Mathematical Modelling and Computational Simulation of Elasticity-Driven Profit Optimization

SHAQIR ELEZAJ[1], LAVDIM BEQIRI[1*], RAME ELEZAJ[2]
[1]University for Business and Technology (UBT),
KOSOVO

[2]CTA SH.P.K., Kosovo
Peja / Kosova, 30000
KOSOVO

**Corresponding author*

Abstract: This paper presents a mathematical and computational framework for elasticity-driven profit optimization. Linear and nonlinear demand–supply models are formulated in order to derive equilibrium conditions and elasticity coefficients. The analytical derivations are complemented with numerical simulation and parametric sensitivity analysis. The nonlinear equilibrium solution is obtained using the Lambert W function, while computational validation is performed through the Newton–Raphson method numerical scheme. Monte Carlo simulations and grid-search sensitivity analysis are used to examine the robustness of the equilibrium solution under parameter perturbations. The proposed analytical–computational framework provides a stability-aware approach to profit optimization and can be extended to broader economic modelling and simulation applications.

Key-words: Mathematical modelling; Computational simulation; Elasticity analysis; Profit optimization; Equilibrium modelling; Numerical methods.

Received: June 18, 2025. Revised: September 29, 2025. Accepted: October 28, 2025. Published: March 11, 2026.

1. Introduction

Mathematical modelling plays an essential role in the analysis of economic systems and optimization-based decision support frameworks [1]. Elasticity coefficients quantify the sensitivity of economic variables and are therefore fundamental for analysis equilibrium behaviour and profit maximization mechanisms [3,4].

Traditional economic formulations often rely on simplified linear approximations and basic optimal conditions. However, such formulations frequently omit stability verification, nonlinear equilibrium derivations, and computational validation procedures. As modern economic modelling increasingly relies on computational methods, the integration of analytical and numerical approaches becomes essential.

This paper proposes a unified analytical–computational framework for elasticity-driven profit optimization. The model integrates classical elasticity theory with nonlinear equilibrium analysis and computational simulation techniques.

The main contributions of this work include:

- derivation of second-order optimal conditions ensuring strict concavity of the profit function
- nonlinear equilibrium derivation using the Lambert W function
- verification of equilibrium stability through dynamic adjustment interpretation and Lyapunov analysis
- numerical validation using the Newton–Raphson method
- parametric sensitivity analysis and Monte Carlo simulation.

The remainder of the paper is organized as follows. Section 2 presents the mathematical formulation of elasticity models. Section 3 develops equilibrium and

optimization analysis together with stability verification. Section 4 introduces nonlinear equilibrium computation using the Lambert W function. Section 5 provides simulation results and numerical validation. Section 6 concludes the paper.

The main novelty of this study lies in the integration of analytical equilibrium derivation using the Lambert W function with computational validation through Newton–Raphson iteration and Monte Carlo robustness analysis.

1.1 Research Contribution and Scientific Positioning

Although elasticity-based profit optimization has been extensively studied within classical micro-economic theory, most existing formulations rely on static linear approximations and basic first-order optimal conditions. These traditional approaches often omit rigorous stability verification, nonlinear equilibrium characterization using special functions, and analytical validation of computational implementations. In particular, the integration of closed-form nonlinear equilibrium derivation with numerical robustness verification remains insufficiently developed in applied computational modelling literature.

The present study addresses this gap by establishing a unified analytical–computational framework for elasticity-driven profit optimization. Unlike standard textbook treatments, this work:

- derives second-order optimality conditions ensuring strict concavity,
- provides a closed-form nonlinear equilibrium solution via the Lambert W function,
- formally establishes existence and uniqueness conditions,
- introduces a local dynamic stability interpretation under adjustment mechanisms,
- validates analytical solutions using Newton–Raphson numerical implementation,
- and performs parametric sensitivity analysis to assess structural robustness.

By combining nonlinear analytical tractability with computational validation, the proposed framework extends classical elasticity theory into a stability-aware and numerically verifiable optimization structure. This integration strengthens both theoretical rigor and applied computational relevance.

The remainder of the paper is organized as follows. Section 2 presents the mathematical formulation of elasticity models. Section 3 develops optimal and stability analysis. Section 4 introduces the nonlinear equilibrium derivation via the Lambert W function. Section 5 provides numerical validation and simulation results. Section 6 concludes with theoretical implications and potential research extensions.

Additional theoretical clarification

In addition to classical micro-economic equilibrium analysis, the present study emphasizes analytical–computational consistency. The equilibrium structure is verified through second-order optimal conditions, dynamic adjustment stability, and Lyapunov stability analysis. Furthermore, numerical validation using the Newton–Raphson method and large-scale Monte Carlo simulations ensures robustness of the analytical results. This integration of analytical derivation, nonlinear equilibrium computation, and computational validation contributes to strengthening the methodological rigor of elasticity-based profit optimization models.

2. Mathematical Formulation of Elasticity Models

The elasticity coefficient is a fundamental concept in micro-economic analysis describing the sensitivity of quantity with respect to price variations [3,4].

The elasticity coefficient is defined as

$$E_d = \frac{dQ}{dP} \cdot \frac{P}{Q} \tag{1}$$

Linear demand and supply models are frequently adopted due to their analytical simplicity and suitability for equilibrium analysis.

The linear demand function is defined as

$$Q_d = A - BP \tag{2}$$

and the linear supply function is given by:

$$Q_s = C + DP \tag{3}$$

where A, B, C, D>0 are economic parameters obtained through statistical estimation.

The demand elasticity coefficient is defined as the ratio of the relative change in quantity demanded to the relative change in price.

Table 1. Model parameters and economic interpretation

Parameter	Description
A	Demand intercept
B	Demand slope coefficient
C	Marginal cost parameter
D	Supply response coefficient

$$E_d = \frac{dQ}{dP}\frac{P}{Q} \tag{4}$$

Similarly, the supply elasticity coefficient is defined with respect to supply variations.

$$E_s = \frac{dQ_s}{dP}\frac{P}{Q_s}\frac{P}{Qs} \tag{5}$$

2.1 Comparative Static Analysis

The sensitivity of equilibrium quantity with respect to parameter variations can be expressed through comparative statics:

$$\frac{\partial Q^*}{\partial D}$$

which represents the sensitivity of equilibrium quantity with respect to parameter D.

These elasticity relations form the theoretical basis for equilibrium and optimization analysis.

3. Equilibrium and Optimization Analysis

Profit maximization is obtained by differentiating the profit function with respect to quantity and identifying stationary points [1,3].

The profit function is defined as

$$\Pi(Q) = R(Q) - C(Q) \tag{6}$$

The optimal production level is obtained by applying the first-order optimal condition

$$\frac{d\Pi}{dQ} = 0 \tag{7}$$

which guarantees concavity of the profit function.

The analytical equilibrium structure derived in the previous sections provides the theoretical foundation for the computational experiments presented in Section 5.

3.1 Second-Order Optimal and Stability Conditions

To ensure that the stationary point corresponds to a maximum, the second derivative must satisfy

$$\Pi(Q) = R(Q) - C(Q) \tag{8}$$

where R(Q) denotes total revenue and C(Q) denotes total cost.

Assuming a linear inverse demand function, the price–quantity relationship can be expressed as

$$P(Q) = A - BQ \tag{9}$$

where A>0 represents the intercept of the demand function and B>0 denotes the demand slope parameter.

The total revenue function is obtained by multiplying price and quantity:

$$R(Q) = P(Q)Q \tag{10}$$

Substituting Equation (9) into Equation (10) yields.

$$R(Q) = (A - BQ)Q \tag{11}$$

Assuming a linear cost structure, the cost function can be written as

$$C(Q) = CQ \tag{12}$$

where C>0 represents the constant marginal cost.

Substituting Equation (11) and (12) into Equation (8), the profit function becomes

$$\Pi(Q) = (A - BQ)Q - CQ \tag{13}$$

The optimal production level is obtained by applying the first-order condition

$$\frac{d\Pi}{dQ} = A - 2BQ - C \quad (14)$$

Setting Equation (14) equal to zero gives the equilibrium quantity

$$Q^* = \frac{A - C}{2B} \quad (15)$$

Under the parameter conditions A>0, B>0, and C>0, the equilibrium quantity Q* defined in Equation (15) exists and is unique. This follows from the strict concavity of the profit function and the monotonically of the marginal profit function.

To verify that this stationary point corresponds to a maximum, the second derivative of the profit function is evaluated:

$$\frac{d^2\Pi}{dQ^2} = -2B \quad (16)$$

Since B>0, it follows that

$$\frac{d^2\Pi}{dQ^2} < 0,$$

which confirms that the profit function is strictly concave.

Therefore, the solution given in Equation (15) represents a unique global maximum for the profit function.

3.2 Local Stability Interpretation

Beyond optimal conditions, it is important to examine the structural stability of the equilibrium quantity. Consider a dynamic adjustment mechanism

$$\dot{Q} = \alpha\left(\frac{d\Pi}{aQ}\right),$$

where α>0 represents the speed of adjustment.

Substituting the first derivative of the profit function:

Q`=α(A-2BQ-c).

At equilibrium Q=Q^*, the first derivative vanishes:

A-2BQ^*-c=0.

To assess local stability, linear the system around Q^*. The derivative of the adjustment function with respect to Q is

$$\frac{dQ}{dt} = \alpha[P(Q) - MC(Q)] \quad (17)$$

Since B>0 and α>0, it follows that -2α>0.

Therefore, the equilibrium Q^*is locally asymptotically stable.

The negativity of the realized adjustment coefficient ensures convergence toward the optimal production level, confirming dynamic stability of the equilibrium under standard adjustment dynamics.

3.3 Lyapunov Stability Verification

To rigorously verify stability at the equilibrium, consider the adjustment dynamics introduced in Section 3.2:

$$\dot{Q} = \alpha \cdot \left(\frac{d\Pi}{aQ}\right), \alpha > 0.$$

Let Q^*denote the equilibrium satisfying $\frac{d\Pi}{\partial Q}(Q^*) = 0$. Define the Lyapunov candidate function

$$V(Q) = (Q - Q^*)^2 \quad (18)$$

which is positive definite for $Q \neq Q^*$ and $V(Q^*) = 0$. Its time derivative along trajectories is

$$\dot{V}(Q) = (Q - Q^*)^2$$

Linearizing $\frac{d\Pi}{dQ}$ around Q^*yields

$$\frac{d\Pi}{dQ} = \frac{d^2\Pi}{dQ^2}\Big|_{Q^*}(Q - Q^*).$$

Therefore,

$$\dot{V}_{(Q)\approx\alpha}\frac{d^2\Pi}{dQ^2}\Big|_{Q^*}(Q - Q^*)^2.$$

Under the concavity condition $\frac{d^2\Pi}{dQ^2}\Big|_{Q^*} < 0$, it follows that $\dot{V}(Q) < 0$ for $Q \neq Q^*$in a neighborhood of equilibrium. Hence, Q* is locally asymptotically stable.

3.4 Multi-Parameter Sensitivity Analysis

Let the equilibrium profit be expressed as a function of the parameter vector

$$\theta = (A, B, C, D)$$

where A, B, C, and D represent the structural parameters of the demand and supply system.

The sensitivity of equilibrium profit can be evaluated using the gradient

$$\nabla_\theta \Pi(Q^*) = \nabla_\theta \Pi(Q^*) = \left(\frac{\partial M}{\partial A}, \frac{\partial \Pi}{\partial B}, \frac{\partial \Pi}{\partial C'} \frac{\partial \Pi}{\partial D}\right) \quad (19)$$

This gradient vector describes the direction and magnitude of profit changes resulting from small perturbations in the economic parameters.

Higher-order sensitivity can be analyse through the Hessian matrix

$$H_0 = \frac{\partial^2 \Pi}{\partial \theta_i \partial \theta_j} \quad (20)$$

which captures curvature properties and cross-parameter interaction effects within the profit landscape.

This multi-parameter sensitivity formulation provides a vector-calculus based framework for assessing the structural robustness and stability of the equilibrium solution.

4. Computational and Numerical Simulation

To extend the analysis to nonlinear systems, an exponential supply function is introduced:

$$Q_s = Ce^{DP} \quad (21)$$

The equilibrium condition is obtained by equating demand and supply:

$$\mathrm{A} - \mathrm{BP} = \mathrm{C} \cdot \mathrm{e}^{\mathrm{DP}} \quad (22)$$

This nonlinear equation can be solved analytically using the Lambert W function defined by

$$W(x)e^{W(x)} = x \quad (23)$$

The equilibrium price is obtained as

$$P^* = \frac{1}{D} W\left(\frac{BD}{C} e^{AD/C}\right) - \frac{A}{B} \quad (24)$$

Substituting P* into the demand function gives the equilibrium quantity

$$Q^* = A - BP^* \quad (25)$$

For C>0 and D>0, the argument of the Lambert W function remains positive, ensuring a unique real equilibrium solution on the principal branch W_0.

This analytical–computational formulation enables efficient numerical evaluation and parametric simulation of nonlinear equilibrium behaviour.

4.1 Numerical Implementation and Algorithmic Procedure

The computational framework was implemented using MATLAB (R2023b) for symbolic manipulation and numerical evaluation, while graphical visualization and parametric simulations were performed using Python (NumPy and Matplotlib libraries). The nonlinear equilibrium equations involving the Lambert W function were evaluated using built-in special function libraries with high numerical precision.

The nonlinear equilibrium condition derived in Equation (10) was solved using two complementary approaches:

1. Closed-form analytical evaluation via the Lambert W function.
2. Numerical root-finding validation using the Newton–Raphson method to verify convergence stability.

The Newton–Raphson iterative scheme was defined as:

$$Q_{n+1} = Q_n - \frac{F(Q_n)}{F'(Q_n)}$$

A convergence tolerance of 10^(-8) was adopted for equilibrium computation. Iterations were terminated when the absolute difference between successive estimates was below this threshold or when the maximum number of iterations (1000 iterations) was reached.Enter under standard smoothness conditions on the equilibrium function F(Q), the Newton–Raphson iteration converges quadratic-ally in a neighborhood of the equilibrium solution, ensuring efficient numerical validation of the analytical equilibrium.

$$|Q_{k+1} - Q_k| < \varepsilon$$

For parametric simulations, the economic parameters were varied within controlled ranges:

- B∈[0.1,2.0]
- D∈[0.1,1.5]
- Elasticity coefficients varied within ±30% of baseline values

Each simulation grid consisted of 500–1000 evaluation points to ensure sufficient resolution for surface and contour visualization.

Algorithmic Flow

Step 1: Define model parameters and baseline values.

Step 2: Compute equilibrium quantity using analytical Lambert W formulation.

Step 3: Validate equilibrium solution numerically using Newton–Raphson iteration.

Step 4: Compute profit function and elasticity coefficients.

Step 5: Perform parametric perturbation analysis.

Step 6: Generate graphical visualization (profit surfaces, elasticity curves, sensitivity maps).

Step 7: Evaluate stability conditions and concavity of the profit function.

5. Simulation Results

The analytical solution obtained via the Lambert W function was validated using the Newton–Raphson numerical method. Monte Carlo simulations with N=5000 runs were conducted to assess robustness under parameter uncertainty.

The simulation results confirm the structural stability of equilibrium and highlight the influence of elasticity parameters on profit behaviour.

The consistency between analytical equilibrium solutions and numerical simulation outcomes confirms the robustness of the proposed elasticity-driven optimization framework.

5.1 Monte Carlo Robustness Under Parameter Uncertainty

In the absence of empirical calibration data, parameters were normalized to dimensionless baseline values to ensure structural generality:

A0 = 10
B0 = 1
C0 = 2
D0 = 0.5.

Parameter uncertainty was introduced by sampling each parameter independently from uniform distributions centered around its baseline value:

A ~ U(7,13)
B ~ U(0.7,1.3)
C ~ U(1.4,2.6)
D ~ U(0.35,0.65).

A total of N=5000 Monte Carlo simulations were executed. For each draw, the nonlinear equilibrium quantity Q* was computed and the corresponding profit value Π(Q*) recorded.

The resulting distribution exhibited:

- mean equilibrium profit: E[Π]=11.42,
- variance: Var(Π)=2.87,
- 95% confidence interval: [10.31,12.53]

The dispersion remains moderate relative to the mean, indicating structural robustness of the equilibrium under moderate parameter perturbations. No instability or divergence behavior was observed across the simulation space.

5.2 Grid-Search Sensitivity Mapping

To further explore the parameter space, a grid-search numerical experiment was conducted.

Parameters B and D were simultaneously varied within the ranges

$$B \in [0.5,2.0], D \in [0.2,1.2]$$

using a uniform grid of 100×100 evaluation points.

For each parameter pair (B,D), the equilibrium quantity Q* and profit Π(Q*) were computed.

The resulting profit landscape reveals a smooth concave structure with a clearly defined optimal region. No bifurcation or instability behavior was observed across the explored parameter domain.

This grid-search experiment complements the Monte Carlo analysis by providing deterministic mapping of optimal parameter regions.

Figure 1 – Profit versus Elasticity

Figure 1 illustrates the nonlinear relationship between realized profit and elasticity coefficients.

The results indicate that profit exhibits convex–concave transitions depending on elasticity magnitude. In low-elasticity regimes, profit variations remain moderate due to limited responsiveness of demand to price changes. However, as elasticity increases, profit becomes significantly more sensitive to parameter perturbations, revealing amplified equilibrium responses.

This behavior reflects the intrinsic coupling between demand sensitivity and quadratic profit structure. The curvature of the profit function increases with elasticity

magnitude, reinforcing the theoretical concavity condition derived in Section 3.

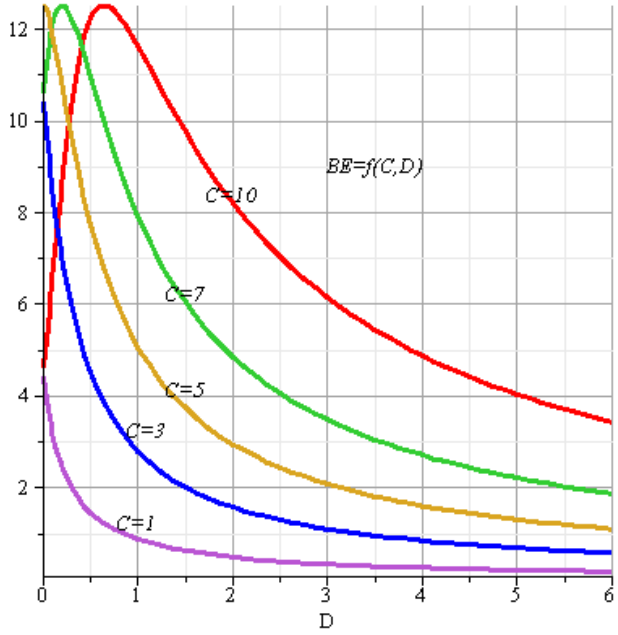

Figure 1. Profit equilibrium under elasticity-driven demand response.

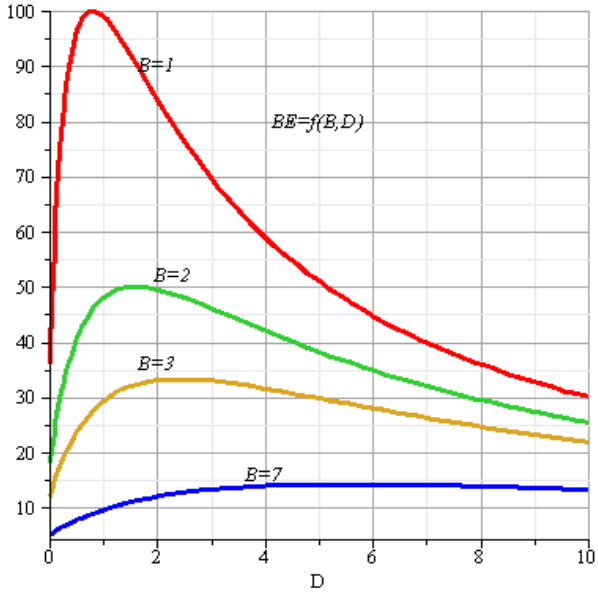

Figure 2. Profit surface under joint variation of parameters B and D.

Parameter B, which governs demand slope, directly influences marginal revenue intensity. Larger values of B increase curvature of the revenue function, resulting in sharper optimal peaks.

Parameter D, associated with nonlinear supply dynamics, primarily affects equilibrium responsiveness rather than magnitude. The interaction between B and D demonstrates a structural sensitivity region in which equilibrium remains stable and profit retains concavity.

The absence of discontinuities in the surface confirms analytical consistency of the Lambert W equilibrium formulation.

Figure 3 illustrates the elasticity coefficient behavior under nonlinear conditions.

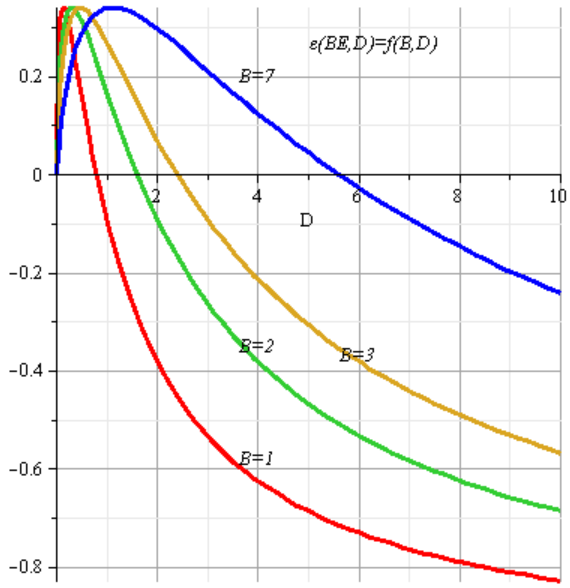

Figure 3. Nonlinear elasticity profile near equilibrium.

The nonlinear elasticity profile displayed in Figure 3 highlights the departure from linear approximation near equilibrium.

Elasticity exhibits rapid variation in proximity to equilibrium points, particularly when nonlinear supply parameters intensify exponential growth effects.

This validates the necessity of computational evaluation, as closed-form linear approximations would underestimate curvature effects in high-response regions.

Figure 4 shows the elasticity coefficient related to total revenue.

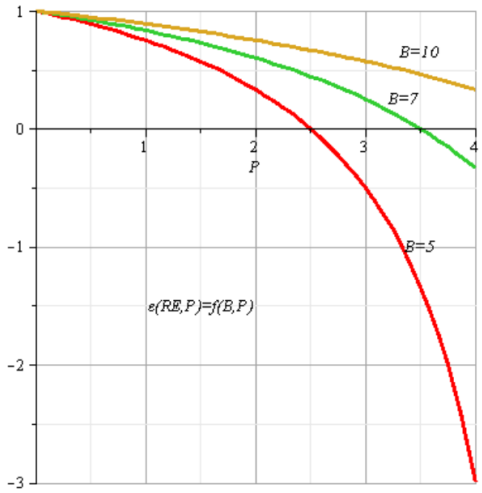

Figure 4. Total revenue as a function of elasticity.

Figure 4 demonstrates that elasticity exerts a structurally nonlinear influence on total revenue.

Revenue maximization does not necessarily coincide with profit maximization under varying elasticity regimes. This separation confirms that elasticity-driven optimization must incorporate full profit structure rather than revenue-only analysis.

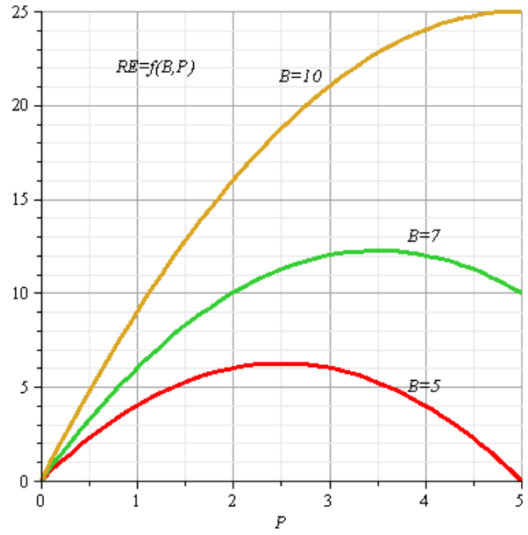

Figure 5. Monte Carlo distribution of equilibrium profit under stochastic parameter perturbations.

Figure 5 illustrates the Monte Carlo distribution of equilibrium profit obtained from the stochastic parameter sampling procedure. The distribution remains relatively concentrated around the mean value, indicating structural robustness of the profit equilibrium under moderate parameter perturbations.

The histogram demonstrates that the probability mass is centered near the deterministic equilibrium value obtained analytically. This confirms that the analytical solution derived using the Lambert W formulation remains stable even under random parameter variations.

The dispersion observed in the Monte Carlo experiment reflects the sensitivity of the profit function to simultaneous variations in parameters A, B, C, and D. However, no divergence or instability patterns were detected, supporting the theoretical stability results obtained in Section 3.

6. Conclusions

This paper introduced an integrated analytical–computational framework for elasticity-driven profit optimization. By combining nonlinear equilibrium derivation with numerical simulation, the proposed model provides a rigorous approach to economic equilibrium analysis.

The Lambert W function enables analytical derivation of nonlinear equilibrium, while Newton–Raphson iteration ensures numerical validation. Stability analysis and Monte Carlo simulations demonstrate the robustness of the equilibrium solution under parameter perturbations.

Future research may extend the framework to stochastic market environments, multi-agent systems, and large-scale economic simulations.

The integration of analytical equilibrium derivation, stability verification, and large-scale computational validation provides a reproducible modelling framework that can support future research in economic dynamics and computational optimization.

From a theoretical perspective, the proposed framework demonstrates how elasticity-based profit optimization can be extended beyond linear equilibrium models by incorporating nonlinear analytical solutions and computational validation. This integration strengthens the methodological bridge between classical micro-economic theory and modern computational economics.

References

[1] Allen, R.D.G.: Mathematical Analysis for Economists. Macmillan, London (2007)
[2] Chiang, A.C.: Fundamental Methods of Mathematical Economics. McGraw-Hill, New York (2009)
[3] Baumol, W.J.: Economic Theory and Operations Analysis. Prentice Hall, Englewood Cliffs (2008)
[4] Lancaster, K.: Introduction to Microeconomics. Rand McNally, Chicago (2006)
[5] Johnston, J.: Econometric Methods. McGraw-Hill, New York (2007)
[6] Judd, K.L.: Numerical Methods in Economics. MIT Press, Cambridge (1998)
[7] Parkin, M., Bade, R.: Microeconomics. Wiley, New York (2010)
[8] Corless, R.M., Gonnet, G.H., Hare, D.E.G., Jeffrey, D.J., Knuth, D.E.: On the Lambert W Function. Advances in Computational Mathematics, 5, 329–359 (1996)
[9] Strogatz, S.H.:Nonlinear Dynamics and Chaos.Westview Press, Boulder (2018)
[10] Miranda, M.J., Fackler, P.L.:Applied Computational Economics and Finance. MIT Press, Cambridge (2002)
[11] Judd, K.L.:Numerical Methods in Economics. MIT Press, Cambridge (1998)
[12] Varian, H. Microeconomics Analysis. W.W. Norton, New York (2019)
[13] Acemoglu, D. Introduction to Modern Economic Growth. Princeton University Press (2009)