Oscillatory Blood Flow in Elastic Artery with Stenosis

D. N. RIAHI
School of Mathematical & Statistical Sciences, University of Texas Rio Grande Valley,
One West University Boulevard, Brownsville, Texas 785520-4933, USA
(Email:daniel.riahi@utrgv.edu)(http://www.utrgv.edu/math/)

Abstract: We investigate oscillatory blood flow in an elastic artery and in the presence of stenosis, whose form is given analytically. The stenosis is a condition where an artery wall thickens as a result of fatty materials such as cholesterol. We consider the case of very low pulse frequency that can be the case for patients whose hearts beat too slowly, but with arbitrary value of the ratio γ of the artery radius to the axial extent of the stenosis. The arbitrary value of such ratio makes the present problem more applicable to more general applications for arterial blood flow systems. The blood flow is assumed to be a suspension of red cells in plasma, and we make use of the variable fluid viscosity modeling by counting the red cells in the plasma. The coupled differential equations for the blood flow velocity and blood pressure are simplified first subjected to reasonable modeling and approximations and then solved mostly theoretically. In this paper, blood speed, blood pressure force and the wall shear stress are computed for different values of γ, axial variable, time variable and the hematocrit due to percentage level of the red cells in the plasma. We find, in particular, that the magnitudes of the oscillations of the blood pressure force and the wall shear stress increases with the hematocrit effect, while such magnitude for the blood speed decreases with increasing the hematocrit effect.

Key-Words: arterial flow, blood flow, stenosis, oscillatory flow, low frequency, blood cells, blood plasma

1. Introduction

Diseases that occur in the blood vessels and in the heart are the major causes of mortality worldwide. The underlying cause for these events is the formation of lesions, known as stenosis. These lesions can grow and occlude the artery and hence prevent blood supply to the distal bed, which could lead to heart attacks or stroke. Srivastava et al. [4] studied arterial blood flow through an overlapping stenosis and in the absence of a catheter. They calculated impedance and shear stress for different stenosis height. Riahi et al. [1] investigated arterial blood flow in the presence of an overlapping stenosis and with no catheter using the variable viscosity model due to Einstein for the blood flow. There have also been studies of the blood flow systems in the catheterized arteries such as the one due to Srivastava and Rastogi [5].

In all the studies that have been carried out so far for the arterial blood flow system, the values of the ratio of the artery radius to the axial extent of the stenosis were given a fixed number less than one. In the present paper we consider arbitrary value of such ratio, but we restrict the oscillatory blood flow to the case of very low frequency, which can be the case for the atherosclerosis patients whose hearts beat too slowly. This can be the case for such patients to have bradycardia illness. In the present study we apply an analytically based shape [1] for the stenosis shape in the artery (Figure 1), where the blood is represented by a variable viscosity model, which is known as Einstein model for the blood viscosity [1]. In figure 1 we provide non-dimensional shape function $R(z)$ versus non-dimensional value of the axial variable of an artery, where the geometry of a segment of the artery containing the stenosis, whose axial extent is designated by L_0, is given in Figure 2.

2. Formulation & Analysis

We consider the problem of axisymmetric flow of blood in an artery in the form of a circular cylindrical annulus tube with the radius R_0 and in the presence of an atherosclerosis whose shape (Figure 1) is based on a mathematical model [1]. The artery length is assumed to be sufficiently large in comparison to its radius so that the end effects can be neglected.

The blood flow system in an artery is based on the original governing equations for the mass conservation and momentum [6] for unsteady axisymmetric form in cylindrical coordinate system with axial direction along the co-axial direction of artery. These equations are simplified under reasonable conditions for mild stenosis, unidirectional flow assumption [6], where the axial velocity component dominates over the radial velocity components, and the inertial nonlinear terms in the momentum equations are small. The simplified non-dimensional form of the unsteady equations for the blood flow are given below

$$\gamma R_e\, \partial u/\partial t+(1/r)\partial(r\tau)/\partial r+\partial P/\partial z =0, \qquad (1a)$$

$$\partial P/\partial r=0, \qquad (1b)$$

$$(1/r)(\partial/\partial r)(r\, v)+\partial u/\partial z=0, \qquad (1c)$$

where

$$\tau=-[1+\lambda(1-r^n)](\partial u/\partial r). \qquad (1d)$$

Here u is the axial component of blood velocity, v is the radial velocity, t is time variable, r and z are the cylindrical coordinates with axial variable z along the tube axis and radial variable r along the direction perpendicular to the tube axis, P is the blood pressure, τ is the shear stress, λ is the parameter representing the maximum hematocrit and is referred to as the hematocrit parameter, $R_e=UR_0/\mu$ (Reynolds number), U is a velocity scale, μ is a constant reference for the kinematic viscosity, $\gamma=R_0/L_0$ is the aspect ratio of the artery radius to the axial extent of the stenosis, and n ($n\geq 2$) is the parameter for the shape of constriction [1], which is taken as n=2 in this paper. The boundary conditions for the blood flow are [2]

$$\partial u/\partial r=0 \quad \text{on } r=0, \qquad (1e)$$

$$(u, v)=(\partial\xi/\partial t, \partial\eta/\partial t), \text{ on } r=R(z), \qquad (1f)$$

where ξ and η are the axial and radial displacements for the elastic artery, respectively [2]. The expression for the shape function $R(z)$ of the internal segment of the artery containing the stenosis is given by

$$R(z)=1-\varepsilon_1[11(z-b)-47(z-b)^2 +72(z-b)^3 -36(z-b)^4]$$

for $b\leq z \leq(b+1)$ and $R(z)=1$ for otherwise. (2)

Here $b=d/L_0$ and $\varepsilon_1= [3\delta/(2\, R_0)]$ is the non-dimensional maximum stenosis height (figure 2). Since the equations for these displacements are rather lengthy and the focus in present paper is on blood flow quantities, we do not provide these equations here and refer the reader to [2] for such equations.

We now consider an artery in the form of a tube with small radius. The inside boundary of the artery is partially structured along a distance L_0 due to the presence of a stenosis (figure 2). In figure 2, where the flow system and the geometry is shown in the cylindrical annulus, the arterial tube is given over a distance $L=2d+L_0$ in the axial direction, δ is the maximum height of the atherosclerosis into the lumen, which appears at particular location in the axial direction.

We now consider the oscillatory blood flow under the restriction that the frequency of the oscillation is small, we introduce a slow time scale $t_s =\varepsilon t$, where $\varepsilon<<1$ is a small quantity. We use such rescaling in the system (1a-f) and find that to the zeroth order in ε, the solutions for this system for blood velocity and pressure admit product of an oscillatory part in time and a function of z and r. It appears a relevant solution to the system can be based on a perturbation procedure, where to order up to $O(\varepsilon^2)$ can be like

$$(u, v, P)=(u_0, v_0, P_0)\, sin(\omega t_s)+\varepsilon(u_1, v_1, P_1)\, cos(\omega t_s)+O(\varepsilon^2), \qquad (3a)$$

where ω is the frequency of the pulse oscillation, and the quantities with subscripts 0 and 1 are only functions of z and/or r.

Using (3a) in the rescaled form of (1a-

f) and keep only terms to zeroth order in ε, we find linear system for u_0, v_0 and P_0, whose expressions were find by integration in radial direction and apply boundary conditions after assigning and using a prescribed volume flow rate Q_0 at this order in the form

$$Q_0 = 2\pi\int_0^R r\, u_0\, dr. \qquad (3b)$$

Next, we apply similar procedure at the first order in ε and find some rather lengthy expressions for u_1, v_1 and P_1.

3. Results and Discussion

We carried out numerical calculations of several expressions for several different values of λ, γ, t_s, z and r for fixed values of Q_0 =1, b=0.5, ε=0.001 and ε_1 =0.1. Our first calculated results are for the case of aspect ratio γ<<1, where the results are qualitatively the same for γ≤0.01. For given value of z=0.7 inside the stenosis zone and at a given instant in time, we find that dP/dz (axial rate change of the blood pressure in the artery) is oscillatory in time with magnitude of oscillation increases with the hematocrit effect (Figure 3). The blood speed also oscillates with the same frequency, but its magnitude of oscillation decreases with increasing the hematocrit parameter. This result is in contrast to the case where the pulse frequency has moderate value [2]. The wall shear stress (negative of the radial derivative of the axial velocity at the artery wall) was found to have similar behavior to that of the blood pressure force and agree with the results for moderate pulse frequency case [1]. Comparing these results with the corresponding results for order one or large values of γ, we find that both blood pressure force, wall shear stress and the blood speed have smaller magnitudes of the oscillations in the case of small value of the aspect ratio. Next we consider the case of moderate values of the aspect ratio, such as a typical value was found to be γ=1. Similar to the case of small aspect ratio, the magnitudes of oscillations of the blood pressure force and the wall shear stress decrease with increasing the hematocrit effect, while the magnitude of the oscillations of the blood speed decreases with increasing the hematocrit effect. Comparing these results for the moderate value of the aspect to those for either small or large values of γ, we find that the magnitudes of the oscillations of the blood flow quantities increase with γ.

We also calculated the values of the blood pressure force, blood speed and the wall shear stress for order one values of γ such as γ=1 and for different values of the hematocrit parameters, axial and radial variables and given instant in time. We find that in contrast to the case of small γ and moderate frequency of oscillations [2], the values of these blood flow quantities are lower in the stenosis zone as compare to the corresponding values of in the artery flow regions outside the stenosis zone.

4. Conclusion

We investigated the unsteady blood flow in an elastic artery with stenosis to calculate blood flow quantities such as blood pressure force, blood speed and the wall shear stress for the case of very low frequency of the oscillation and for arbitrary values of the ratio γ of the artery radius to the axial extent of the stenosis. We found, in particular, that the blood pressure force, blood speed and the wall shear stress are oscillatory in time. The magnitudes of the blood pressure force, blood speed and the wall shear stress are smaller for γ<<1. The magnitudes of oscillations of blood pressure force and the wall shear stress increase with the hematocrit effect, while the magnitude of the oscillation of the blood speed decreases with increasing the hematocrit parameter regardless of the imposed value of the aspect ratio. For moderate values of the aspect ratio, the magnitudes of the flow oscillations for pressure force, velocity and wall shear stress are lower in the stenosis zone than the corresponding ones outside the stenosis region.

The present paper and the corresponding results are consider the first part of our ongoing research work, and we plan to investigate other flow quantities in the elastic artery, extend to two-phase blood flow, and the more general cases to include the nonlinear effects for both steady and unsteady cases.

Another important extension of the present study can be for the unsteady stenosis evolution and for medically realistic finite systems with cases conforming more to the medically generated data in order to identify the components of elastic arterial blood flow diseases which can improve the health conditions of the corresponding patients.

Reference:

[1] Riahi, D. N., Roy, R. and Cavazos, S., On arterial blood flow in presence of an overlapping stenosis, Mathematical and Computer Modelling, **54**, 2011, pp.2999-3006.
[2] Riahi, D. N., Modeling unsteady two-phase blood flow in catheterized elastic artery with stenosis, Engineering Science & Technology, an Int. Journal, **19**, 2016, pp. 1233-1243
[3] Srivastava, V. P., Two-phase model of blood flow through stenosed tubes in the presence of a peripheral layer: Applications, Journal of Biomechanics, **29**, 1996, pp.1377-1382
[4] Srivastava, V P, Rastogi, R and Mishra, S, Non-Newtonian arterial blood flow through an overlapping stenosis, Applications and Applied Math: An Int. Journal, **5** (1), 2010, pp.225-238.
[5] Srivastava, V. P. and Rastogi, R., Blood flow through a stenosed catheterized artery: Effects of hematocrit and stenosis shape, Computer and Mathematics with Applications, **59**, 2010, pp.1377-1385
[6]. White, F. M., 1991, Viscous Fluid Flow, Second Edition, McGraw-Hill, Inc., New York

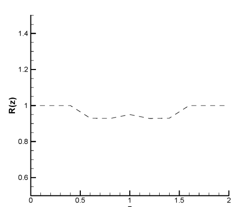

Figure 1. Shape of the stenosis

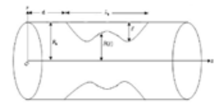

Figure2.Flow geometry with stenosis

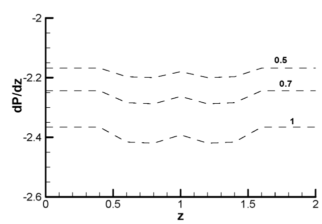

Figure3.Blood pressure gradient versus *z* and different values of hematocrit