$$=\frac{2-n}{\bar{a}_n^2}\left[\frac{1}{\lambda}\frac{\partial T_1}{\partial r}\frac{\partial p_1}{\partial r}+\frac{p_1}{\omega}\left(\frac{m}{r^2}\frac{\partial T_1}{\partial \varphi}+k\frac{\partial T_1}{\partial x}\right)\right].$$

Here $\{u_1, v_1, w_1\}, p_1$- amplitudes of perturbations of velocity and pressure, $\delta_n^2 = k^2 - \lambda / a_n^2$.

From the first four equations of the system (31) yields the modified Bessel equation for pressure perturbation:

$$\frac{d^2 p_1}{dr^2}+\frac{1}{r}\frac{dp_1}{dr}-\left(k^2+\frac{m^2}{r^2}\right)p_1=0,$$

which has the following solution

$$p_1 = A_1 I_m(kr) + A_2 K_m(kr), \qquad (32)$$

where $A = const$. The boundary conditions (28) with account (32) and conditions of unperturbed distributed along the axis heat source ($r \le s_0$) [4]:

$$r = s_0,\ \vec{v}=0;\quad r=1,\ \frac{dp_1}{dr}=\left(1-\rho_{21}\right)\bar{\rho}_1\omega\zeta, \qquad (33)$$

which result in a system of boundary conditions for differential equations (31). From (32)-(33) is got:

$$A_1 I_m(ks_0)+A_2 K_m(ks_0)=0,\ A_1 I_m'(ks_0)+A_2 K_m'(ks_0)=0,$$
$$A_1 = A_2 = 0,\ I_m(ks_0)K_m'(ks_0) \ne I_m'(ks_0)K_m(ks_0).$$

As the modified Bessel and Hankel functions have only positive values, and the first of them is monotonously increasing, the second - monotonously decreasing, then at all values of argument other than zero, this inequality is incorrect only at $k=0, m\ne 0$. However in the latter case, owing to properties of $I_m, K_m$, obviously $A_1 = A_2 = 0$ and $p_1 \equiv 0$, as well. The system is steady against rather small perturbations of its parameters, i.e. parametric oscillations fade in time here.

The boundary condition can be replaced with more general: $u_1(s_0)=0$. Two other velocity components of the unperturbed melt can be other than zero (for example, in a case when the entered axial area of constant temperature isn't motionless). Then unlike considered above it turns out: $A_1 I_m'(ks_0)+A_2 K_m'(ks_0)=0,\quad A_1 I_m'(k)+A_2 K_m'(k)=\alpha\omega^2,$ where is $\alpha=(1-\rho_{21})\bar{\rho}\zeta<\zeta$. As shown above, $\omega$ is small. The value $\alpha$ is small too for small-amplitude perturbations, therefore in a linear approach value $\alpha\omega^2$ can be neglected, where from:

$$A_1 I_m'(ks_0)+A_2 K_m'(ks_0)=0,\ A_1 I_m'(k)+A_2 K_m'(k)=0.$$

Investigation of the algebraic equation array (AEA) of two equations shows that its solution is non-trivial in case $s_0 = 1$ or $k=0$. But both cases are physically unreal because by $s_0 = 1$ all channel is filled with a melt of constant temperature $T_0 = T_*$, and by $k=0$ it is $p_1 \equiv 0$, e.g. the system is unperturbed (stability).

# 4 Conclusion

The model for solidification boundary interface has been developed. An analysis shown that the front of melt crystallization under neglect influence of the channel wall (the metallurgical unit) is steady against rather low-amplitude perturbations of parameters of physical system if only in an equilibrium state a melt was immovable.

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