Some considerations regarding the Analysis of Pneumatic Actuators by Mathematical Models

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Abstract: The paper presents a comparative analysis of mathematical models of linear pneumatic actuators. The non-linear mathematical model of the linear pneumatic actuator is deduced in the first part of the paper and presented for comparison with the linearised mathematical model. The approximation rate of the linearised model, comparative to the non-linear model, is specified on the ground of the comparative analysis of the indicial response obtained by numeric simulation. In the end the validity areas of the presented mathematical models are identified.

Key- words: pneumatic actuator, mathematical modulations, numerical simulation.

1 Introduction

Characteristic to hydraulic drives is the fact, that in concordance with the perfect gases law, the density inside the considered control volume depends on the pressure *P* and the temperature *T* from inside this volume:

$$\rho = \frac{P}{R\,T} \qquad (1)$$

where R is the universal constant of perfect gases.

With these considerations, at the mathematical modeling of the pneumatic actuator the variation in time of the air density must be taken into account. Therefore, the weight rates and the transformation type of air inside the actuator will be taken into account.

Work hypothesis:

- the air volumes inside the motor chambers undergo isentropic transformations,
- the thermal regime is stationary, characterized by the absence of heat exchange with the outside, $\overset{o}{Q}_C$ = O, and the constant temperature of the air inside the actuator, T_A = *const.*,
- zero leakage at the motor α_M = 0,
- the temperature at which the air enters the actuator is constant $T_{i0} = const.$,
- the zero position of the pneumatic actuator is its median position, characterized by the volumes $V_{A0} = V_{B0} = 0.5\,V_M$, the pressures $P_{A0} = P_{B0} = 0.5\,P_0$ the temperature $T_{A0} = T_{B0} =$ 295,15 ^0K and the density $\rho_{A0} = \rho_{B0} = \dfrac{P_{A0}}{RT_O}$,
- between the variations in time of the pressures inside the actuator chambers, we have the following dependence formula [1]:

$$\frac{dP_B}{dt} = -\frac{0{,}5\,L_M + z_M}{0{,}5\,L_M - z_M}\,\frac{P_B}{P_B}\,\frac{dP_A}{dt} \qquad (2)$$

The input of the pneumatic actuator is the weight rate Q_{mA}, or the flow Q_A, which enter the motor chamber of the pneumatic actuator.

As output, there can be considered the displacement velocity of the actuator's mobile element, or the displacement z_M, in relation with its reference position.

2 The non-linear mathematical model

The mass conservation equations written for the two chambers of the pneumatic actuator on the ground of the work hypothesis are [2], [3]:

$$\begin{cases} \dfrac{T_{i0}}{T_A} Q_{mA} = \dfrac{T_{i0}}{T_A} \rho_A Q_A = \rho_A \overset{o}{V}_A + \dfrac{\rho_A V_A}{\chi P_A} \overset{o}{P}_A \\ -Q_{mB} = -\rho_B Q_B = \rho_B \overset{o}{V}_B + \dfrac{\rho_B V_B}{\chi P_B} \overset{o}{P}_B \end{cases} \quad (3)$$

where P_A, P_B are the pressures inside the motor chambers, ρ_A, ρ_B - the air densities from the two motor chambers, $V_A = 0.5\, V_M + A_M\, z_M$, $V_B = 0.5\, V_M - A_M\, z_M$ - the momentary volumes of the two chambers, A_M - the utile surface area of the actuator, χ - isentropic exponent (for air χ = 1,4), $Q_{mA} = \rho_A Q_A$, $Q_{mB} = \rho_B Q_B$ - the weight rate which enters the motor's motor chamber, respectively which exits the motor's motor chamber, Q_A and Q_B - the flows at the motor.

With this explanatory notes and taking into account formula (1) of the air density, the systems equations (2) can be rewritten as follows:

$$\begin{cases} \dfrac{T_{i0}}{\rho_A T_A} Q_{mA} = A_M v_M + \dfrac{0.5\, V_M + A_M z_M}{\chi P_A} \overset{o}{P}_A \\ -\dfrac{1}{\rho_B} Q_{mB} = -A_M v_M + \dfrac{0.5\, V_M - A_M z_M}{\chi P_B} \overset{o}{P}_B \end{cases} \quad (4)$$

where $v_M = \overset{o}{z}_M$ is the velocity of the motor's piston, and Q_A, Q_B are the flows which enter, respectively exit the chambers of the pneumatic actuator.

If we take into account the formula (2), the continuity equations of the flow at the motor can be written as:

$$\begin{cases} Q_B = \dfrac{T_{i0}}{T_A} Q_A \\ Q_B = A_M v_M + \dfrac{0.5\, V_M + A_M z_M}{\chi P_A} \overset{o}{P}_A \end{cases} \quad (5)$$

The equation of the pneumatic actuator's dynamic balance is:

$$A_M (P_A - P_B) = m_{LM} \overset{o}{v}_M + c_M v_M + F_{RM} \quad (6)$$

where F_{RM} is the resistance force at the actuator, m_{LM} - the mass of the load reduced at the stroke of the pneumatic actuator and c_M - coefficient of viscous friction. Therefore, the dynamics, in time range, of the symmetrical linear pneumatic actuator is described by the equation system from below, obtained by associating the equations (5), (2) and (6):

$$\begin{cases} Q_B = \dfrac{T_{i0}}{T_A} Q_A \\ \dfrac{0.5\, V_M + A_M z_M}{\chi P_A} \overset{o}{P}_A = Q_B - A_M v_M \\ \overset{o}{P}_B = -\dfrac{0.5\, V_M + A_M z_M}{0.5\, V_M - A_M z_M} \overset{o}{P}_A \\ m_{LM} \overset{o}{v}_M + c_M v_M + F_{RM} = A_M (P_A - P_B) \\ z_M = \int v_M \, dt \end{cases} \quad (7)$$

The non-linear character of the mathematical model can be noticed, introduced by the form of the first three equations, as well as the dependence of the mathematical model of the temperature T_A of the air inside the motor chamber of the pneumatic actuator, this having a input status for the pneumatic actuator.

3 Linearized mathematical model

The linearised mathematical model, presented in the references has been deduced on the ground of the similarity with the linear hydraulic actuator.

The mathematical model of the pneumatic actuator, in time range, respective in complex range, is described by the equation systems:

$$\begin{cases} Q_M = A_M\, v_M + C_{PM}^0\, \overset{o}{P}_L \\ A_M\, P_L = m_{RM}\, \overset{o}{v}_M + c_M\, v_M + F_{RM} \\ z_M = \int v_M\, dt \end{cases} \tag{8}$$

$$\begin{cases} P_L(s) = \dfrac{1}{T_1\, s}\left[Q_M(s) - K_1\, v_M(s)\right] \\ v_M(s) = \dfrac{K_2}{T_2\, s+1}\left[K_1\, P_L(s) - F_{RM}(s)\right] \\ z_M(s) = \dfrac{1}{s}\, v_M(s) \end{cases} \tag{9}$$

$$K_1 = A_M;\ K_2 = \frac{1}{c_M};\ T_1 = C_{PM}^0;\ T_2 = \frac{M_{LM}}{c_M}.$$

The dependence of the pneumatic actuator velocity upon inputs, flow and resistance force, deduced on the ground of the block diagrams' algebra;

$$v_M(s) = \frac{K_M}{\dfrac{s^2}{\omega_M^2} + 2\,\dfrac{\zeta_M}{\omega_M}\, s + 1}\left[Q_M(s) - T_F\, s\, F_{RM}(s)\right]$$

(10)

highlights the final transfer function of the actuator, characterized by the amplification coefficient K_M, the time constant T_F, the natural pulsation ω_M and the damping factor ζ_M;

$$K_M = \frac{1}{A_M};\ T_F = \frac{C_{PM}^0}{A_M};$$

$$\zeta_M = \frac{c_M}{2\, A_M}\sqrt{\frac{C_{PM}^0}{M_{LM}}};$$

$$\omega_M = A_M\sqrt{\frac{1}{C_{PM}\, M_{LM}}}.$$

According to the final value theorem, the stationary position error (the actuator's derivation of zero) Δz_M produced by a step variation $\Delta F_{RM}(s) = \dfrac{\Delta F_{RM}}{s}$ of the actuator's load at no flow, ($Q_M = 0$), results:

$$\Delta z_M = -\frac{\Delta F_{RM}}{K_{gM}^0} = -\frac{L_M}{4\, E_g^0\, A_M}\,\Delta F_{RM} \tag{11}$$

4 Comparative analyses of mathematical models

In the following we present the analysis of the indicial response of the pneumatic actuator with the following characteristics: D_M = 60 mm, d_M = 20 mm, L_M = 300 mm, P_0 = 4,5 bar, T_{i0} = 230 °K, T_A = 315 °K, , Q_B = 30 dm³ (step signal in the origin), F_{RM} = 200 N (step signal at t = 0,25 seconds).

The indicial response has been obtained by numerical simulation using Simulink/Matlab. Simulations, for the following variants of the mathematical models of the pneumatic actuator, have been completed:

1. The elasticity modulus and the chambers volumes have been considered constant.

2. Only the chambers volumes have been considered constant.

3. Both the variation of the air elasticity modulus and of the actuator's chambers volumes has been taken into account.

Fig.1 and 2 represent the indicial responses for the following parameters: the actuator's velocity v_M, the pressures of the motor chambers P_A and P_B, the output flow Q_B and the relative pressure error $\Delta P_r\%$. The models **1** and **2** have been noted with the indices 1 and 2, and model 3 has been used for reference.

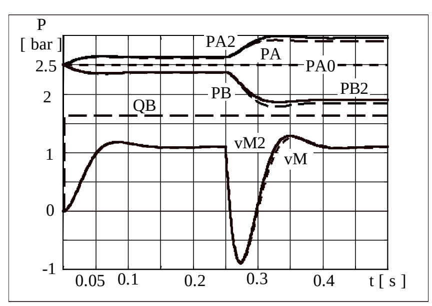

Fig.1 The indicial responses for actuator's velocity.

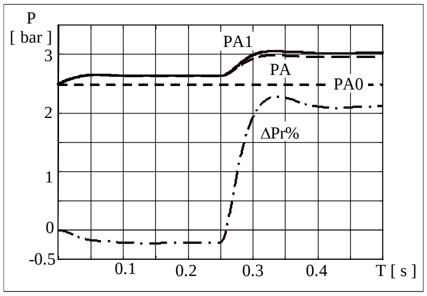

Fig.2 The indicial responses for the pressures of the motor chambers

The comparative analysis of the mathematical models of the pneumatic actuator, completed on the ground of the indicial response, makes possible to highlight the following notes:

- At the flow's conservation equation writing, it is mandatory to consider the temperature at which the air enters the motor and the room temperature in which it enters, respectively the report T_{i0}/T_A of these.
- The natural pulsation of the pneumatic actuator is as higher, as the utile surface area is higher and its length is lower, the motor's dumping being influenced in the opposite way.
- The natural pulsation and the damping are influenced by the value of the working pressure.
- For regular values of the compressed air pressure, the air elasticity module $E_g = \chi P$, and therefore the rigidity of the pneumatic spring, represents approximately $4 \cdot 10^{-3}$ from the oil's elasticity module. Therefore, the pneumatic actuators are damped much stronger and their natural pulsation is much smaller.
- The actuator's derivation by zero (the stationary position error) caused by the resistance load variation at the actuator is much higher, due to the powerful air compressibility.
- The modulation errors are below 6% in regard with the perturbation - the resistance force at the actuator, respectively under 1% in regard with the flow.
- These errors are slightly lower when the variation of the elasticity modulus depending on pressure is considered.
- The linerarised model of the pneumatic actuator can be used with errors below 1%, when only inertial forces solicit the pneumatic actuator.

Conclusions

The linearised model is valid, to describe the operation of the pneumatic actuator only in the following conditions:

- in the immediate vicinity of the stationary operating point,
- when the actuator is solicited only by inertial forces,
- the temperature at which the air enters the actuator must be always considered.

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