Supersonic boundary layer stability with vectored mass transfer through a porous surface

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Abstract: The study continues the cycle of investigations concerned with the modeling of the methods of controlling flow regimes in compressible boundary layers. The effect of distributed heat and mass transfer on the stability parameters of a supersonic boundary layer is considered at a moderate supersonic Mach number $M = 2$. Emphasis is placed on the modeling of both the normal injection, when only the V component of the mean velocity is nonzero, and injection in other directions, including the tangential injection, when only the U component is nonzero on the wall. The formulation of the problem is similar with that of the gas curtain influence on the small fluctuation development. It is assumed that the effect of the injection of a similar gas with different temperatures is analogous to the injection of a gas with different densities, namely, the cold gas injection mimics the heavy gas injection, and vice versa. For this reason, in this study this modeling is realized by means of varying the temperature factor (wall heating or cooling). The case, in which the so-called "cutoff" regime is realized, that is, the velocity disturbances on a porous surface can be taken to be zero, is also considered.

Key-Words: compressible boundary layer, hydrodynamic stability, laminar-turbulent transition.

1 Introduction

In numerically investigating the methods of controlling the regimes of supersonic flow past bodies considerable attention has been given to the distributed mass transfer effect [1–3]. The use of injection or suction has a considerable influence on both the properties of the original flow and its stability. The suction effect on the stability and laminar-turbulent transition is well studied for a subsonic boundary layer [4]. The employment of mass transfer produces certain characteristics of boundary layers that ensure the given disturbance parameters on the range of the loss of stability and the transition. The suction withdraws low velocity gas masses from wall regions; as a result, the mean velocity profiles become more inflated and have greater transverse gradients in the wall regions, which lead to an increase in their stability, the rise of the critical Reynolds numbers, and transition delay. The boundary layers become thinner with the result that the tendency to transition in the turbulent state reduces [5]. Contrariwise, gas injection leads to the boundary layer thickening and a decrease in its inflation with the tendency to the appearance of local inflections which accelerates the disturbance growth, reduces the stability margin, and makes the transition nearer. Nevertheless, gas injection through a permeable wall is widely applied in the practice. First of all, this is due to the use of cold gas injection as a means of thermal protection of thermally stressed elements of engineering devices. Its main mechanism consists in the absorption of the thermal energy of the hot gas by a cooler arriving through the permeable surface. In this case, the direction of the cold gas injection relative to the surface in a flow can be different, from normal to tangential. In [6] many aspects of using the slot injection in the form of gas curtains are presented and their effect on heat transfer and the thermal parameters of boundary layers is discussed. Typical of the gas curtains is that the cold gas is injected along the surface. The theoretical modeling of the slot injection requires the use of the complete Navier–Stokes equations which makes the problem solution considerably more difficult.

The problem can be considerably simplified in the case of gas injection through a permeable surface. However, the techniques of tangential mass transfer through permeable walls were for a long time absent. In the few recent years a certain progress in this direction has been achieved which makes it possible to realize injection/suction at different angles to the surface. By way of illustration, we will note the paper [7] in which an insert ensuring the tangential injection is described. In this connection, a theoretical investigation of the boundary layer in the presence of gas injection through porous walls at different angles to the surface has become particularly topical. Injection of a heavy or light foreign gas can be used for the purpose of controlling the friction drag and heat fluxes. In [8] it was theoretically shown that injection of a cooled similar gas influences the drag and the heat fluxes in the same fashion as injection of a heavy foreign gas. Along with the thermal protection problem and the control of the drag and the heat fluxes, there exists another important problem concerned with the control of laminar-turbulent transition. With increase in the gas density near the wall the boundary layer stability increases which is achieved by heavy gas injection through a porous wall. In the case of a subsonic boundary layer the theoretical possibility of its stabilization by means of heavy gas injection normal to the wall was confirmed in [9]. In the present study gases with different densities are modeled by varying the temperature factor (wall heating or cooling) [10, 11]. Unfortunately, the possibility of the disturbance suppression using this means has its own restrictions. The injection normal to the surface favors the appearance of an inflection point in the velocity profile which leads to flow destabilization. To reduce this effect an attempt to inject the gas at a certain angle to the main flow direction can be made. Here, the joint effect of the injection and the temperature factor is studied in the case of flow past a permeable porous surface, whose pore radii are much smaller than the boundary layer thickness scales. In this case, the so-called "cutoff" regime [2] is realized, when the velocity disturbances on the body surface can be taken to be zero. Emphasis is placed on the modeling of distributed injection and an analysis of its effect on the disturbance development scenario at a moderate supersonic Mach number M= 2. Both the normal injection, where only the mean velocity component V perpendicular to the surface is nonzero, and injection in different directions, including the tangential injection, where only the longitudinal component U is nonzero, are modeled. This formulation can be similar with the problem of the gas curtain influence on the small fluctuation development scenario. In the case of compressible gas flow theoretical investigations are made more difficult by the necessity of taking the temperature and density disturbances into account. At M = 2 only vortex disturbances, or traveling Tollmien–Schlichting waves (first mode), are considered. At present it is well established that the normal injection destabilizes both vortex and acoustic disturbances. At the same time, in the case of flow past a surface with heat transfer the influence of this factor on disturbances can be different: in cooling the vortex disturbances are stabilized (the rate of their streamwise growth diminishes), whereas the acoustic disturbances are destabilized. The same difference in the injection effect exists in the case of heating: the vortex disturbances are destabilized, while the acoustic disturbances are stabilized. The situation is made considerably more complicated in the case of the joint influence of mass and heat transfer. Thus, the purpose of the study is an investigation of the influence of injection and heat transfer on the mean velocities, the critical Reynolds numbers of vortex disturbances, and their frequency dependences (frequency cuts).

2 Basic equations and methods of solution

2.1 Equations for Disturbances.

The method of determining the disturbance parameters in compressible supersonic boundary layers is based upon the classical perturbation method. We will present certain necessary facts. The flow field is represented in the form of the sum of the mean and fluctuating quantities [5]

$$\mathbf{u}=|U(Y)+\varepsilon u', \varepsilon v', \varepsilon w'|, \qquad \rho(Y)+\varepsilon\varsigma', \;\; P(Y)+\varepsilon p',$$
$$T(Y)+\varepsilon\Theta', \; p'/P=\varsigma'/\rho+\Theta'/T,$$

where $\varepsilon << 1$ is the fluctuation field scale. We will consider the disturbed fields of the compressible gas velocity, density, pressure, and temperature in the dimensionless Cartesian coordinate system $X, Y, Z = (x,y,z)/\delta$, where δ is the scale length, $\delta=\sqrt{x\nu_e/U_e}$. The primed and primeless quantities are the fluctuating and mean components of the corresponding quantities divided by their values at the outer edge of the boundary layer (U_e, ρ_e, T_e, P_e).

The wave solutions are sought in the form:

$$Z'=Z(Y)\exp(i\theta), \tag{1.1}$$

where $Z'=|u',v',w',p',\Theta'|$ are the perturbations of the longitudinal, normal, and transverse velocities, the pressure, and the temperature, $\theta=\alpha X+\beta Z-\omega t$, $\alpha=\alpha_r+i\alpha_i$, $\alpha_i<0$ is the growth rate, the real frequency $\omega=2\pi f$; and the wavenumbers α, β and the frequency are related by the dispersion equation $\alpha=\alpha(\omega,\beta)$ in accordance with the linear theory. The spectral parameters and the structural forms of the perturbations are determined from the Dan–Lin system:

$$\begin{gathered}
\rho(Gu+U_Y v)+i\alpha p/\gamma \mathrm{M}^2-\mu u_{YY}/\mathrm{Re}=0,\\
\rho G w+i\beta p/\gamma \mathrm{M}^2-\mu w_{YY}/\mathrm{Re}=0,\\
\rho G v+p_Y/\gamma \mathrm{M}^2=0 \qquad (1.2)\\
G\varsigma+\rho_Y v+\rho(i\alpha u+v_Y+i\beta w)=0,\\
\rho(G\Theta+T_Y v)+(\gamma-1)(i\alpha u+v_Y+i\beta w)-\frac{\mu\gamma}{\sigma \mathrm{Re}}\Theta_{YY}=0,\\
\varsigma=\rho(p/P-\Theta/T), \;\; G=i(-\omega+\alpha U).
\end{gathered}$$

Here M is the Mach number at the outer edge, $\gamma = c_p/c_v$ is the adiabatic exponent, $\sigma = c_p\mu_e/k$ is the

Prandtl number and k is the temperature conductivity coefficient. All the parameters are nonimensionlized on their values at the outer edge of the boundary layer and the following normalization of the eigenfunctions is chosen: $v(Y_k)=1$.

The homogeneous system (1.2) is the basic system for determining the eigenvalues α at given β and ω and the Reynolds numbers $\mathrm{Re}=u_e\delta/\nu$, and for constructing the amplitude eigenfunctions of the linear waves (1.1). The system was integrated using the orthogonalization method [5].

As noted above, the problem was formulated for the "cutoff" regime ($r/\delta << 1$), when, in view of the smallness of the pore radii r, the compressibility effects in the pore itself can be neglected. Then the following conditions hold:

$$u, v, w, \Theta = 0, \quad Y = 0; \tag{1.3}$$

at the outer edge and in the far field ($Y = \infty$) the usual conditions of decay (boundedness) of the amplitude functions are preassigned

$$u, v, w, \Theta = 0 \tag{1.4}$$

The solution of the eigenvalue problem for system (1.2) under the boundary conditions (1.3) and (1.4) closes the stage of constructing small linear perturbations (1.1).

2.2 Average Parameters

The boundary layer equations for the dimensional asterisked quantities are used in the Cartesian coordinate system (x, y)

$$\rho^*(u^*\frac{\partial u^*}{\partial x}+v^*\frac{\partial u^*}{\partial y})=\frac{\partial}{\partial y}\left(\mu^*\frac{\partial u^*}{\partial y}\right)$$

$$\rho^* C_p(u^*\frac{\partial T^*}{\partial x}+v^*\frac{\partial T^*}{\partial y})=\mu^*\left(\frac{\partial u^*}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(k\frac{\partial T^*}{\partial y}\right)$$

$$\frac{\partial \rho^* u^*}{\partial x}+\frac{\partial \rho^* v^*}{\partial y}=0, \quad p^*=p_e=\rho^* R^* T^*$$

In accordance with [12], we introduce the scale length $\delta=\sqrt{x^*\nu_e/U_e}$ and the dimensionless self-similar coordinate $Y=y/\delta$. In this system the dimensionless equations of the boundary layer go over into the system of ordinary differential equations

$$\frac{d}{dY}\left(\mu\frac{dU}{dY}\right)+g\frac{dU}{dY}=0$$

$$\frac{d}{dY}\left(\frac{\mu}{\sigma}\frac{dT}{dY}\right)+g\frac{dT}{dY}=-(\gamma-1)\mathrm{M}^2\mu\left(\frac{dU}{dY}\right)^2 \tag{1.5}$$

$$\frac{dg}{dY}=\frac{U}{2T}$$

When a similar gas is injected through the wall at an angle φ, the curtain angle is clockwise measured from the direction opposite to that of the main flow. The schematics of the injection are shown below. Thevelocity components at the wall are determined as follows: $V(0)=G \sin\varphi$, $U(0)=-G\cos\varphi$, where G is the absolute value of the injected gas velocity. In view of the fact that $g(0) = -V(0)Re/Tw$ [5], we obtain $g(0) = -GRe/Tw \sin\varphi$.

We will introduce the parameter $Cq =- G\mathrm{Re}/Tw$ characterizing the injection/suction intensity. In this case, the boundary conditions can be written in the form:

on the body surface ($Y = 0$) $g = Cq \sin\varphi$,

$$U = CqTw/\mathrm{Re}\cos\varphi, T' = 0 \text{ or } T = Tw, \tag{1.6}$$

where the former temperature condition corresponds to a thermally insulated wall and the latter condition to a constant wall temperature.

on the outer edge $(Y=\infty)$ $U = 1$; $T = 1$ (1.7)

The system of equations (1.5) with the boundary conditions (1.6) and (1.7) was integrated using the Runge–Kutta method from the wall to Ym, where Ym is certainly greater than the boundary layer thicness. The temperature dependence of μ was taken in acordance with the Sutherland law.

For any regime the dimensionless parameter of the wave frequency F related with the frequency by the equation $F= \omega/\mathrm{Re}$ and the dimensionless wave parameter $b = \beta\, 10^3/\mathrm{Re}$ were introduced. Both two-dimensional (plane) waves with $\beta = 0$ and three-dimensional (oblique) waves with $\beta \neq 0$ were considered. For vortex disturbances the three-dimensional components are most growing and their growth rates are considerably greater than those of two-dimensional waves.

3 Discussion of the results

Calculations have been performed in the assumption of a perfect gas with constant values of specific heat ratio γ=1.4 and Prandtl number σ=0.72. It is helpful to recall how the distributed injection/suction influences the vortex disturbance parameters at $M = 2$ [2]. In Fig. 1 the evolution of the disturbance growth rates is shown for four frequencies; the case of the impermeable wall corresponds to the $Cq = 0$ line. In the case of suction ($Cq > 0$) the disturbances of all the frequencies stabilize and become decaying at Cq

= 0.15, whereas in the case of injection (only the normal injection with $U(0) = 0$ is considered) all the disturbances become or remain growing. The value $Cq = -0.3$ was the last value for which the disturbance parameters could be obtained. At this value the mean velocity profiles with an inflection point are formed and there arise some difficulties in solving the boundary value problem for $Cq < -0.3$.

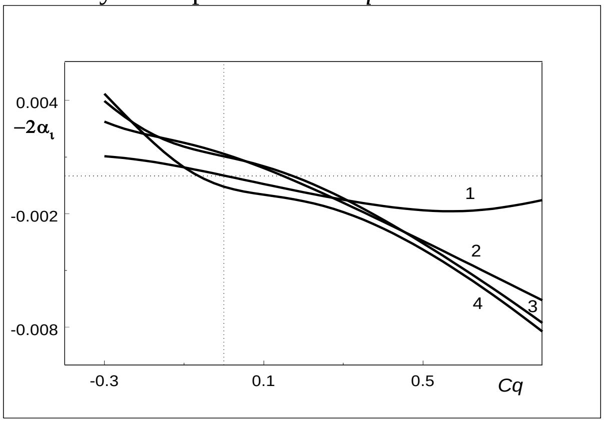

Fig.1. Growth rates of the three-dimensional vortex waves at M = 2, Re = 600, and 2b = 0.225 as functions of the mass transfer intensity Cq for the frequencies $F = (0.19, 0.38, 0.57, 0.76)\times10^{-4}$ (curves (*1*–*4*)).

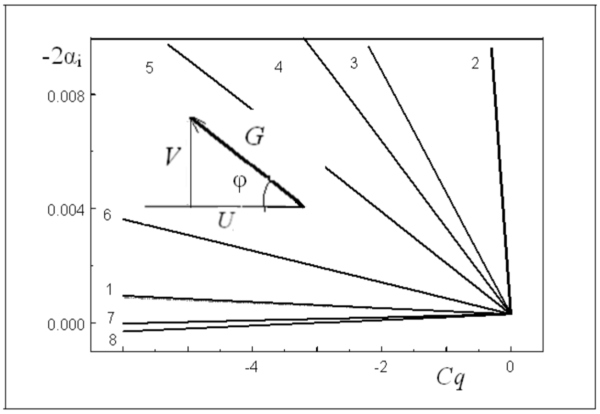

Fig. 2. Effect of the angle φ of inclination of the injected gas velocity on the growth rates of the three-dimensional vortex waves at M = 2, Re = 600, 2b = 0.225, and $F = 0.192\times10{-4}$; (*1*–*8*) relate to φ= 0, 1.57, 3.0, 3.04, 3.08, 3.12, 3.14, and 3.1416; in the same figure the schematics of the arrangement of the longitudinal U and normal V velocity components are presented.

In Fig. 2 injection is schematically represented. The angle φ is measured from the wall, when the injected-gas and main-flow velocity vectors are oppositely directed. In Fig. 2 it can be seen how the disturbance growth rates vary with the angle φ the value φ = π /2 corresponds to the normal injection, the $0 \le \varphi < \pi/2$ range to the counter-stream injection, and the $\pi/2 < \varphi \le \pi$ range to the classical streamwise injection. Here, to eliminate variant readings, the limiting positions $\varphi = 0$ and π are respectively called the counterstream and streamwise "curtains". Clearly visible is that the growth rates strongly decrease only in the vicinities of these limiting values. While the value $Cq = -0.3$ is limiting for the normal injection, in the case of the curtains Cq can increase almost without bounds. In Fig. 2 it is also well seen that the difference between the growth rates of the two different curtains increases with $|C_q|$, the streamwise curtain favoring the greater stabilization of the vortex disturbances.

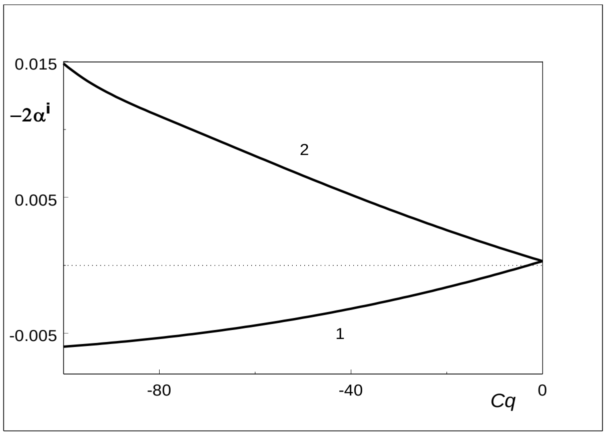

Fig. 3. Effect of the injection intensity Cq on the growth rates of the three-dimensional vortex waves at $M = 2$, Re = 600, $2b = 0.225$, and $F = 0.192\times10{-4}$ for the streamwise φ= π (*1*) and counterstream φ = 0 (*2*) curtains

In Fig. 3 this difference is illustrated for fairly large injections. The mean velocities near the wall can be readily evaluated from Eq. (1.6). Since in the case of the streamwise curtain the gas with only the velocity component U is tangentially injected into the boundary layer and this component naturally increases with $|Cq|$, the overall effect leads to an increase in the wall velocity. To some extent this is equivalent to the situation occurring in the case of suction, when near-wall low-velocity layers vanish. Another explanation of this effect is that in the wall region the difference between the outer flow and boundary layer velocities diminishes, which leads to the reduction in the effective Reynolds number Reeff and the growth rates. The increase in the growth rates in the case of the counterstream curtain can also be explained. In certain situations (when the gas is tangentially injected in a limited region) the flow pattern can be treated as a flow with boundary layer separation, which leads to an increase in Re_{eff}. The profiles of the mean longitudinal velocities and temperatures for different curtains are plotted in Fig. 4. In the case of the counterstream curtain an increase in $|Cq|$ leads to the transverse lengthening of the region of negative U. It is clearly visible that in the case of the streamwise curtain the mean temperatures in the boundary layer decrease, which favors the disturbance stabilization, and, contrariwise, in the case of the counterstream curtain these temperatures increase. It is worthwhile to remind that the temperature of the thermally insulated impermeable wall Tw = 1.78.

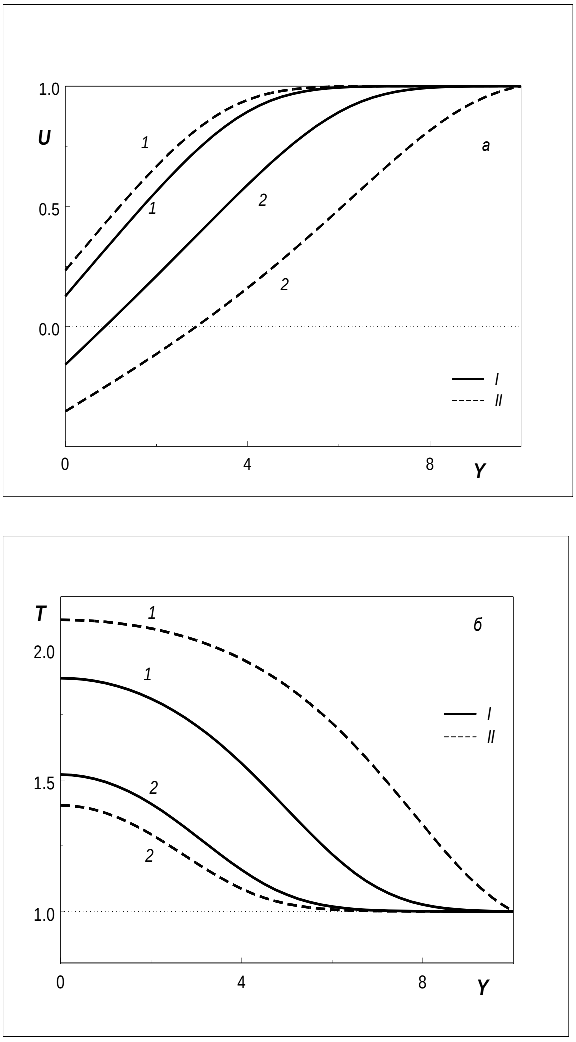

Fig. 4. Effect of the curtain angle and the injection intensity Cq on the profiles of the mean longitudinal velocity (a) and the mean temperature (b) for $Cq = -50$ (I) and -100 (II); (*1*, *2*) correspond to φ= π and 0.

The effect of injection and curtain regimes is well illustrated in Fig. 5 for the neutral curves on the thermally insulated surface and in Fig.6 for the frequency cuts. In the case of the normal injection with

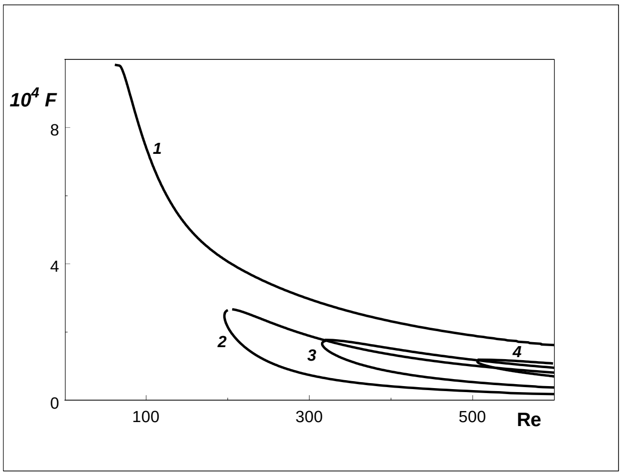

Fig. 5. Neutral curves of the vortex mode on thermally insulated surfaces for different Cq in the case of the normal injection $\varphi=\pi/2$, $Cq = -0.3$ (curve *1*) and the streamwise curtain $\varphi=\pi$, $Cq = -0.3$, -50, and -100 (*2*–*4*).

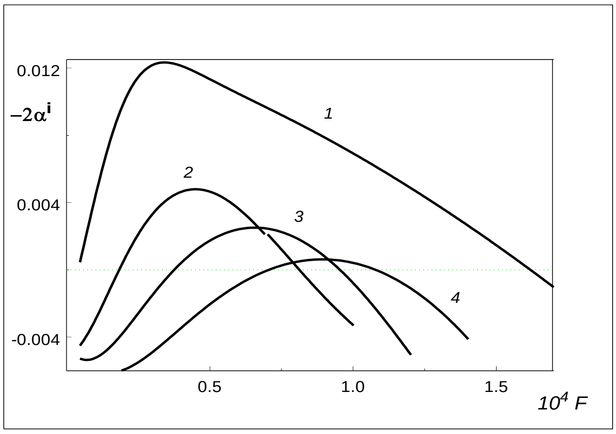

Fig. 6. Frequency cuts illustrating the growth rates of three-dimensional disturbances ($2b$ = 0.225) at Re = 600 and the same parameters as in Fig. 5

$Cq = -0.3$ the critical Reynolds number is very small ($Re_c \sim 80$) and considerably smaller than Re_c for the tangential injection (Rec = 190 for the streamwise and counterstream curtains and for the impermeable wall). It is necessary to recall that in the $Cq = -0.3$ case under consideration the variations in U and the mean temperature are so small that all the disturbance parameters coincide in the plots. With increase in $|Cq|$ in the case of the streamwise curtain the disturbances are considerably stabilized, the critical Reynolds number Rec increases, and the hazardous frequency range sharply shrinks. As shown in Fig. 6, in the case of the normal injection growing fluctuations exist within an extremely broad frequency band from 0 to 90 kHz, with very high growth rates; in the case of the streamwise curtain with the same *Cq* this range reduces down to 10 to 40 kHz, while the maximum growth rates decrease by a factor of three. The increasingly stronger variations are observable at higher |*Cq*|.

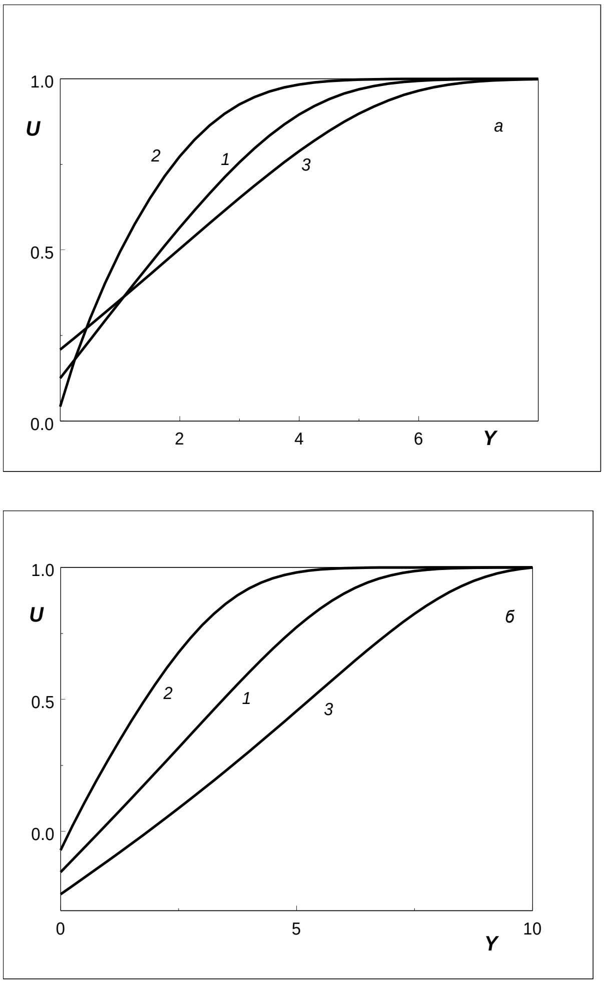

Fig. 7. Effect of the temperature factor on the mean velocity profiles at $Cq = -50$ for different streamwise (a) and counterstream curtains (b); (*1*) thermally insulated wall, (*2*) cooling, and (*3*) heating.

In considering the temperature factor influence (modeling gases with different densities) it is worthwhile to make the reference to [10, 11], where the nature of the variations in the mean velocities and temperatures under heating and cooling are considered in detail. The common feature is the thickening of the boundary layer, both dynamic and thermal, under heating and its thinning under cooling and the greater (smaller) inflation of the *U* profiles under cooling (heating). In the case of the tangential injection some interesting features appear in the proximity of the wall, as illustrated in Fig. 7.

In the case of cooling at a given *Cq* the longitudinal velocity component of the injected gas diminishes for both curtains due to the wall temperature decrease (in accordance with Eq. (1.6)), which must have a destabilizing (stabilizing) effect on the disturbance in the cases of streamwise (counterstream) curtains. There appears the competing influence on the disturbances which can manifest itself in certain regimes.

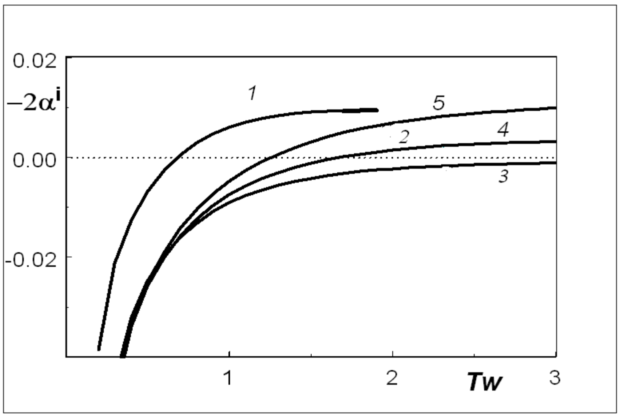

Fig. 8. Temperature (*Tw*) dependence of the three-dimensional disturbance growth rates ($2b = 0.225$) at Re = 600 and $F = 0.192\times10^{-4}$ in the case of the normal injection ($Cq = -0.3$ and $\varphi=\pi/2$, curve *1*) and at $Cq = 0.3$ and -50 for the streamwise (*2*, *3*) and counterstream (*4*, *5*) curtains.

The case is possible, in which the temperature is so low that it, as it were, blocks the injection, the initial value being $U(0)=0$. This interesting wall temperature effect on both the mean characteristics and the small fluctuations is confirmed in Fig. 8 in which the dependences of the growth rates on the wall temperature *Tw* are plotted. At $Tw < 1$ the dependences on both the type of the curtain and the injection intensity degenerate, while the disturbance decay rates are the same in all the regimes and completely coincide with the decay rates on the impermeable wall. It can also be noted that in the case of heating the domains of a weak *Tw* influence on the growth rates appear. The strongest influence can be observable on the low temperature range. The similar $\alpha_i(Tw)$ dependences exist for the normal injection which is also presented in the figure.

In Fig. 9 the wall temperature effect on the neutral curve positions is presented for the streamwise curtain. Clearly visible is that the heating and, therefore, a decrease in the injected gas density, leads to the vortex disturbance destabilization; the critical Reynolds number decreases and the hazardous frequency range broadens. Contrariwise, the cold gas injection (high density) leads to the disturbance stabilization, an increase in Re*c*, and the hazardous frequency range shrinking. It should be noted that the nature of the dependences is very regular, without any jumps.

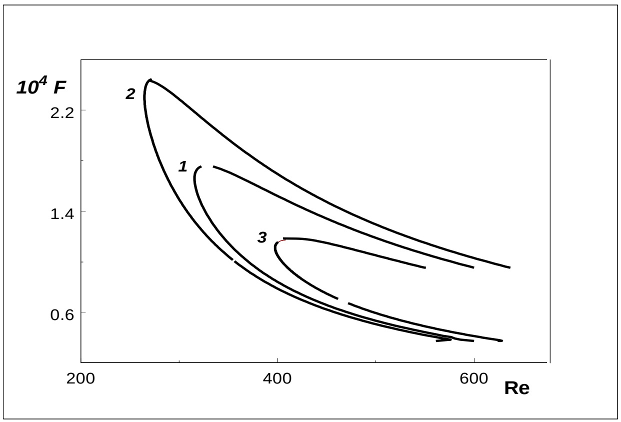

Fig. 9. Neutral curves of the vortex mode on noninsulated surfaces for the streamwise curtains at $Cq = -50$ in the case of heating (Tw =1.62, *2*) and cooling (Tw =1.42, *3*) in comparison with the case of the thermally insulated wall (Tw =1.52, *1*).

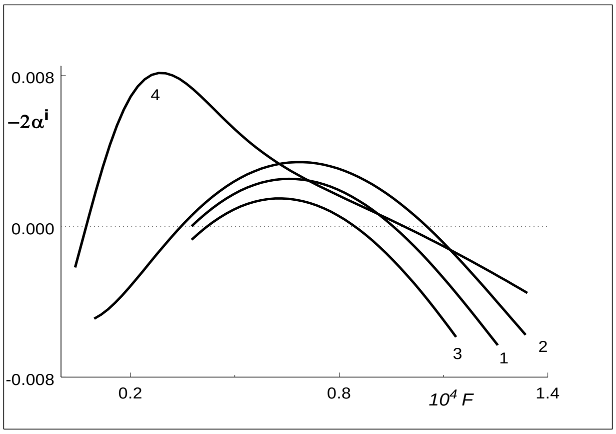

Fig. 10. Frequency cuts illustrating the growth rates of three-dimensional disturbances ($2b$ = 0.225) at Re = 600 and $Cq = -50$; for the streamwise curtain the same parameters as in Fig. 9 and for the counterstream curtain thermally insulated wall with Tw = 1.889 (*4*).

This regularity is also clearly visible on the frequency cuts (Fig. 10) plotted for the same temperatures. For the lower temperatures the shrinking of the domain of existence of growing fluctuations is accompanied by a considerable decrease in the growth rates. This is in complete agreement with the inferences made in [9, 13]. For the sake of comparison in the same figure the frequency cut of the counterstream curtain is also plotted. Clearly that in this regime the hazardous frequency range is considerably wider, the growth rates are high, and their maximum is displaced toward the lower frequencies (about 15 kHz in the case under consideration.

4 Conclusions

The methods of controlling flow regimes in compressible boundary layers by means of distributed injection are modeled.

The variations in the mean parameters of the boundary layers are studied for different versions of the tangential injection. It is shown that in the case of the streamwise curtain the near-wall velocities increase, which leads to considerable stabilization of the vortex modes. At large injection intensities the disturbances can become decaying. Contrariwise, in the case of the counterstream curtain a reverse flow can be realized in the wall regions, which considerably destabilizes small fluctuations.

The influence of the temperature factor modeling the injection of gases with different densities is considered. Along with the previously investigated behavior of the mean temperatures and velocities under heating and cooling, in the case of the tangential injection some interesting features are revealed in the wall region.

At a given injection intensity in the cooling regime the longitudinal near-wall velocities of the injected gas decrease for both curtains due to the wall temperature reduction. This wall temperature effect on both the mean parameters and the small fluctuations is confirmed by the dependences of the growth rates on the wall temperature Tw. At low temperatures, $Tw < 1$, the dependences on both the type of the curtain and the injection intensity degenerate and the disturbance decay rates are the same in all the regimes. The strongest Tw effect is observable on the low temperature range. The wall temperature effect manifests itself both in the neutral curve positions and in the form of frequency cuts. For the lower temperatures the domains of existence of growing fluctuations and increasing disturbance growth rates shrink. This has a positive effect on the stabilization of flow regimes. The comparison with the frequency cuts for the counter stream curtains shows that in this regime the hazardous frequency range is considerably wider, the growth rates are high, and their maximum is displaced toward the lower frequencies. The general conclusion is that at supersonic velocities the flow regimes can actually be controlled. Using the distributed tangential injection the thermal protection of the surface in a flow can be realized with conserving the laminar nature of the flow.

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