# Grey Wolf Optimizer Based Design of Reinforced Concrete Retaining Walls Considering Shear Key

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*Abstract:* - To build a structural member, all safety issues must be provided and the essential goal of engineering is to develop that design with the minimum cost. The optimization of reinforced concrete members contains non-linear design constraints and these constraints are developed according to the geotechnical and structural state limits in optimum design of retaining walls. The presented study proposes a detailed optimum design of reinforced concrete (RC) retaining walls employing grey wolf optimizer imitating the leadership hierarchy and hunting behavior of grey wolves. The detailed optimization considers the dimension and reinforcement design variables of stem, toe, heel and key parts of walls. The results of the proposed method were compared with other metaheuristic based methods and novel approach is competitive with the others in minimum objective function and reliability in finding optimum values.

*Key-Words:* Reinforced concrete cantilever retaining walls, Optimum design, Grey wolf optimizer.

## 1 Introduction

In engineering designs, one of the most important objectives is to find the best design with maximum profit. By reduction of earth resources, the economy in materials became the most important factor. In addition to that, engineering designs must be safe and durable. In the area of structural engineering, there are a lot of optimum structural design proposals including trusses [1-3], frames [4-6] and reinforced concrete (RC) elements [7-9]. The design of RC elements contains several constraints and the optimization of RC members is highly non-linear problem. In addition to that, RC retaining walls contain geotechnical and structural design limits and these limits are considered as constraints. The non-linear optimum design of RC retaining walls has been done by using metaheuristic based methods. The employed metaheuristic algorithms in optimum design of RC retaining walls are simulated annealing [10, 11], particle swarm optimization (PSO) [12] and genetic algorithm [13] as classical ones and harmony search [14], big bang big crunch (BB-BC) [15], firefly algorithm [16], charged system search [17], teaching learning based optimization (TLBO) [18], biogeography-based optimization algorithm (BBO) [19], and grey wolf optimizer (GWO) [20] as new generation algorithms. The optimum design of RC retaining walls is still an active research area by proposing new novel methods for different metaheuristic algorithms and developing detailed optimum design of members including shear key.

In this study, grey wolf optimizer (GWO) was used to develop a design methodology for optimum design of RC retaining walls including the dimensions and reinforcements of stem, toe, heel, and key part of walls.

## 2 The optimization method - Grey wolf optimizer

The Gray wolf (*Canis lupus* as Latin name) is one of the species of Canidae family. They are on the top of the food chain since grey wolfs are apex predators. They live in a group with strict social dominant hierarch. At the top of a hierarchy, the leader called alpha ($\alpha$) is found. The decisions such as hunting, sleeping place, waking time is done by alpha. The second in the hierarchy is beta and he or she helps alpha in decisions. Beta is the best candidate for being an alpha if he or she passes away.

The lowest rank is omega ($\omega$). The role of omega is scapegoat and they are the last wolves to allowed to eat. If a wolf is none of $\alpha$, $\beta$ and $\omega$, he or she is called subordinate or delta ($\delta$). Scout, sentials, elders, hunters and caretaker are in delta group and these wolfs submit to $\alpha$, $\beta$, and dominate $\omega$.

The social hierarchy and hunting technique of grey wolves are mathematically modeled by Mirjalili et al. [21].

In grey wolf optimizer (GWO), the best solution assigned for α. The second and third best ones are β and δ, respectively. The other solutions are ω. The results are updated for each iteration. During the optimization, several parameters are updated according to exploitation and exploration imitating attacking prey and search for prey stages, respectively.

The steps of the optimization methodology can be seen in Figure 1. The iterations continue for a constant number defined by user as the stopping criteria. The objective function is the total cost of RC retaining wall.

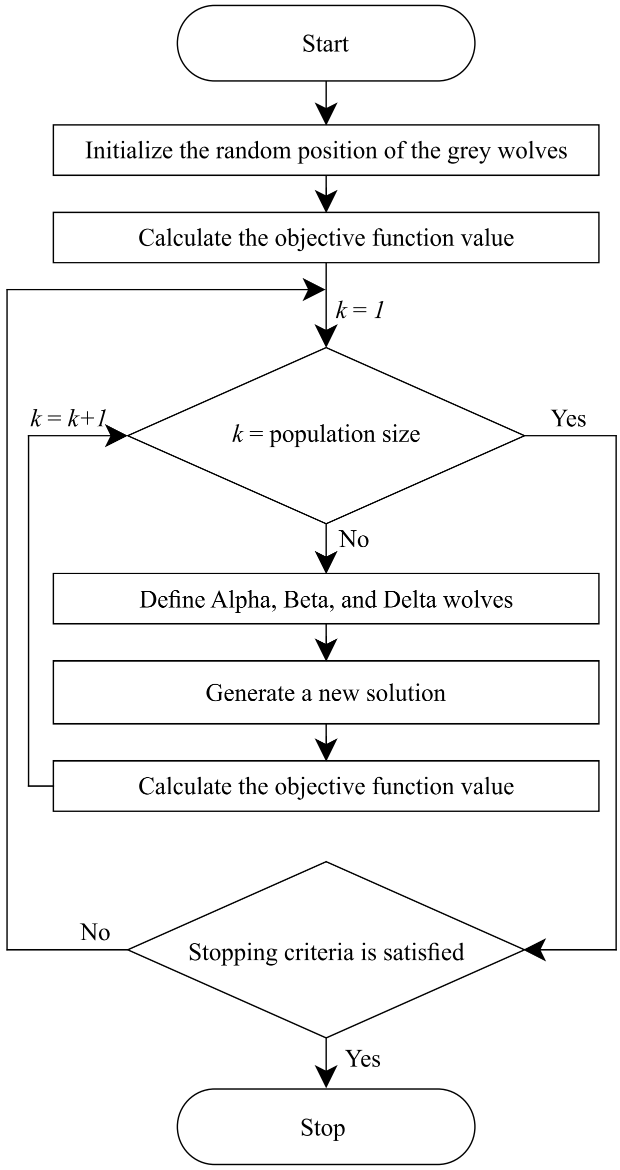


Figure 1. The flowchart of the GWO algorithm

## 3 Design of RC retaining walls

In Figure 2, active earth pressure passive earth pressure surcharge load and bearing stress are shown with concentrated loads such as $P_A$, $P_P$, $P_q$, and $P_B$, respectively. The design constants are listed in Table I.

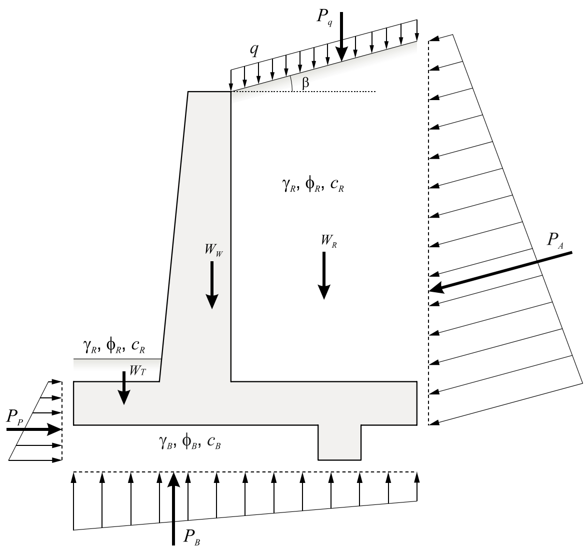


Figure 2. Loads acting on a cantilever retaining wall

TABLE I. THE DESIGN CONSTANTS

| | Input parameter | Symbol |
|---|---|---|
| *Problem* | Height of stem | $H$ |
| | Backfill slope angle | $\beta$ |
| | Surcharge load | $q$ |
| | Depth of the soil in front of wall | $D$ |
| *Retained soil* | Internal friction angle of retained soil | $\phi_R$ |
| | Cohesion of retained soil | $c_R$ |
| | Unit weight of retained soil | $\gamma_R$ |
| *Base soil* | Internal friction angle of base soil | $\phi_B$ |
| | Cohesion of base soil | $c_B$ |
| | Unit weight of base soil | $\gamma_B$ |
| *Reinforcing steel* | Yield strength of steel | $f_y$ |
| | Unit weight of steel | $\gamma_s$ |
| | Unit cost of steel | $C_s$ |
| *Concrete* | Compressive strength of concrete | $f'_c$ |
| | Unit weight of concrete | $\gamma_c$ |
| | Unit cost of concrete | $C_c$ |
| *Reinforced concrete* | Concrete cover | $c_c$ |
| | Design load factor | LF |
| | Unit cost of formwork | $C_f$ |
| *Factors of safety* | Safety factor for overturning stability | $SF_{O,design}$ |
| | Safety factor for sliding stability | $SF_{S,design}$ |
| | Safety factor for bearing capacity | $SF_{B,design}$ |

The geotechnical state limits are overturning stability, sliding stability and bearing stability. The safety factor of overturning ($SF_o$) is defined as Eq. (1). In Eq. (1), $M_R$ defines the resisting moment for overturning including the moments of surcharge loads, self-weight of the wall ($W_w$) and weight of backfill soil ($W_R$), while $M_o$ is the moment that overturn the wall resulting from active earth pressure. The passive loads are not considered for possible removing of soil.

$$SF_O = \frac{\sum M_R}{\sum M_O} \tag{1}$$

$$P_A = \frac{1}{2}\cdot\gamma_R\cdot H^2\cdot K_A - 2\cdot c_R\cdot H\cdot\sqrt{K_A} \tag{2}$$

$$P_P = \frac{1}{2}\cdot\gamma_B\cdot D^2\cdot K_p + 2\cdot c_B\cdot D\cdot\sqrt{K_p} \tag{3}$$

The concentrated $P_A$, and $P_P$ are given as Eqs. (2) and (3), respectively. The active ($K_A$) and passive ($K_P$) coefficient are written according to Rankine theory as follows:

$$K_A = \frac{\cos^2(\phi-\theta)}{\cos^2\theta\cdot\cos(\delta+\theta)\cdot\left[1+\sqrt{\frac{\sin(\delta+\phi)\cdot\sin(\phi-\beta)}{\cos(\delta+\theta)\cdot\cos(\beta-\theta)}}\right]^2} \tag{4}$$

$$K_P = \frac{\cos^2(\phi+\theta)}{\cos^2\theta\cdot\cos(\delta-\theta)\cdot\left[1+\sqrt{\frac{\sin(\delta+\phi)\cdot\sin(\phi+\beta)}{\cos(\delta-\theta)\cdot\cos(\beta-\theta)}}\right]^2} \tag{5}$$

The second geotechnical state limit that is the sliding stability is formulized as Eq. (6).

$$SF_S = \frac{\sum F_R}{\sum F_D} \tag{6}$$

In Eq. (6), $F_R$ and $F_D$ are the resisting and sliding forces that formulated as Eqs. (7) and (8), respectively.

$$\sum F_R = \sum V\cdot\tan\left(\frac{2\cdot\phi_B}{3}\right) + \frac{2\cdot L_F\cdot c_B}{3} + P_P \tag{7}$$

$$\sum F_D = P_A\cdot\cos\beta \tag{8}$$

In the equations, B is the length of base slab and $\Sigma V$ is the total weight of the wall.

The bearing stability is formulated as the ratio of ultimate bearing capacity ($q_u$) and maximum intensity of soil pressure ($q_{max}$) as sesn in Eq. (9). The maximum and minimum intensity of soil stresses are calculated according to Eq. (10).

$$SF_B = \frac{q_u}{q_{\max}} \tag{9}$$

$$q_{\substack{min\\max}} = \frac{\sum V}{B}\cdot\left(1\pm\frac{6\cdot e}{B}\right) \tag{10}$$

The eccentricity of moment resulting from the difference between sum of resisting and overturning moment is written as Eq. (11).

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V} \tag{11}$$

In the calculations, the active and passive loads are calculated according to active ($K_a$) and passive ($K_p$)

coefficients given as Eqs. (12) and (13), respectively.

$$K_a = \cos\beta \cdot \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\theta}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\theta}} \quad (12) \qquad K_p = \tan^2\left(45 + \frac{\theta}{2}\right) \quad (13)$$

In the Figure 3, the design variables of the problem are shown. The variables; $x_i$ ($i$=1-8) related with the dimensions and variables $R_j$ ($j$=1-16) are related to the reinforcement bars. The full names of design variables are given as Table II.

The dimension constraints are listed with lower and upper bounds in Table III. The other constraints are given as Eqs. (14-21). The stability constraints (geotechnical state limits) are the first four constraints. The capacity constraints for moment and shear forces are formulated as Eqs. (18) and (19), respectively and these constraints are calculated for four sections of RC retaining wall.

The geometry constraints are given as Eqs. (20) and (21). $M_u$ and $V_u$ are the ultimate internal forces such as moment and shear forces, respectively. $M_d$ and $V_d$ are the moment and shear force capacity of the wall, respectively.

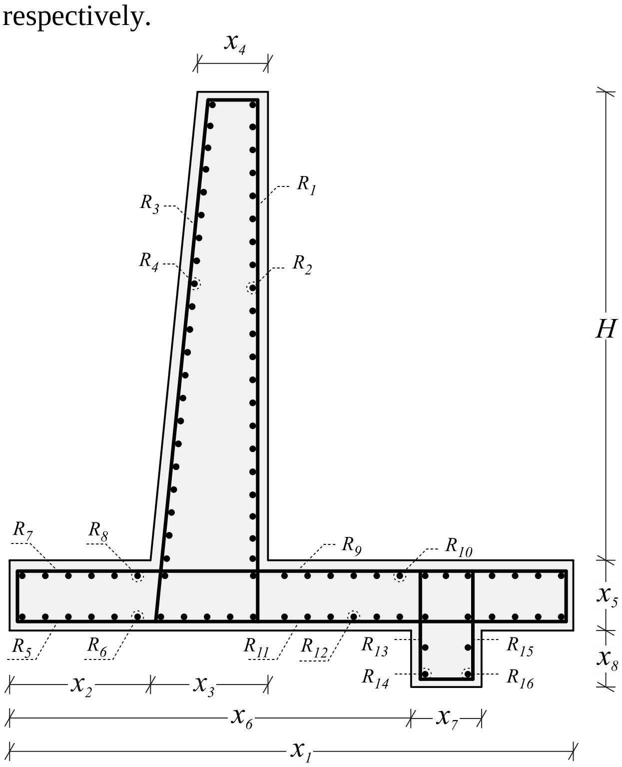


Figure 3. Design variables for cantilever retaining wall

$$g_1(x) = \frac{SF_{O,design}}{SF_O} - 1 \le 0 \tag{14}$$

$$g_2(x) = \frac{SF_{S,design}}{SF_S} - 1 \le 0 \tag{15}$$

$$g_3(x) = \frac{SF_{B,design}}{SF_B} - 1 \le 0 \tag{16}$$

$$g_4(x) = q_{min} \ge 0 \tag{17}$$

$$g_{5-8}(x) = \frac{M_u}{M_d} - 1 \le 0 \tag{18}$$

$$g_{9-12}(x) = \frac{V_u}{V_d} - 1 \le 0 \tag{19}$$

$$g_{13}(x) = \frac{x_2 + x_3}{x_1} - 1 \le 0 \tag{20}$$

$$g_{14}(x) = \frac{x_6 + x_7}{x_1} - 1 \le 0 \tag{21}$$

TABLE II. DESIGN VARIABLES

| | Description | Design variable |
|---|---|---|
| Variables related to cross-section dimension | Total base width | $x_1$ |
| | Toe projection | $x_2$ |
| | Thickness at the bottom of the stem | $x_3$ |
| | Thickness at the top of the stem | $x_4$ |
| | Thickness of the base slab | $x_5$ |
| | Distance of the shear key from the toe | $x_6$ |
| | Thickness of the shear key | $x_7$ |
| | Height of the shear key | $x_8$ |
| Variables related to reinforcing steel area per unit length of the wall | Tension bars of stem | $R_1$ |
| | Shrinkage and temperature bars of stem | $R_{2-4}$ |
| | Tension bars of the toe | $R_5$ |
| | Shrinkage and temperature bars of toe | $R_{6-8}$ |
| | Tension bars of the heel | $R_9$ |
| | Shrinkage and temperature bars of heel | $R_{10-12}$ |
| | Tension bars of the key | $R_{13}$ |
| | Shrinkage and temperature bars of key | $R_{14-16}$ |

TABLE III. DESIGN CONSTRAINTS

| Design variable | Lower bound | Upper bound |
|---|---|---|
| $x_1$ | 0.4*$H$*(12*11) | (0.7*$H$)/0.9 |
| $x_2$ | [0.4*$H$*(12*11)]/3 | [(0.7*$H$)/0.9]/3 |
| $x_3$ | 0.2 | ($H$/0.9)/10 |
| $x_4$ | 0.2 | 0.5 |
| $x_5$ | [$H$*(12*11)]/12 | ($H$/0.9)/10 |
| $x_6$ | 0.5 | 0.8*$H$ |
| $x_7$ | 0.2 | 0.4 |
| $x_8$ | 0.2 | 0.9 |

# 4 Numerical examples

The numerical investigation is done according to numerical values given as Table IV.

TABLE IV. THE NUMERICAL VALUES OF DESIGN CONSTANTS

| Symbol | Unit | Value |
|---|---|---|
| $H$ | m | 4.5 |
| $\beta$ | degree | 15 |
| $q$ | kPa | 30 |
| $D$ | m | 0.75 |
| $\phi_R$ | degree | 36 |
| $c_R$ | kPa | 0 |
| $\gamma_R$ | kN/m$^3$ | 17.5 |
| $\phi_B$ | degree | 34 |
| $c_B$ | kPa | 100 |
| $\gamma_B$ | kN/m$^3$ | 18.5 |
| $f_y$ | MPa | 400 |
| $\gamma_s$ | kg/m$^3$ | 7849 |
| $C_s$ | \$/kg | 0.40 |
| $f'_c$ | MPa | 21 |
| $\gamma_c$ | kN/m$^3$ | 23.5 |
| $C_c$ | \$/m$^3$ | 40 |
| $c_c$ | mm | 70 |
| $LF$ | - | 1.7 |
| $C_f$ | \$/m$^2$ | 4.29 |
| $SF_{O,design}$ | - | 1.5 |
| $SF_{S,design}$ | - | 1.5 |
| $SF_{B,design}$ | - | 3.0 |

The optimum results of design variables are given as Table V for GWO based method. The total cost of the retaining wall that is the objective function of the problem to be minimized is 257.65$ (Table VI). Also, several methods employing BB-BC [15], BBO [19], HS [14], PSO [12] and TLBO [18] are also tested for the same example.

TABLE V. OPTIMUM RESULTS

| | BB-BC | BBO | IHS | PSO | TLBO | GWO |
|---|---|---|---|---|---|---|
| $X_1$ | 2.94 | 2.93 | 2.93 | 2.93 | 2.93 | 2.93 |
| $X_2$ | 0.73 | 0.76 | 0.76 | 0.76 | 0.76 | 0.76 |
| $X_3$ | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |
| $X_4$ | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| $X_5$ | 0.48 | 0.49 | 0.49 | 0.49 | 0.49 | 0.49 |
| $X_6$ | 2.72 | 2.69 | 2.36 | 2.61 | 2.55 | 2.68 |
| $X_7$ | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| $X_8$ | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| $R_1$ | ϕ16/170 | ϕ16/170 | ϕ16/170 | ϕ16/170 | ϕ16/170 | ϕ16/170 |
| $R_5$ | ϕ16/140 | ϕ14/100 | ϕ14/100 | ϕ14/100 | ϕ14/100 | ϕ14/100 |
| $R_9$ | ϕ16/120 | ϕ14/100 | ϕ14/100 | ϕ14/100 | ϕ14/100 | ϕ14/100 |
| $R_{15}$ | ϕ10/170 | ϕ10/170 | ϕ10/170 | ϕ10/170 | ϕ10/170 | ϕ10/170 |

# 5 Discussion and Results

The results presented in Section 3 are the best optimum results of 100 independent runs of the optimization methodology. In the Table VI, the total cost of the optimum design is given for the maximum and minimum of 100 runs. Also, mean and standard deviation values of 100 runs are given.

TABLE VI. THE COMPARISON OF METHODS

| Algorithm | Stem Height | Total Cost ($) | | | |
|---|---|---|---|---|---|
| | | Min | Max | Mean | St. Dev. |
| BB-BC | 4.5 | 257.57 | 328.70 | 279.18 | 14.254 |
| BBO | 4.5 | 257.56 | 277.04 | 259.40 | 3.192 |
| IHS | 4.5 | 257.79 | 260.62 | 259.20 | 0.560 |
| PSO | 4.5 | 257.54 | 300.16 | 263.27 | 8.230 |
| TLBO | 4.5 | 257.48 | 260.47 | 258.07 | 0.793 |
| GWO | 4.5 | 257.65 | 260.35 | 258.73 | 0.655 |

According to the results, the best method minimizing the objective function is TLBO, but the proposed method employing GWO outperforms TLBO for maximum costs and standard deviation value.

Although PSO, BBO and BB-BC have lower minimum cost than GWO, the reliability of these methods are not the best one since the maximum cost, mean cost and standard deviation values are high. Especially, BB-BC have a high standard deviation value such as 14.254.

As a conclusion, the GWO based method is competitive with the other methods and effective one in finding best value for different runs of the optimization methodology for the solved optimum design of RC retaining walls that considers the detailed optimization of dimensions and reinforcements of stem, toe, heel and key parts of wall.

*References:*

[1] Degertekin SO, Hayalioglu MS. "Sizing truss structures using teaching-learning-based optimization". *Computers & Structures,* 119, 177-188, 2013.

[2] Dede T, Ayvaz Y. "Combined size and shape optimization of structures with a new meta-heuristic algorithm". *Applied Soft Computing,* 28, 250-258, 2015.

[3] Bekdaş G, Nigdeli SM, Yang X-S. "Sizing optimization of truss structures using flower pollination algorithm". *Applied Soft Computing,* 37, 322-331, 2015.

[4] Alberdi R, Khandelwal K. "Comparison of robustness of metaheuristic algorithms for steel frame optimization". *Engineering Structures,* 102, 40-60, 2015.

[5] Pezeshk S, Camp CV, Chen D. "Design of nonlinear framed structures using genetic

optimization". *Journal of Structural Engineering-Asce*, 126(3), 382-388, 2000.

[6] Murren P, Khandelwal K. "Design-driven harmony search (DDHS) in steel frame optimization". *Engineering Structures,* 59, 798-808, 2014.

[7] Lee C, Ahn J. "Flexural design of reinforced concrete frames by genetic algorithm". *Journal of Structural Engineering,* 129(6), 762-774, 2003.

[8] Govindaraj V, Ramasamy JV. "Optimum detailed design of reinforced concrete frames using genetic algorithms". *Engineering Optimization,* 39(4), 471-494, 2007.

[9] Nigdeli SM, Bekdas G, Kim S, Geem ZW, *A novel harmony search based optimization of reinforced concrete biaxially loaded columns*. 2015, pp. 1097-1109.

[10] Ceranic B, Fryer C, Baines RW. "An application of simulated annealing to the optimum design of reinforced concrete retaining structures". *Computers & Structures,* 79(17), 1569-1581, 2001.

[11] Yepes V, Alcala J, Perea C, Gonzalez-Vidosa F. "A parametric study of optimum earth-retaining walls by simulated annealing". *Engineering Structures,* 30(3), 821-830, 2008.

[12] Ahmadi-Nedushan B, Varaee H. "Optimal Design of Reinforced Concrete Retaining Walls using a Swarm Intelligence Technique". *The first International Conference on Soft Computing Technology in Civil, Structural and Environmental Engineering,* Stirlingshire, Scotland, 2009.

[13] Kaveh A, Kalateh-Ahani M, Fahimi-Farzam M. "Constructability optimal design of reinforced concrete retaining walls using a multi-objective genetic algorithm". *Structural Engineering and Mechanics,* 47(2), 227-245, 2013.

[14] Kaveh A, Abadi ASM. "Harmony search based algorithms for the optimum cost design of reinforced concrete cantilever retaining walls". *International Journal of Civil Engineering,* 9(1), 1-8, 2011.

[15] Camp CV, Akin A. "Design of Retaining Walls Using Big Bang-Big Crunch Optimization". *Journal of Structural Engineering-Asce,* 138(3), 438-448, 2012.

[16] Sheikholeslami R, Gholipour Khalili B, Zahrai SM. "Optimum Cost Design of Reinforced Concrete Retaining Walls Using Hybrid Firefly Algorithm". *International Journal of Engineering and Technology,* 6(6), 465-470, 2014.

[17] Talatahari S, Sheikholeslami R. "Optimum design of gravity and reinforced retaining walls using enhanced charged system search algorithm". *KSCE Journal of Civil Engineering,* 18(5), 1464-1469, 2014.

[18] Temur R, Bekdaş G. "Teaching learning-based optimization for design of cantilever retaining walls". *Structural Engineering and Mechanics,* 57(4), 763-783, 2016.

[19] Aydogdu I. "Cost optimization of reinforced concrete cantilever retaining walls under seismic loading using a biogeography-based optimization algorithm with Levy flights". *Engineering Optimization,* 49(3), 381-400, 2017.

[20] Bekdas G, Temur R. "*Grey Wolf Optimizer for Optimum Design of Reinforced Concrete Cantilever Retaining Walls*". Editörler: International Conference of Numerical Analysis and Applied Mathematics, (AIP Conference Proceedings, Melville: *Amer Inst Physics*, 2018.

[21] Mirjalili S, Mirjalili SM, Lewis A. "Grey Wolf Optimizer". *Advances in Engineering Software,* 69, 46-61, 2014.