Multiple Solutions of Slip Flow and Heat Transfer over an Exponential Shrinking Sheet with Stability Analysis

RAJESH SHARMA
Department of Mathematics, National Institute of Technology Hamirpur,
Hamirpur, Himachal Pradesh, India, 177005
raj.juit@gmail.com

Abstract: - The steady viscous flow with heat transfer over a permeable exponential shrinking sheet with partial slip at the boundary is studied. Similarity equations are obtained using similarity transformation in exponential form, which are then solved numerically using MATLAB routine boundary value problem solver based on finite difference method. Numerical results show that dual solutions exist for a certain range of mass suction. A stability analysis has been performed to show that first solution branch is stable while the other is always unstable.

Key-Words: - Heat transfer, multiple solutions, shrinking sheet slip flow, stability analysis

1 Introduction

The boundary layer flow on a stretching/shrinking sheet with heat transfer has many practical applications in industrial manufacturing processes such as in the polymer industry, where one deals with production of plastic sheet. The main aim is to generate better quality sheet, which depends upon the rate of cooling. The stretching sheet flow problem was first investigated by Crane [1] and reported an exact analytical solution to the Navier-Stokes equations. Following this study, the problem was then extended by other researchers [2-9].

Wang [10] was the first to investigate the unsteady shrinking sheet film and gave only little information on this type of flow. Later, shrinking sheet problem was investigated by Miklavcic and Wang [11] and established the existence and uniqueness criteria that there may be similarity solutions for this problem, if adequate suction on the surface is applied to confine vorticity, Further, this problem was investigated by Fang and Zhang [12], Cortell [13], Merkinand Kumaran [14], Sharma et al. [15] and many others. Sharma et al. [16] have investigated the stagnation point flow of a micropolar fluid over a stretching/shrinking sheet with second-order velocity slip. Recently, Fauzi et al. [17] have studied the flow and heat transfer over a stretching and shrinking sheet with slip and convective boundary condition.

In most of circumstances, fluid normally sticks to the boundary and no slip condition is consistent with the flow problem. Many fluids with particulates, such as emulsions, suspensions, foams, polymer solution, rarefied gas etc., where there may be a slip between the fluid and the boundary [18]. In the present article, we have investigated the boundary layer flow and heat transfer over an exponentially shrinking sheet with velocity and thermal slip effects as proposed by Beavers and Joseph [19]. The mathematical model of the problem is non-linear whose analytical solution is very hard to find out. Therefore, in this study, MATLAB routine boundary value problem (BVP) solver is used as a tool for the numerical simulation and the flow characteristics are discussed.

2 Problem formulation

Consider the two-dimensional boundary layer flow of a viscous and incompressible fluid past a permeable exponentially shrinking sheet coinciding with the plane $y = 0$, the flow being confined in the region $y \geq 0$ as shown in Fig. 1. Two equal and opposite forces are applied along the x – axis towards the origin O of the coordinate system, so that the wall shrinks keeping the origin fixed. It is assumed that the mass flux velocity is $v_w(x)$ with $v_w(x) < 0$ for suction and $v_w(x) > 0$ for injection or withdrawal of the fluid.

Under the assumption of boundary layer approximation, the governing equations of continuity, motion and energy are.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

where t is the time, u and v are the components of velocity in the x and y directions, T is the fluid temperature, α is the thermal diffusivity, ν is the kinematic viscosity.

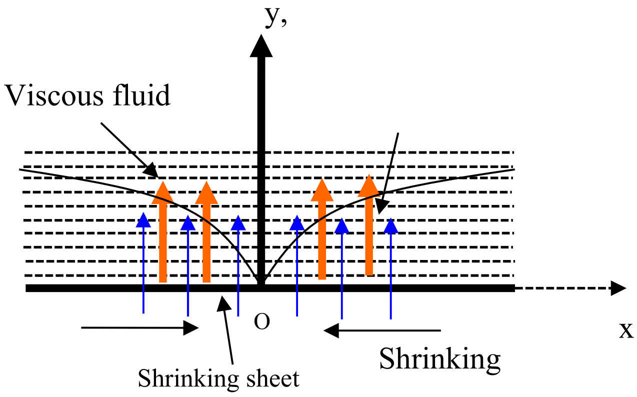

Fig. 1: Physical Model and Coordinate System

We assume that the initial boundary conditions of these equations are given by

$$\begin{aligned}
&t<0: \quad v=0, \quad u=0, \quad T=0 \quad \text{for any} \quad x, y\\
&t\geq 0: \quad v=v_w(x), \quad u=U+N\nu\frac{\partial u}{\partial y}, \quad T=T_w+D\frac{\partial T}{\partial y} \quad \text{at} \quad y=0\\
&\qquad u\to 0, \quad T\to T_\infty \quad \text{as} \quad y\to\infty
\end{aligned} \qquad (4)$$

where $U=-U_0\exp(x/L)$ is the shrinking velocity, $T_w=T_0\exp(x/2L)$ is the variable temperature at the sheet and $v_w(x)=V_0\exp(x/2L)$. Here L, U_0, T_0 and V_0 are the length, velocity, temperature and mass flux velocity characteristics, respectively, with $V_0<0$ for suction and $V_0>0$ for injection or withdrawal of the fluid. Further, we assume that the slip velocity factor N and the thermal slip factor D change with x and are given by $N=N_1\exp(-x/2L)$ and $D=D_1\exp(-x/2L)$, where N_1 is the initial value of velocity slip factor and D_1 is the initial value of the thermal slip factor (see Mukhopadhyay and Andersson [20]). The no-slip case is recovered for $N=D=0$.

We introduce now the following similarity variables (see Mukhopadhyay and Gorla [21]),

$$\begin{aligned}
&\eta=y\sqrt{\frac{U_0}{2\nu L}}\exp(x/2L), \qquad u=U_0\exp(x/L)f'(\eta,\tau),\\
&v=-\sqrt{\frac{\nu U_0}{2L}}\exp(x/2L)\left[f(\eta,\tau)+\eta f'(\eta,\tau)\right]\\
&T=T_0\exp(x/2L)\theta(\eta,\tau), \qquad \tau=\frac{U_0}{2L}\exp(x/L)t
\end{aligned} \qquad (5)$$

where prime denotes differentiation with respect to η. Substituting (5) into Eqs. (2) and (3), we obtain the following ordinary differential equations

$$f'''+ff''-2f'^2-\frac{\partial f'}{\partial \tau}(1+2\tau f')=0 \qquad (6)$$

$$\frac{1}{Pr}\theta''+(f\theta'-f'\theta)-\frac{\partial\theta}{\partial\tau}(1+2\tau f')=0 \qquad (7)$$

and the boundary conditions (4) become

$$\begin{aligned}
&\tau<0: \quad f=0, \quad f'=0, \quad \theta=0 \quad \text{for any} \quad \eta\\
&\tau\geq 0: \quad f(0)=s, \quad f'(0)=-1+\lambda f''(0), \quad \theta(0)=1+\delta\theta'(0)\\
&\qquad f'(\eta)\to 0, \quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty
\end{aligned} \qquad (8)$$

Here $\lambda=N_1\sqrt{U_0\nu/2L}\,(>0)$ is the velocity slip parameter, $\delta=D_1\sqrt{U_0/2\nu L}\,(>0)$ is the thermal slip parameter, $Pr=\nu/\alpha$ is the Prandtl number and $s=-V_0\sqrt{U_0/2\nu L}$ is the suction $(s>0)$ or blowing $(s<0)$ parameter.

3 Steady flow Solution

Taking the steady flow situation $\tau=0$ and $\partial/\partial\tau=0$ in Eqs. (6) and (7), we obtain

$$f'''+ff''-2f'^2=0 \qquad (9)$$

$$\frac{1}{Pr}\theta''+(f\theta'-f'\theta)=0 \qquad (10)$$

with boundary conditions

$$\begin{aligned}
&f(0)=s, \quad f'(0)=-1+\lambda f''(0), \quad \theta(0)=1\\
&f'(\eta)\to 0, \quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty
\end{aligned} \qquad (11)$$

The set of coupled nonlinear differential Eqs. (9) and (10), along with the boundary conditions (11) form a two point boundary value problem and is solved numerically using MATLAB routine BVP solver based on finite difference method with fourth order accuracy.

4 Results and discussion

Numerical solutions to the governing ordinary differential equations (9) and (10), along with the boundary conditions (11) are obtained using MATLAB routine solver for various values of the velocity slip parameter, thermal slip parameter and suction parameter. To solve this BVP with MATLAB routine BVP solver, we need the initial guess values. It is found that with different initial guess will result in different two solutions. Duality nature of the solution is consistent with the previous analysis for the shrinking sheet case (Miklavcic and Wang [11]; Bhattacharyya [16]).

The Variation of the reduced skin friction coefficient (or the surface shear stress) $f''(0)$ and the reduced local Nusselt number $-\theta'(0)$ with s for different values of the velocity slip parameter λ and thermal slip parameter δ are presented in Figs. 2-3 and Fig. 4. It is observed that when s is equal to a certain $s_c (>0)$, there is only one solution and when $s > s_c$, there is no solution. Based on our computations, the value of s_c decreases as λ increases and does not depend upon the values of δ. Hence, velocity slip parameters widen the range of s for which the solution exists.

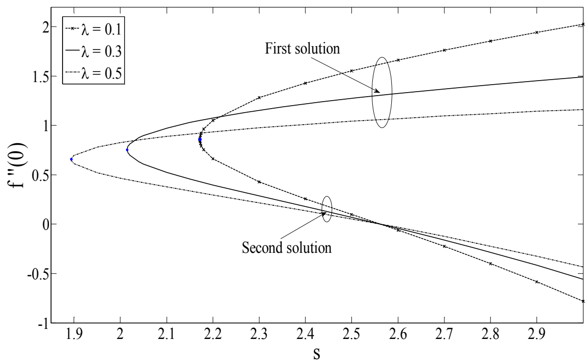

Fig. 2: Variation of $f''(0)$ with s for various values of λ

In Figs. 2-3, for the first solution, $f''(0)$ decreases but $-\theta'(0)$ increases with increasing λ. Thus, the surface shear stress decreases but the heat transfer at the surface increases with λ. The second solution shows complicated and quite different behaviors compared with the first solution. For the second solution, with the increase in λ, both $f''(0)$ and $-\theta'(0)$ decrease, while for $s > 2.54$ the pattern is reversed. In Figs. 4 , it is seen that for the first solution branch, the value of $-\theta'(0)$ is consistently higher with lower values of δ, while reverse pattern is observed for the second solution branch.

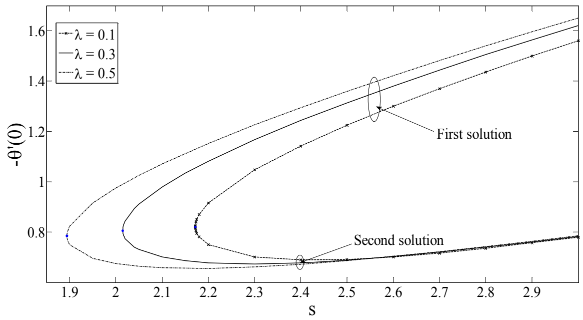

Fig. 3: Variation of $-\theta'(0)$ with s for various values of λ

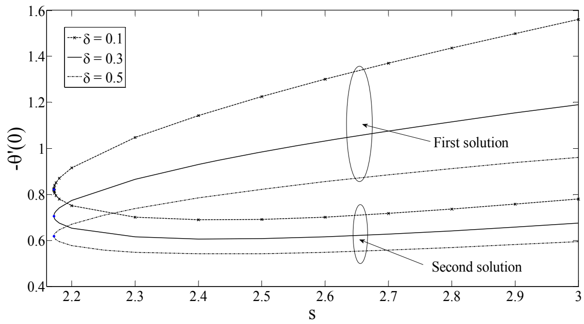

Fig. 4: Variation of $-\theta'(0)$ with s for various values of δ

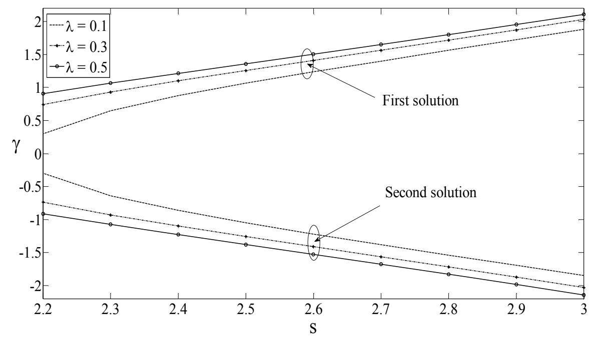

Fig. 5: Variation of γ with s for various values of λ

Following Merkin [22], we have test the linear stability of the steady flow solution. According to Merkin [22], stability is determined by the sign of the smallest eigenvalue. The positive minimum eigenvalue determines the stable flow. Based on this approach, we have converted the problem to eigenvalue problem and find out the minimum eigenvalue for both the solution shown in figure 5 & 6. For first solutions, the eigenvalues are always positive, while negative for second solutions. Thus, we conclude that first solutions are linearly stable, while the second solutions are linearly unstable for these particular parameter values. It is also observed from Figs. 5 & 6 that stability of first solution branch increases with the increase of λ and s, while δ has no effect on flow stability.

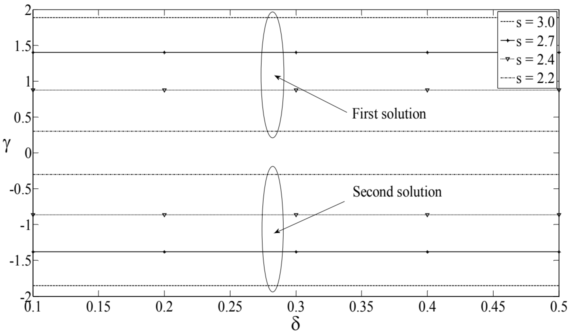

Fig. 6: Variation of γ with s for various values of δ

5 Conclusions

In summary, the slip viscous flow and heat transfer over an exponentially shrinking sheet with wall mass transfer has been solved numerically using MATLAB BVP solver to exhibit the effects of velocity slip parameter λ, thermal slip parameter δ and mass suction parameter s. It is found that multiple solutions exist in a certain range of mass suction parameter and the range of mass suction parameter for which the solution exists expands with the velocity slip parameter. It is observed that first solution is linearly stable while second solution is unstable. The flow stability increases with increasing velocity slip parameter λ and mass suction parameter s.

6 Acknowledgements

The financial support received from the SERB, DST, Govt. of India (Project Code: ECR/2016/000368) is gratefully acknowledged.

References:

1. L. J. Crane, Flow past a stretching plate, *J. Appl. Math. Phys. (ZAMP)*, Vol. 21, 1970, pp. 645-647.
2. P. S. Gupta and A. S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, *Can. J. Chem. Eng.*, Vol. 55, 1977, pp. 744-746.
3. L. J. Grub ka and K. M. Bo bba, Heat transfer characteristics of a continuous, stretching surface with variable temperature, *Trans. ASME J. Heat Transfer*, Vol. 107, 1985, pp. 248-250.
4. H. I. Andersson, J. B. Aarseth and B. S. Dandapat, Heat transfer in a liquid film on an unsteady stretching surface, *Int. J. Heat Mass Transfer*, Vol. 43, 2000, pp. 69-74.
5. A. Ali and A. Mehmood, Homotopy analysis of unsteady boundary layer flow adjacent to permeable stretching surface in a porous medium, *Commun. Nonlinear Sci. Numer. Simulat.*, Vol. 13, 2008, pp. 340-349,.
6. A. Ishak, R. Nazar and I. Pop, "Heat transfer over an unsteady stretching permeable surface with prescribed wall temperature," *Nonlinear Anal. Real World Appl.*, Vol. 10, 2009, pp. 2909-2913,.
7. S. Kazem, M. Shab an and S. Abbasbandy, Improved analytical solutions to a stagnation-point flow past a porous stretching sheet with heat generation, *J. Franklin Inst.*, Vol. 348, 2011, pp. 2044–2058.
8. R. Sharma, Effect of viscous dissipation and heat source on unsteady boundary layer flow and heat transfer past a stretching surface embedded in a porous medium using element free Galerkin method, *Appl. Math. Comput.*, Vol. 219, 2012, pp. 976-987.
9. R. Sharma, A. Ishak and I. Pop, Partial slip flow and heat transfer over a stretching sheet in a nanofluid, *Mathematical Problem in Engineering*, Vol. 2013, 2013, Article ID 724547, 7 pages.
10. Y. Wang, Liquid film on an unsteady stretching sheet, *Quart. Appl. Math.*, Vol. 48, 1990, pp. 601-610.
11. M. Miklavcic and C. Y. Wang, Viscous flow due to a shrinking sheet, *Quart. Appl. Math.*, Vol. 64, 2006, pp. 283–290.
12. T. Fang and J. Zhang, "Closed-form exact solutions of MHD viscous flow over a shrinking sheet, *Commun. Nonlinear Sci. Numer. Simulat.*, Vol. 14, 2009, pp. 2853-2857.
13. R. Cortell, On a certain boundary value problem arising in shrinking sheet flows, *Appl. Math. Comput.*, Vol. 217, 2010, pp. 4086-4093.

14. J. H. Merkin and V. Kumaran, The unsteady MHD boundary-layer flow on a shrinking sheet, *Eur. J. Mech. B Fluids*, Vol. 29, 2010, pp. 357-363.
15. R. Sharma, A. Ishak, I. Pop, Stability analysis of magnetohydrodynamics stagnation-point flow towards a stretching/shrinking sheet, *Comp. Fluids*, Vol. 102, 2014, pp. 94-98.
16. R. Sharma, A. Ishak, I. Pop, Stagnation point flow of a micropolar fluid over a stretching/shrinking sheet with second-order velocity slip, *J. Aerospace Eng.*, Vol. 29, No. 5, 2016, 04016025.
17. E. L. A. Fauzi, S. Ahmad, I. Pop, Flow and heat transfer over a stretching and shrinking sheet with slip and convective boundary condition, *AIP Conf. Pro.,* Vol. 1830, 2017, 020019.
18. V. P. Shidlovskiy, *Introduction to the Dynamics of Rarefied Gases*. American Elsevier Publishing Company Inc, New York, 1967.
19. G. S. Beavers and D. D. Joseph, Boundary condition at a naturally permeable wall, *J. Fluid Mech*., Vol. 30, 1967, pp. 197–207.
20. S. Mukhopadhyay and H. I. Andersson, Effects of slip and heat transfer analysis of flow over an unsteady stretching surface, *Heat Mass Transfer,* Vol. 45, 2009, pp. 1447–1452.
21. S. Mukhopadhyay and R. S. R. Gorla, Effects of partial slip on boundary layer flow past a permeable exponential stretching sheet in presence of thermal radiation, *Heat Mass Transfer,* Vol. 48, 2012, pp. 1773-1781.
22. J. H. Merkin, On dual solutions occurring in mixed convection in a porous medium, *J. Eng. Math.*, Vol. 20, 1985, pp. 171-179.