Mass ratio factor on optimum TMD design in frequency domain

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Abstract: - In the design of tuned mass dampers (TMDs), the mass of the TMD is an important performance factor. Generally, the best optimum performance is seen for the maximum mass which is restricted by the loading capacity of the main system. In the presented study, the optimum period and damping ratio of TMD for various single degree of freedom (SDOF) structures are investigated for different mass ratios. In the methodology, a metaheuristic algorithm called flower pollination algorithm is employed. The optimization objective is the minimization of maximum amplitude of the transfer function of the system, which is generally seen at the critical frequency of the structure. According to the results, the frequency or period ratio is generally only related to the mass ratio, but the main structure period plays a great role on the optimum damping ratio.

Key-Words: Tuned mass dampers, mass factor, metaheuristic algorithms, flower pollination algorithm, frequency domain, transfer function.

1 Introduction

For the vibration of mechanical systems, a mass attached to a springlike element (stiffness element) and a damper is used to reduce undesired mechanical vibrations. This device is called tuned mass damper (TMD) and it is used with all mechanical systems including robots, vehicles, machines and civil structures.

For example, TMDs can be used in structures for reduction of vibrations resulting from different sources such as wind, earthquake and traffic. For example, a TMD was added for comfort to Berlin TV Tower (Figure 1). For seismic safety, Los Angeles Theme Building-Lax Theme Building (Figure 2) was retrofitted with a TMD 20% mass ratio.

Tuned mass dampers can only effective when the device is tuned according to the critical natural frequency of the structure. For that reason, several closed-form expressions were developed for single degree of freedom structures. These formulations are for the frequency ratio of TMD and structure and damping ratio of the TMD. These optimum tuning equations are only variable of the mass ratio of TMD and structure and the period of structure, the content of excitation and non-critical vibration modes are not considered. By considering the critical mode, these equations can be approximately used for multiple degrees of freedom systems [2-5].

Figure 1 Berlin TV Tower

Figure 2 Lax Theme Building

It is impossible to derive a formula by including the inherent damping of the main system. For that reason, numerical optimization techniques have been developed [6-9]. Also, the formulations of Sadek et al. [4] were found according to the numerical trial results to include damping in the optimization. The most popular methods in recent years are to use metaheuristic algorithm. Leung and Zhang derived their formulations according to Particle Swarm Optimization based methodology [5].

The first metaheuristic based TMD optimization studies employed Genetic Algorithm (GA). The GA applications include regular structures [10-11], asymmetric plan structures [12-13] and active tuned mass dampers including fuzzy logic controller [14]. Another bio-inspired algorithm called bionic algorithm was employed by Steinbuch [15] for optimization of TMDs.

The music inspired Harmony Search (HS) has been employed in many TMD optimization studies to investigate time domain optimum results [16], mass ratio factor [17], preventing brittle fracture [18] and frequency domain optimum results [19]. Bat algorithm (BA) based methodology has been proposed by Bekdaş et al. [20]. Also, soil-structure interaction was considered by Bekdaş and Nigdeli employing HS and BA [21]. Teaching Learning Based Optimization [22] and Flower Pollination Algorithm (FPA) [23] were also employed in the optimum TMD design.

In the present study, the mass ratio factor is investigated on single degree of freedom systems. The optimum tuning of mass dampers was done by employing FPA using frequency domain analyses of the system.

2 Methodology

In this section, the equation of a single degree of freedom structural system combined with a TMD is presented. Then, the methodology is briefly explained.

In the Figure 3, a two degree of freedom system with the freedom of main system and TMD is shown.

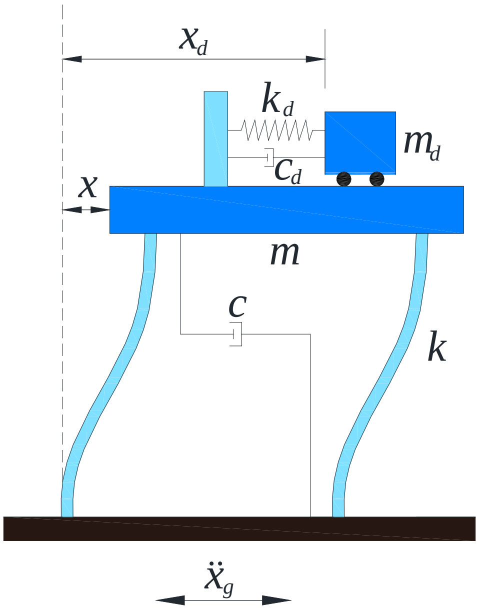

Figure 3 The structure-TMD model

Under a ground excitation, the equation of structure can be written as Eq. (1).

$$\mathrm{M}\ddot{x}(t)+\mathrm{C}\dot{x}(t)+\mathrm{K}x(t)=-\mathrm{M}\{1\}\ddot{x}_g(t) \tag{1}$$

The *M*, *C* and *K* are the mass, damping and stiffness matrices multiplied by the acceleration($\ddot{x}(t)$), velocity ($\dot{x}(t)$) and displacement vectors (x(t)), respectively. The acceleration and velocity are derivative of displacements respect to time. The matrices are as follows:

$$\mathrm{M}=\begin{bmatrix} m & 0 \\ 0 & m_d \end{bmatrix} \tag{2}$$

$$\mathrm{K}=\begin{bmatrix} k+k_d & -k_d \\ -k_d & k+k_d \end{bmatrix} \tag{3}$$

$$\mathrm{C}=\begin{bmatrix} c+c_d & -c_d \\ -c_d & c+c_d \end{bmatrix} \tag{4}$$

$$x(t)=\begin{bmatrix} x \\ x_d \end{bmatrix} \tag{5}$$

m, c, k and x are the mass, stiffness, damping coefficient and displacement of the main structure. The parameters and displacement of TMD denoted with a subscript, d. The ground excitation is shown with $\ddot{x}_g(t)$.

During the optimization the mass of TMD (md) is taken as a constant, because it is optimum in maximum values. The other optimized parameters are the period (Td) and damping ratio (ξd) which are formulated as follows:

$$T_d = 2\pi\sqrt{\frac{m_d}{k_d}} \tag{6}$$

$$\xi_d = 2c_d m_d \sqrt{\frac{k_d}{m_d}} \tag{7}$$

The optimization objective is to minimize the transfer function (TF) value of the acceleration of structure which the ratio of Laplace transforms of and $\ddot{x}_g(t)$. It is calculated as given in Eq. (8) in frequency domain.

$$\mathrm{TF(w)} = \begin{bmatrix} \mathrm{TF} \\ \mathrm{TF_d} \end{bmatrix} = \left[-\mathrm{M}\omega^2 + \mathrm{C}\omega\mathrm{j} + \mathrm{K}\right]^{-1} \mathrm{M}\omega^2 \{1\} \tag{8}$$

TF has imaginary (j) and real part. The amplitude is considered as objective function (f) and it is given in Eq. (9) in decibel (dB).

$$\mathrm{f} = 20\mathrm{Log}_{10}\left|\max(\mathrm{TF_N}(\omega))\right| \tag{8}$$

The main goal of the optimization is to minimize the value of f. For that reason, an iterative analysis is done according to the rules of FPA developed by Yang [24].

The algorithm inspired from the pollination process of flowering plants uses two types of phases called global and local pollination. In global, the rules of Lévy flight are considered to express the pollen transfer process of pollinators. In local pollination, the existing results are used with linear random distribution to express the self-pollination process of flowering plants.

A switch probability is used to choose the type in each iteration and this value is taken as 0.5 to give an equal probability for two types of optimization. The steps of the methodology can be generalized in the following topics.

1- Define structural parameters and design variable ranges.
2- Generate initial solutions randomly
3- Choose a type according to probability
4- Generate and update existing solutions
5- Continue 3 and 4 until maximum iteration number.

3 Numerical examples

As the numerical investigations, 10 different SDOF structure with periods from 0.1s and 5s were investigated. The optimum results are found for 10 mass ratio value for all structures. The investigated mass ratio values are between 1% and 40%. The inherent damping of structures was taken as 5%. The maximum allowed damping ratio of TMD is 50%. The optimizations of all TMDs were done 10^4 maximum iterations.

In the Table I, the optimum results are presented for different structure periods (T) and mass ratio (μ). The maximum transfer function value of the structure without TMD is 20 dB and it reduced with 68.3% for TMD with 40% mass.

In Figure 4, TMD vs. structure period is plotted. By the increase of the mass ratio, the period of TMD is also increasing. For different periods of structure, the period ratio of TMD and structures are close to each other and these values are changing between 1.0066 and 1.2850 according to the mass ratio values.

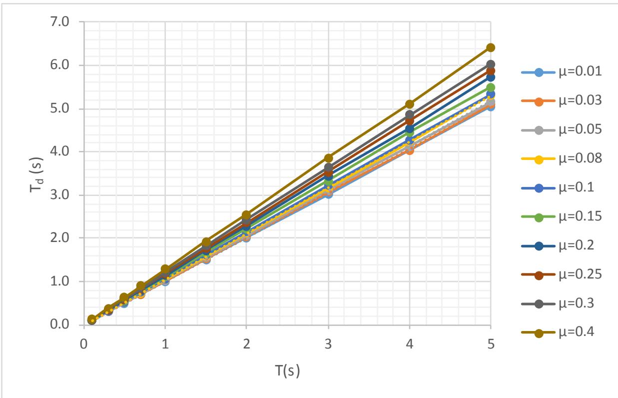

Figure 4. T_d vs T plot

In the Figure 5, the optimum ratio of TMD vs. the period of structure is plotted. As seen, the optimum damping ratio is similar for structure periods lower than 1s, but it is complex for the other structures.

Lastly, the objective function vs. the structure period is plotted in Figure 6 and the values are nearly same as mass ratios and the performance increases by the increase of the mass ratio.

TABLE I. THE OPTIMUM RESULTS

T(s)	0.1	0.3	0.5	0.7	1	1.5	2	3	4	5
μ	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
T_d(s)	0.1007	0.3020	0.5034	0.7043	1.0068	1.5102	2.0136	3.0204	4.0613	5.0512
x_d	0.0677	0.0677	0.0696	0.0649	0.0713	0.0713	0.0713	0.0713	0.0578	0.0500
f(dB)	15.8968	15.8948	15.8931	15.8836	15.8926	15.8926	15.8926	15.8926	15.4080	15.7146
μ	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
T_d(s)	0.1022	0.3067	0.5112	0.7155	1.0224	1.5307	2.0469	3.0696	4.0407	5.1058
x_d	0.1120	0.1120	0.1125	0.1074	0.1074	0.1028	0.1161	0.1068	0.0985	0.1061
f(dB)	13.5845	13.5828	13.5813	13.5826	13.5768	13.5623	13.5430	13.4209	13.2848	13.0512
μ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
T_d(s)	0.1038	0.3112	0.5187	0.7263	1.0369	1.5565	2.0719	3.1199	4.1381	5.1590
x_d	0.1411	0.1398	0.1424	0.1393	0.1362	0.1391	0.1458	0.1503	0.1584	0.1466
f(dB)	12.3010	12.3009	12.2981	12.2974	12.2873	12.2744	12.2625	12.2575	12.2450	11.9544
μ	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
T_d(s)	0.1060	0.3180	0.5301	0.7421	1.0603	1.5879	2.1167	3.1705	4.2300	5.3186
x_d	0.1743	0.1741	0.1778	0.1751	0.1778	0.1656	0.1711	0.1799	0.1567	0.1933
f(dB)	11.0246	11.0238	11.0242	11.0210	11.0242	11.0066	10.9744	10.9561	10.8581	10.9625
μ	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
T_d(s)	0.1075	0.3225	0.5375	0.7525	1.0751	1.6125	2.1517	3.2241	4.2806	5.3386
x_d	0.1930	0.1929	0.1951	0.1950	0.1923	0.1880	0.1999	0.1756	0.1852	0.1660
f(dB)	10.3931	10.3921	10.3915	10.3901	10.3878	10.3850	10.3771	10.3813	10.1939	10.3503
μ	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
T_d(s)	0.1112	0.3334	0.5558	0.7779	1.1113	1.6642	2.2220	3.3271	4.4553	5.5042
x_d	0.2297	0.2291	0.2307	0.2295	0.2267	0.2185	0.2218	0.2338	0.2337	0.2268
f(dB)	9.2162	9.2159	9.2146	9.2117	9.2138	9.2101	9.2135	9.1532	9.1643	9.0548
μ	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
T_d(s)	0.1147	0.3442	0.5738	0.8029	1.1469	1.7190	2.2945	3.4468	4.5460	5.7350
x_d	0.2584	0.2595	0.2613	0.2568	0.2591	0.2539	0.2576	0.2645	0.2378	0.2370
f(dB)	8.3681	8.3679	8.3678	8.3673	8.3629	8.3536	8.3540	8.3473	8.2971	8.3492
μ	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
T_d(s)	0.1182	0.3548	0.5913	0.8282	1.1822	1.7739	2.3610	3.5427	4.7249	5.8912
x_d	0.2834	0.2841	0.2848	0.2876	0.2812	0.2848	0.2813	0.2910	0.2991	0.2666
f(dB)	7.7091	7.7087	7.7087	7.7088	7.7074	7.7017	7.6892	7.6681	7.6526	7.6235
μ	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
T_d(s)	0.1217	0.3652	0.6085	0.8518	1.2160	1.8240	2.4321	3.6481	4.8641	6.0402
x_d	0.3049	0.3063	0.3050	0.3036	0.3007	0.3007	0.3007	0.3007	0.3007	0.2950
f(dB)	7.1736	7.1735	7.1725	7.1727	7.1717	7.1717	7.1717	7.1717	7.1717	7.0430
μ	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
T_d(s)	0.1284	0.3852	0.6421	0.8985	1.2839	1.9271	2.5617	3.8608	5.1180	6.4254
x_d	0.3382	0.3380	0.3382	0.3355	0.3388	0.3444	0.3282	0.3583	0.3449	0.3349
f(dB)	6.3407	6.3405	6.3404	6.3404	6.3372	6.3353	6.3298	6.3320	6.2951	6.3068

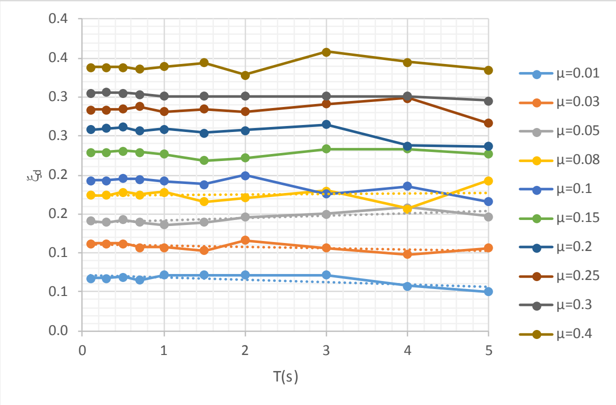

Figure 4. ξ_d vs T plot

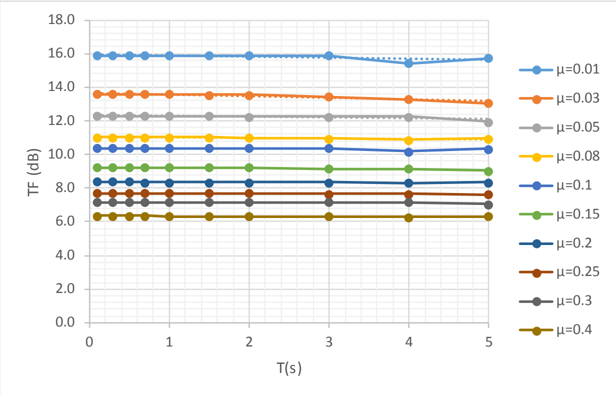

Figure 4. TF vs T plot

4 Conclusions

According to the results, the frequency or the period ratios of the TMD and structure is not related to the period of the main structure. In that case, the closed form formulations of the optimum frequency ratio [2-5] are useful in optimum tuning. A perfect optimum value can be only found with numerical optimization because the optimum damping values of TMD are different and related with the period of the structure. In that case, closed form equations are not a perfect optimum for the damping ratio.

By using a mass ratio between 1% and 40%, it is possible to reduce the TF value by 20.5% and 63.8%, respectively.

In the future studies, the study can be widened with different inherent damping ratios of the main structure.

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