

Investigation of cross-sectional dimension on optimum carbon fiber reinforced polymer design for shear capacity increase of reinforced concrete beams

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Abstract: - In retrofit of reinforced concrete (RC) structures, the use of carbon fiber reinforced polymers (FRP) is an alternative way which is easily applied without destruction. In this study, the cross-section dimensions of beams are investigated for optimum parameters of CFRP such as spacing of CFRP, width of CFRP and angle of CFRP. The different combinations of cross-section dimension such as breadth (b_w), height (h) and thickness of flange of T-shaped cross-sections (h_f). The rules of ACI-318: Building code requirements for structural concrete was used in development of the total area of CFRP which is the objective function and penalized with three design constraints. The optimization method using Jaya Algorithm was used in the investigation. According to the results, the rise of b_w increases the required CFRP area, but in the cases with big additional shear force, it provides a design without violation of design constraints. The increase of h has always a reduction of required areas.

Key-Words: Carbon fiber reinforced polymer, optimum design, Jaya algorithm, cross-sectional dimensions.

1 Introduction

The capacity of reinforced concrete (RC) member for shear forces and flexural moments can be increased by different retrofit method. These methods generally need partial destruction of existing member and the use of structures may not be possible in that case. Whereas, non-destructive methods such as wrapping with carbon fiber reinforced polymer (CFRP) provide the use of the structure during application.

The optimization is to find a balanced solution between safety and economy. For that reason, an objective is minimized or maximized while the required design constraints are provided. Metaheuristic methods are the algorithms which are inspired by the imitation of a happening from nature and life. These algorithms have been used as numerical optimization methods in structural engineering problems such as weight optimization of trusses [1-3], optimum tuning of tuned mass dampers [4-6], optimum design of RC members [7-9], optimum design of RC retaining walls [10-12] and optimum design of CFRP [13-14].

In this study, an investigation is done for the different combination of cross-sectional dimensions

of RC beams are done for the optimum parameters of CFRP applied to increase the shear force capacity. During the investigation, discrete optimization is done for CFRP parameters for optimum design and the area of CFRP is minimized for the unit meter of the beam. Also, the design constraints were considered according to ACI318: Building code requirements for structural concrete [15].

2 The optimum CFRP design for shear capacity

A beam with a T-section is shown in Figure 1. In this figure, the CFRP strips are shown with the design variables such as spacing of CFRP (s_f), width of CFRP (w_f) and angle of CFRP (β). In Figure 1, d_f is the depth of beam where the wrapping of CFRP is done. It is calculated as Eq. (1).

The objective function, which is the area of CFRP per unit meter, is given as Eq. (2). The design constraints provided according to ACI318 [15] are given as Eqs. (3-5). The design constants and variables shown in equations are listed in Table I.

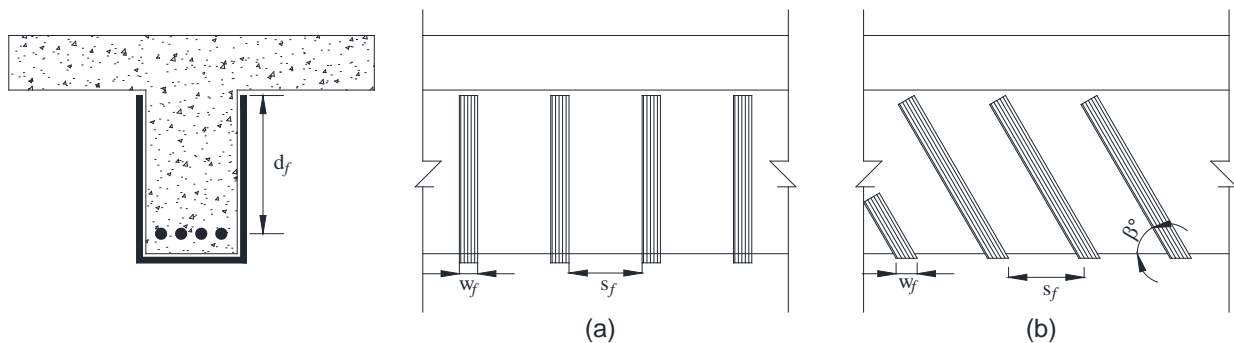


Figure 1. The T-shaped beam with CFRP [16]

$$d_f = d - h_f \tag{1}$$

$$A = \frac{w_f \left(\frac{2d_f}{\sin \beta} + b \right)}{s_f} \times 1000 \tag{2}$$

$$g_1(x) : s_f \leq \frac{d}{4} \tag{3}$$

$$g_2(x) : V_{\text{additional}} < 0.7R \frac{(2t_f w_f f_{fe})(\sin \beta + \cos \beta)d_f}{s_f + w_f}$$

$$g_3(x) : \frac{(2t_f w_f f_{fe})(\sin \beta + \cos \beta)}{s_f + w_f} \leq \frac{2\sqrt{f'_c} b_w d}{3} - V_s$$

The optimum results are found according to the method using Jaya Algorithm (JA) [14]. The

algorithm was developed by Rao and Jaya means victory [17]. It is a single-phase algorithm and it has no user defined variables. In iterative optimization, a candidate design variable solution used in the next step (x_i^{t+1}) for a population ($i=1$ to N population number) is found according to existing solution (x_i^t), two random number between 0 and 1 (r_1 and r_2), the best (g^*) and the worst (g^w) existing design variables in population as formulated in Eq.(6).

$$x_i^{t+1} = x_i^t + r_1(g^* - |x_i^t|) - r_2(g^w - |x_i^t|) \tag{6}$$

The best and worst solutions are found according to the objective function and the iterations continue for a desired maximum number of iterations. If a set of design variables provides a violated solution of design constraints, the objective function is assigned with a big value.

TABLE I. THE DESIGN CONSTANTS AND VARIABLES

Definition	Symbol
Breadth	b_w
Height	h
Effective depth	d
Thickness of CFRP	t_f
Reduction factor	R
Thickness of slab	h_f
Comp. strength of concrete	f'_c
Effective tensile strength of CFRP	f_{fe}
Width of CFRP	w_f
Spacing of CFRP	s_f
Angle of CFRP	β
Additional shear force	$V_{\text{additional}}$
Shear force capacity of rebar	V_s

3 Numerical examples

The investigation is done for 6 values of h (300-800mm), 4 values b_w (200-500mm) and 3 values of h_f (80-120mm) are done by considering 4 $V_{\text{additional}}$ cases (50-200kN). The optimum values were obtained for the best of 20 independent runs to prevent trapping to local optimum values. The ranges of design variables and the values of design constants are shown in Table II.

TABLE II. THE NUMERICAL VALUES

Symbol	Unit	Value
b_w	mm	200
h	mm	500
d	mm	450
t_f	mm	0.165
R	-	0.5
h_f	mm	100
f'_c	MPa	20
f_{fe}	MPa	3790
w_f	mm	0-1000
s_f	mm	0-d/4
β	°	0-90
V_s	kN	50

4 Results and Conclusions

In the Table III, the optimum results for 50 kN additional shear force capacity is given for $h_f=80$ mm. In this table, 24 combinations of b_w and h_f are given. None of these optimum results have not penalized objective function. According to the results, the rise of b_w increases the optimum value of area of CFRP. This situation is vice versa for h . The optimum β angles are generally 60° or 65° . The dimension design variables are assigned with values with 10mm increment while β is optimized with 5° increment. When $\beta=45^\circ$, the shear force capacity is maximum, but the optimum values are different since the area increases when CFRP is applied with 45° . The optimum value of w_f reduces by the increase of h .

TABLE III. THE OPTIMUM RESULTS (50kN, $H_f=80$ MM)

h	b_w	w_f	s_f	β	A_{best}
300	200	50	60	65	281492.5
300	300	50	60	65	326947.1
300	400	50	60	65	372401.6
300	500	40	50	60	417238.3
400	200	40	90	65	251659.0
400	300	40	90	65	282428.2
400	400	40	90	65	313197.4
400	500	40	90	65	343966.7
500	200	30	100	60	243341.2
500	300	30	100	60	266418.1
500	400	30	100	60	289495.0
500	500	20	70	55	311860.7
600	200	30	130	65	227832.7
600	300	30	130	65	246582.7
600	400	30	130	65	265332.7
600	500	20	90	60	284059.0
700	200	20	110	60	226180.1
700	300	20	110	60	241564.7
700	400	20	110	60	256949.3
700	500	20	110	60	272333.9
800	200	30	180	70	223163.9
800	300	20	130	60	237068.9
800	400	20	130	60	250402.2
800	500	20	130	60	263735.6

The optimum results for 100 kN shear force increase and $h_f=80$ mm were shown in Table IV. The optimum results for $b_w=200$ mm and $h=300$ mm has a penalized objective function and the application of CFRP is not enough to provide a 100 kN shear force increase by wrapping the area with 220 mm

effective depth and 200 mm breadth. The optimum β angles are similar while the optimum required areas of CFRP are nearly double of the case of 50 kN.

TABLE IV. THE OPTIMUM RESULTS (100kN, $H_f=80\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
300	300	570	60	65	650780.4
300	400	370	50	60	738930.7
300	500	370	50	60	827025.9
400	200	80	50	65	503317.9
400	300	80	50	65	564856.4
400	400	80	50	65	626394.9
400	500	80	50	65	687933.3
500	200	70	80	65	474366.5
500	300	70	80	65	521033.2
500	400	70	80	65	567699.8
500	500	70	80	65	614366.5
600	200	60	100	65	455665.4
600	300	60	100	65	493165.4
600	400	60	100	65	530665.4
600	500	40	70	60	568118.0
700	200	60	130	65	446436.5
700	300	60	130	65	478015.5
700	400	60	130	65	509594.4
700	500	60	130	65	541173.4
800	200	70	180	70	437401.3
800	300	70	180	70	465401.3
800	400	70	180	70	493401.3
800	500	50	140	60	520530.7

The optimum results of the case with $h_f=80\text{mm}$ and $V_{additional}=150\text{ kN}$ are presented in Table V. For these results, all cases with 300 mm height are penalized. For $h_f=80\text{mm}$ and 200 kN additional shear force, the optimum results are shown in Table VI.

TABLE V. THE OPTIMUM RESULTS (150kN, $H_f=80\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
400	300	350	30	65	845426.5
400	400	350	30	65	937531.8
400	500	690	80	60	1027501.7
500	200	230	100	65	708469.5
500	300	230	100	65	778166.4
500	400	230	100	65	847863.4
500	500	230	100	65	917560.4
600	200	90	70	65	683498.1
600	300	90	70	65	739748.1
600	400	90	70	65	795998.1
600	500	60	50	60	852177.0
700	200	80	90	65	665278.0
700	300	80	90	65	712336.8

700	400	80	90	65	759395.6
700	500	80	90	65	806454.5
800	200	130	180	70	655094.1
800	300	130	180	70	697029.6
800	400	110	170	60	737792.3
800	500	110	170	60	777078.0

All cases lower than 500 mm height and 300 mm breadth is not enough to increase the shear force with 200 kN. For the cases with 500 mm height, the width of CFRP is nearly 1 meter per unit meter.

TABLE VI. THE OPTIMUM RESULTS (200kN, $H_f=80\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
500	300	920	70	65	1037555.2
500	400	920	70	65	1130484.5
500	500	920	70	65	1223413.8
600	300	210	70	65	986330.8
600	400	360	120	65	1061330.8
600	500	240	90	60	1136236.0
700	200	220	130	65	888621.3
700	300	220	130	65	951478.4
700	400	220	130	65	1014335.6
700	500	220	130	65	1077192.7
800	200	210	180	65	868174.3
800	300	210	180	65	922020.5
800	400	210	180	65	975866.6
800	500	210	180	65	1029712.8

For the other cases with 100 mm and 120 mm flange height, the optimum results are given in Tables 7-14 in Appendix. By the increase of h_f , the effective depth of beam reduces. In that case, the required area of CFRP increases. Also, penalized objective functions are obtained in more cross-sectional dimension combinations.

Appendix

TABLE VII. THE OPTIMUM RESULTS (50kN, $H_f=100\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
300	200	60	60	60	296299.1
300	300	60	60	60	346299.1
300	400	40	40	60	396299.1
300	500	30	30	60	446299.1
400	200	20	40	65	257918.8
400	300	10	20	65	291252.2
400	400	40	80	65	324585.5
400	500	40	80	65	357918.8
500	200	30	90	65	243091.1
500	300	30	90	65	268091.1
500	400	30	90	65	293091.1
500	500	30	90	65	318091.1
600	200	30	120	65	234194.5
600	300	30	120	65	254194.5
600	400	30	120	65	274194.5
600	500	30	120	65	294194.5
700	200	10	50	65	228263.4

700	300	30	150	65	244930.1
700	400	30	150	65	261596.8
700	500	30	150	65	278263.4
800	200	30	180	65	224026.9
800	300	30	180	65	238312.7
800	400	30	180	65	252598.4
800	500	30	180	65	266884.1

700	400	130	70	65	1020227.4
700	500	130	70	65	1085227.4
800	200	200	160	65	871215.9
800	300	150	120	65	926771.5
800	400	50	40	65	982327.0
800	500	150	120	65	1037882.6

TABLE VIII. THE OPTIMUM RESULTS (100kN, $H_f=100\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
300	300	610	10	60	681427.2
300	400	610	10	60	779814.3
300	500	610	10	60	878201.4
400	200	160	80	65	515837.7
400	300	120	60	65	582504.3
400	400	110	60	60	647346.3
400	500	110	60	60	712052.2
500	200	50	50	65	486182.3
500	300	100	100	65	536182.3
500	400	50	50	65	586182.3
500	500	100	100	65	636182.3
600	200	80	130	60	463290.1
600	300	80	130	60	501385.3
600	400	80	130	60	539480.6
600	500	80	130	60	577575.8
700	200	60	130	60	449678.7
700	300	60	130	60	481257.7
700	400	60	130	60	512836.6
700	500	60	130	60	544415.5
800	200	50	130	65	435607.9
800	300	50	130	65	463385.7
800	400	50	130	65	491163.5
800	500	50	130	65	518941.3

TABLE XI. THE OPTIMUM RESULTS (50kN, $H_f=120\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
300	200	70	50	65	309757.8
300	300	70	50	65	368091.1
300	400	50	40	55	425684.7
300	500	50	40	55	481240.2
400	200	40	70	65	265316.9
400	300	40	70	65	301680.5
400	400	40	70	65	338044.1
400	500	40	70	65	374407.8
500	200	30	80	70	246097.5
500	300	30	80	70	273370.2
500	400	30	80	70	300642.9
500	500	30	90	55	326427.8
600	200	30	120	60	233989.7
600	300	30	120	60	253989.7
600	400	30	120	60	273989.7
600	500	30	120	60	293989.7
700	200	30	140	70	226846.1
700	300	30	140	70	244493.2
700	400	30	140	70	262140.2
700	500	30	150	60	279632.4
800	200	30	170	70	221552.0
800	300	30	170	70	236552.0
800	400	30	170	70	251552.0
800	500	30	170	70	266552.0

TABLE IX. THE OPTIMUM RESULTS (150kN, $H_f=100\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
400	300	550	20	60	868849.7
400	400	550	20	60	965341.0
400	500	550	20	60	1061832.2
500	200	140	50	65	716479.1
500	300	140	50	65	790163.3
500	400	140	50	65	863847.6
500	500	280	100	65	937531.8
600	200	170	120	65	686432.2
600	300	170	120	65	745052.9
600	400	170	120	65	803673.6
600	500	170	120	65	862294.3
700	200	90	100	60	674518.1
700	300	90	100	60	721886.5
700	400	90	100	60	769254.9
700	500	90	100	60	816623.3
800	200	100	140	65	653411.9
800	300	100	140	65	695078.6
800	400	100	140	65	736745.3
800	500	100	140	65	778411.9

TABLE XII. THE OPTIMUM RESULTS (100kN, $H_f=120\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
400	200	230	90	65	524415.4
400	300	230	90	65	596290.4
400	400	140	60	60	667979.4
400	500	70	30	60	737979.4
500	200	120	110	65	484293.6
500	300	120	110	65	536467.5
500	400	120	110	65	588641.4
500	500	120	110	65	640815.4
600	200	70	100	65	463991.9
600	300	70	100	65	505168.4
600	400	70	100	65	546344.8
600	500	70	100	65	587521.3
700	200	70	130	70	449911.5
700	300	70	130	70	484911.5
700	400	70	130	70	519911.5
700	500	70	130	70	554911.5
800	200	60	140	70	443104.0
800	300	70	180	60	471979.4
800	400	70	180	60	499979.4
800	500	70	180	60	527979.4

TABLE X. THE OPTIMUM RESULTS (200kN, $H_f=100\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
500	300	560	10	65	1053551.1
500	400	560	10	65	1151796.7
500	500	560	10	65	1250042.4
600	300	430	120	65	993669.5
600	400	430	120	65	1071851.3
600	500	430	120	65	1150033.1
700	200	260	140	65	890227.4
700	300	130	70	65	955227.4

TABLE XIII. THE OPTIMUM RESULTS (150kN, $H_f=120\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
500	200	180	50	65	726440.4
500	300	360	100	65	804701.3
500	400	360	100	65	882962.2
500	500	350	110	60	960295.3
600	200	80	50	65	693438.4
600	300	160	100	65	754976.9
600	400	160	100	65	816515.4

600	500	160	100	65	878053.8
700	200	100	90	70	676558.6
700	300	100	90	70	729190.2
700	400	100	90	70	781821.8
700	500	100	90	70	834453.3
800	200	130	170	65	660423.2
800	300	130	170	65	703756.5
800	400	130	170	65	747089.9
800	500	130	170	65	790423.2

TABLE XIV. THE OPTIMUM RESULTS (200KN,
 $H_f=120\text{MM}$)

h	b_w	w_f	s_f	β	A_{best}
500	300	990	0	55	1105711.2
500	400	880	0	55	1205711.2
500	500	890	0	55	1305711.2
600	300	590	130	65	1005325.1
600	400	590	130	65	1087269.6
600	500	590	130	65	1169214.0
700	200	290	140	65	893905.1
700	300	290	140	65	961346.9
700	400	290	140	65	1028788.8
700	500	290	140	65	1096230.7
800	200	230	170	65	876330.8
800	300	230	170	65	933830.8
800	400	230	170	65	991330.8
800	500	230	170	65	1048830.8

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