis

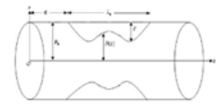

Figure2.Flow geometry with stenosis

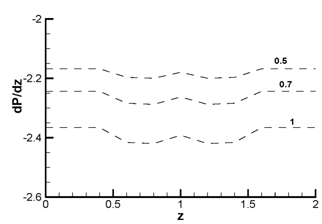

Figure3.Blood pressure gradient versus *z* and different values of hematocrit