Non-linear programing for sizing optimization of truss structures

GEBRAİL BEKDAŞ
Department of Civil Engineering
Istanbul University
34320 Avcılar, Faculty of Engineering, Istanbul, Turkey
TURKEY
bekdas@istanbul.edu.tr

SİNAN MELİH NİGDELİ
Department of Civil Engineering
Istanbul University
34320 Avcılar, Faculty of Engineering, Istanbul, Turkey
TURKEY
melihnig@istanbul.edu.tr

OSMAN HÜROL TÜRKAKIN
Department of Civil Engineering
Istanbul University
34320 Avcılar, Faculty of Engineering, Istanbul, Turkey
TURKEY
turakin@istanbul.edu.tr

Abstract: - In civil engineering, the sizing optimization of truss structures is a widely used practice. The aim of the optimization is to minimize the total weight of truss members by considering the constraints of several members and nodes. Two types of constraints are important in design and these constraints are stress and displacement limitations. In the recent study, a non-linear programming tool employing the interior-point algorithm was integrated with the analyses of truss structures. As numerical examples, two space structures and a plane structure were optimized. The results were compared with the documented methods. As a conclusion the proposed method is more effective on computation time compared to other methods.

Key-Words: Truss structures, Mon-linear programming, Design constraints, Size optimization, Optimization.

1 Introduction

In optimization theory, optimum sizing design of truss structures is an important engineering practice. This problem has non-linear constraints. Firstly, the stress of the members of the truss structures must not exceed the fracture limits. For the calculation of the stress, the area of truss members must be defined and the analyses of internal forces must be done. These analyses can be only done after the cross-sectional areas of the structural members are known. These areas are the design variables and the problem is non-linear. Also, the nodal displacements can be only calculated after the assignment of design variables. In that case, numerical algorithms are used in the optimum design. Another option is to use non-linear programming tools.

In the documented methods, several algorithms have been modified for the optimum design of truss structures. In the Table 1, the employed algorithms and references presented.

In the recent study, a non-linear programming tool is proposed for the sizing optimization of truss structures. The fmincon function of Matlab [24] was integrated into the analyses of truss structures. The function employs the interior point algorithm developed by Fiacco and McCornick [25]. The proposed method was tested on space and plane truss structures.

TABLE I. THE DOCUMENET METHODS FOR THE TRUSS OPTIMIZATION

Method	Reference
A dual simplex algorithm (DSA)	[1]
Genetic algorithm (GA)	[2-4]
Ant colony optimization (ACO)	[5]
Big bang-big crunch (BB-BC)	[6]
Particle Swarm optimizer with passive congregation (HPSO)	[7]
Particle swarm optimization (PSO)	[8]
Simulated annealing (SA)	[9]
Hybrid of BB-BC and PSO (HBB-BC)	[10]
Artificial bee colony (ABC)	[11]
Harmony search (HS)	[12-13]
Teaching learning based optimization (TLBO)	[14-16]
Hybrid particle swallow optimization (HPSO)	[17]
Chaotic swarming of particle (CSP)	[18]
Colliding bodies optimization (CBO)	[19-20]
Flower pollination algorithm (FPA)	[21]
Ray Optimization	[22]
Hybrid of PSO, ACO and HS (HPSACO)	[23]

2 The Optimization Problem

The number of degrees of freedom of truss structure (n) is defined as

$$n = dN - s \tag{1}$$

if N>2 nodes and s≥0 fixed nodal coordinate directions. The number of nodal freedoms (d) is 2 for planar trusses and 3 for space trusses. In that case, the number of bars (m) is defined as follows since long bars overlapping small bars must be prevented;

$$m \geq n \text{ and } m \leq \frac{N(N-1)}{2}. \tag{2}$$

The normalized weight of i^th^ bar (λ_i) can be shown as $\lambda_i \geq 0$ for i∈ (1,....,m). If the material properties of bars are equal,

$$\lambda_i = \gamma L_i A_i, \; (A_i \in \mathbb{R}). \tag{3}$$

The density of the bars is shown with γ while $L_i \geq 0$ is the length of the i^{th} bar. The cross-sectional area of the i^{th} bar is shown with A_i. The design variables of the optimization problem are A_1,..., A_i,....,A_m for i=1,....,m. The elastic equation of equilibrium can be written as

$$K(A)u_A = P, \tag{4}$$

where $u_A \in \mathbb{R}^n$, $K(A) \in \mathbb{R}^{nxn}$ and $P \in \mathbb{R}^n$ are the displacement vector of nodes in global reduced coordinates , the stiffness matrix of the truss and external force vector, respectively. The stiffness matrix of the truss is obtained by merging the element stiffness matrix in global coordinates ($K_i(A) \in \mathbb{R}^{2dx2d}$). For a space truss structure,

$$K_i(A) = EA_i \begin{bmatrix} a^2 & ab & ac & -a^2 & -ab & -ac \\ ab & b^2 & bc & -ab & -b^2 & -bc \\ ac & bc & c^2 & -ac & -bc & c^2 \\ -a^2 & -ab & -ac & a^2 & ab & ac \\ -ab & -b^2 & -bc & ab & b^2 & bc \\ -ac & -bc & c^2 & ac & bc & c^2 \end{bmatrix} \tag{5}$$

where,

$$a = \frac{L_{xi}}{L_i},\; b = \frac{L_{yi}}{L_i} \text{ and } c = \frac{L_{zi}}{L_i}. \tag{6}$$

After the truss stiffness matrix is generated, the corresponding rows and columns of the fixed nodes are eliminated. L_{xi}, L_{yi}, and L_{zi} are the length i^{th} bar in global x,y and z coordinates, respectively. E is the modulus of elasticity.

The design variables are searched between upper (A^U) and lower (A^L) bounds,

$$A^L \leq A_i \leq A^U \;\; i = 1,......m. \tag{7}$$

One of the two design constraints is $g_1(A) \leq 0$. It related to the limitation of stress of the i^{th} bar (σ_i) with tensile limit (σ^L) and compression limitation (σ^U) as seen in (8).

$$g_1(A): \sigma^L \leq \sigma_i \leq \sigma^U \;\; i = 1,......m \tag{8}$$

The stress on the global coordinates are found according to (9).

$$\sigma_i^G = \frac{K_i(A)u_i}{A_i} \;\; i = 1,.....,m \tag{9}$$

The other constraint is related with the limitation of displacements defined in a nodal displacement vector of i^{th} bar (u_i). It is shown as

$$g_2(A): u^L \leq u_i \leq u^U \;\; i = 1,......N, \tag{10}$$

where u^L and u^U are limit of ranges defined as;

$$\left|u^L\right| = \left|u^U\right|;\ u^L \le 0, u^U > 0\,. \tag{11}$$

The objective function (f(A)) can be written as follows;

$$\min f(A) = \sum_{i=1}^{m} \lambda_i\ . \tag{12}$$

The aim of the optimization is to minimize the total weight of the bars. In the proposed method, the code provided for the analyses of the design constraints were integrated to the fmincon function of Matlab [24]. The results with the comparison with the other methods are presented in the following section.

3 Numerical Example

The numerical studies cover three truss structures; two space (25 bar and 72 bar) and a planar (200 bar) [21].

3.1 25 bar truss structure

The model of the structure is shown in Figure 1. The loading cases are shown in Table 2. The elasticity modulus and density are taken as 10 Msi and 0.1 lb/in^3. The ranges of design variables are between 0.01 and 3.4 in^2. The constraints of bars are presented in Table 3 for tensile and compressive stresses. The displacement is limited to 0.35 in. The optimum results are given in Table 4 with the results of other methods.

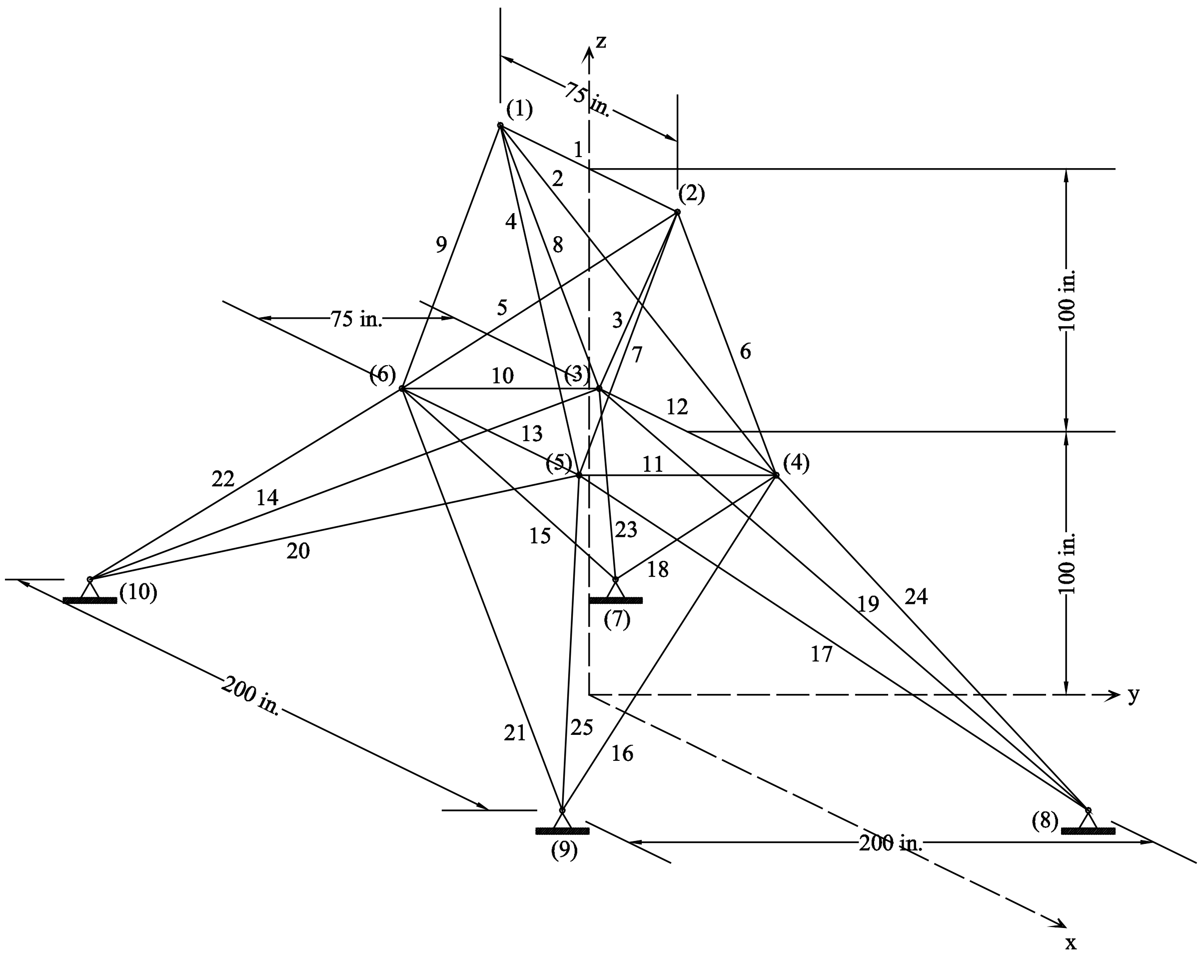

Figure 1 25-bar truss structure.

TABLE II. THE LOADING CASES OF 25-BAR STRUCTURE

Case	Node	P_x (kips)	P_y (kips)	P_z (kips)
1	1	1.0	10.0	-5.0
	2	0.0	10.0	-5.0
	3	0.5	0.0	0.0
	6	0.0	0.0	0.0
2	1	0.0	20.0	-5.0
	2	0.0	-20.0	-5.0

TABLE III. THE DESIGN CONSTRAINT LIMITS OF 25-BAR STRUCTURE

Element group	Members	Compression (ksi)	Tension (ksi)
1	1	35.092	35
2	2-5	11.590	35
3	6-9	17.305	35
4	10,11	35.092	35
5	12,13	35.092	35
6	14-17	6.759	35
7	18-21	6.959	35
8	22-25	11.082	35

TABLE IV. THE OPTIMUM RESULTS OF THE 25-BAR STRUCTURE

Group	GA [2]	ACO [5]	HPSO [7]	BB-BC [6]	SA [9]	HBB-BC [10]	ABC [11]	TLBO [14]	HPSO [17]	CBO [19]	FPA [21]	Present study
1	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0110	0.0100	0.0100	0.0100	0.0100	0.0100
2	2.0119	2.0000	1.9700	2.0920	1.9870	1.9930	1.9790	1.9878	1.9907	2.1297	1.8308	1.9891
3	2.9493	2.9660	3.0160	2.9640	2.9935	3.0560	3.0030	2.9914	2.9881	2.8865	3.1834	2.9905
4	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0102	0.0100	0.0100	0.0100	0.0100
5	0.0295	0.0120	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
6	0.6838	0.6890	0.6940	0.6890	0.6840	0.6650	0.6900	0.6828	0.6824	0.6792	0.7017	0.6835
7	1.6798	1.6790	1.6810	1.6010	1.6769	1.6420	1.6790	1.6775	1.6764	1.6077	1.7266	1.6766
8	2.6759	2.6680	2.6430	2.6860	2.6621	2.6790	2.6520	2.6640	2.6656	2.6927	2.5713	2.6635
Best Weight (lb)	545.80	545.53	545.19	545.38	545.16	545.16	545.19	545.18	545.16	544.31	545.16	545.16
Number of structural analyses	-	16500	125000	20566	400	12500	500000	12199	13326	9090	8149	Duration 2.69s

3.2 72 bar truss structure

The second example (Figure 2) is a 72 bar truss structure. The loading cases of 72 bar truss structure can be seen in Table 5. The material properties are the same as the first numerical example, but the maximum displacement is 0.25 and the stress limits ∓25 ksi for all members. The cross sectional areas must be between 0.1 and 3.0 in^2. The optimum results with design groups are presented in Table 6.

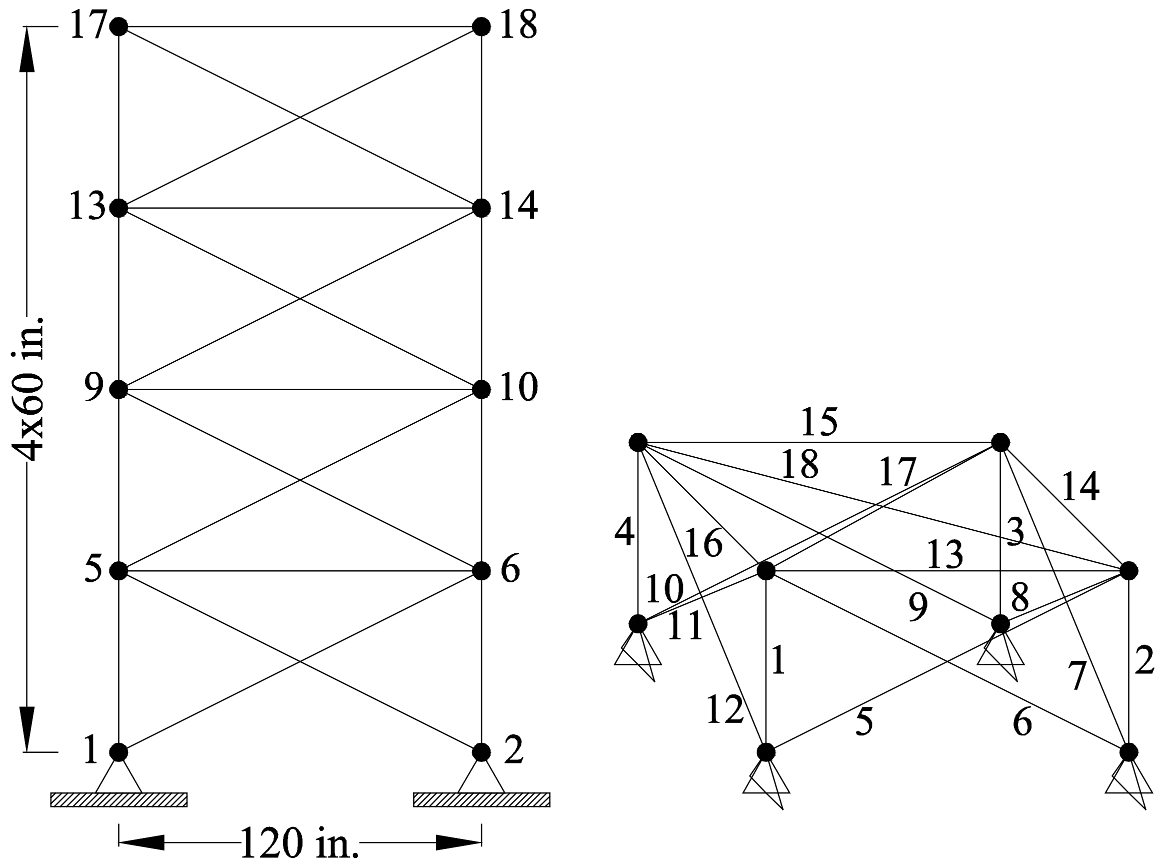

Figure 2 72-bar truss structure.

TABLE V. THE LOADING CASES OF 72-BAR STRUCTURE

Case	Node	Px (kips)	Py (kips)	Pz (kips)
1	17-20	-5.0	-5.0	-5.0
2	17	5.0	5.0	-5.0

3.3 200 bar truss structure

The last problem is a large planar truss structure. As seen in Figure 3, the structure has 200 elements and 77 nodes. The material properties such as the elasticity modulus and density of the material are taken as 30 Msi and 0.283 lb/in3, respectively. The problem has no displacement constraint. The stress constraints are 10 ksi for all members and directions. The range of design variables is between 0.1 and 20 in^2. The members are grouped in 29 sizing variables as seen in Table 7.

The structure is subject to three loading cases. In the first case, +1 kip load is applied in X-direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62 and 71. Secondly, -10 kips load is applied in Y-direction at nodes 1-6, 8, 10, 12, 14-20, 22, 24, 26, 28-34, 36, 38, 40, 42-48, 50, 52, 54, 56-62, 64, 66, 68, 70-75. The final case is the combination of first and second loading cases. The optimum design must be suitable for all cases. The optimum results are presented in Table 8.

TABLE VI. THE OPTIMUM RESULTS OF THE 72-BAR STRUCTURE

Element group	Members	DSA [1]	GA [2]	GA [3]	ACO [5]	PSO [8]	BB-BC [6]	HBB-BC [10]	HS [12]	TLBO [12]	CSP [18]	CBO [19]	CBO [20]	RO [22]	FPA [21]	Present study
1	1–4	2.0259	1.8562	1.7550	1.9480	1.7430	1.8577	1.9042	1.8600	1.8807	1.9446	1.90280	1.8519	1.8365	1.8758	1.8862
2	5–12	0.5332	0.4933	0.5050	0.5080	0.5180	0.5059	0.5162	0.5210	0.5142	0.5026	0.51800	0.5141	0.5021	0.5160	0.5123
3	13–16	0.1000	0.1000	0.1050	0.1010	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.10010	0.1000	0.1000	0.1000	0.1000
4	17–18	0.1000	0.1000	0.1550	0.1020	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.10030	0.1000	0.1004	0.1000	0.1000
5	19–22	1.1567	1.2830	1.1550	1.3030	1.3080	1.2476	1.2582	1.2710	1.2711	1.2676	1.27870	1.2819	1.2522	1.2993	1.2684
6	23–30	0.5689	0.5028	0.5850	0.5110	0.5190	0.5269	0.5035	0.5090	0.5151	0.5099	0.50740	0.5091	0.5033	0.5246	0.5117
7	31–34	0.1000	0.1000	0.1000	0.1010	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.10030	0.1000	0.1002	0.1001	0.1000
8	35–36	0.1000	0.1000	0.1000	0.1000	0.1000	0.1012	0.1000	0.1000	0.1000	0.1000	0.10030	0.1000	0.1002	0.1000	0.1000
9	37–40	0.5137	0.5177	0.4600	0.5610	0.5140	0.5209	0.5178	0.4850	0.5317	0.5067	0.52400	0.5312	0.5730	0.4971	0.5237
10	41–48	0.4791	0.5227	0.5300	0.4920	0.5460	0.5172	0.5214	0.5010	0.5134	0.5165	0.51500	0.5173	0.5499	0.5089	0.5171
11	49–52	0.1000	0.1000	0.1200	0.1000	0.1000	0.1004	0.1000	0.1000	0.1000	0.1075	0.10020	0.1000	0.1004	0.1000	0.1000
12	53–54	0.1000	0.1049	0.1650	0.1070	0.1090	0.1005	0.1007	0.1000	0.1000	0.1000	0.10150	0.1000	0.1001	0.1000	0.1000
13	55–58	0.1579	0.1557	0.1550	0.1560	0.1610	0.1565	0.1566	0.1680	0.1565	0.1562	0.15640	0.1560	0.1576	0.1575	0.1565
14	59–66	0.5501	0.5501	0.5350	0.5500	0.5090	0.5507	0.5421	0.5840	0.5429	0.5402	0.54940	0.5572	0.5222	0.5329	0.5456
15	67–70	0.3449	0.3981	0.4800	0.3900	0.4970	0.3922	0.4132	0.4330	0.4081	0.4223	0.40290	0.4259	0.4356	0.4089	0.4104
16	71–72	0.4984	0.6749	0.5200	0.5920	0.5620	0.5922	0.5756	0.5200	0.5733	0.5794	0.55040	0.5271	0.5972	0.5731	0.5697
Best Weight (lb)		379.310	380.320	383.120	380.240	381.910	379.850	379.660	380.620	379.632	379.970	379.6943	379.77	380.458	379.095	379.615
Number of structural analyses		-	15000	-	18500	8000	19679	13200	13742	21542	10500	15600	-	19084	9029	Duration 11.64 s

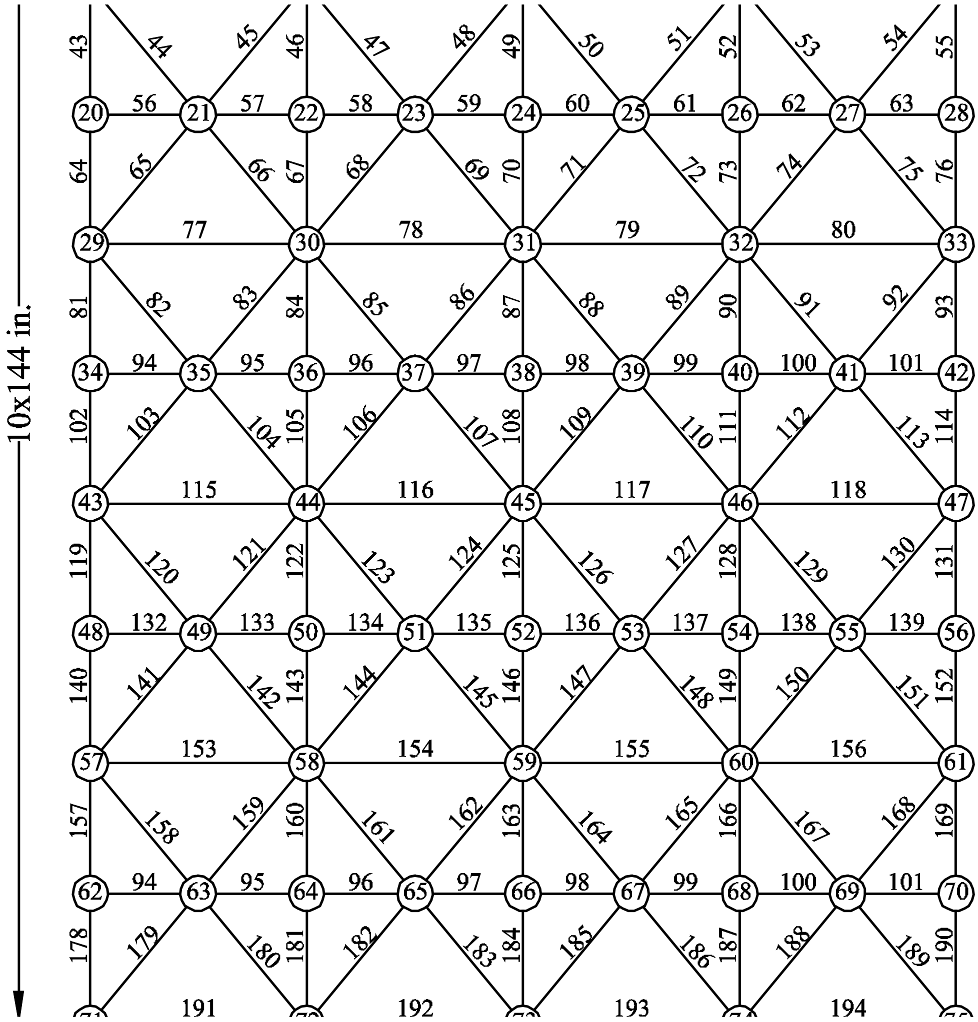

Figure 3 200-bar truss structure.

TABLE VII. THE MEMBER GROUPING OF THE 200-BAR STRUCTURE

Element group	Members	Element group	Members
1	1, 2, 3, 4	16	82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113
2	5, 8, 11, 14, 17	17	115, 116, 117, 118
3	19, 20, 21, 22, 23, 24	18	119, 122, 125, 128, 131
4	18, 25, 56, 63, 94, 101, 132, 139, 170, 177	19	133, 134, 135, 136, 137, 138
5	26, 29, 32, 35, 38	20	140, 143, 146, 149, 152
6	6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33, 34, 36, 37	21	120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151
7	39, 40, 41, 42	22	153, 154, 155, 156
8	43, 46, 49, 52, 55	23	157, 160, 163, 166, 169

9	57, 58, 59, 60, 61, 62	24	171, 172, 173, 174, 175, 176
10	64, 67, 70, 73, 76	25	178, 181, 184, 187, 190
11	44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75	26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189
12	77, 78, 79, 80	27	191, 192, 193, 194
13	81, 84, 87, 90, 93	28	195, 17, 198, 200
14	95, 96, 97, 98, 99, 100	29	196, 199
15	102, 105, 108, 111, 114		

4 Conclusions

According to the results of the proposed method integrated with analyses of truss structures, the proposal is a comparative method according to the other documented methods. Comparing to the other methods using metaheuristic algorithms, the method is effective in the computational effort.

For example, the total optimization period is only 2.69 s for the proposed method. At the same computer system, a structural analysis in metaheuristic based methods is 0.1512 s long. In that case, the FPA based method [21] is effective to find the similar optimum value in 1232.13 s. Similarly, the optimum result of the second example is found in 11.64 s while FPA [21] is effective to find an optimum value in 1067.23 s by using the same equipment. It must be noted that the other methods need more analyses than the FPA based method.

For the last example, the duration of the optimization is 550.99 s which is significantly more than the other examples for the proposed method. Since the problem is big, the duration of an analysis (0.2138 s) is nearly two times of the second example. In that case, the FPA based method [21] is effective to find the optimum value in 2284.453 s. For this example, several methods may be effective in reduction of optimum weight, but minor constraint violations may occur in the metaheuristic algorithm based methods.

As a conclusion, the proposed strategy for the optimum sizing of truss structures is a quick and effective tool. By using this rapid method, it will be possible to find better member grouping options than the proposed ones in the documented methods. In that case, economical and practical solutions can be found. This issue will be considered in the future studies.

TABLE VIII. THE OPTIMUM RESULTS OF THE 200-BAR STRUCTURE

Element group	HS [13]	GA [4]	SA [9]	HPSACO [23]	HS [12]	TLBO [15]	CSP [18]	HPSO [17]	TLBO [16]	FPA [21]	Present study
1	0.1253	0.3469	0.1468	0.1033	0.1540	0.1460	0.1480	0.1213	0.1135	0.1425	0.1069
2	1.0157	1.0810	0.9400	0.9184	0.9410	0.9410	0.9460	0.9426	0.9484	0.9637	0.9154
3	0.1069	0.1000	0.1000	0.1202	0.1000	0.1000	0.1010	0.1220	0.1078	0.1005	0.2094
4	0.1096	0.1000	0.1000	0.1009	0.1000	0.1010	0.1010	0.1000	0.1000	0.1000	0.1000
5	1.9369	2.1421	1.9400	1.8664	1.9420	1.9410	1.9461	2.0143	1.9345	1.9514	1.9154
6	0.2686	0.3470	0.2962	0.2826	0.3010	0.2960	0.2979	0.2800	0.2889	0.2957	0.3175
7	0.1042	0.1000	0.1000	0.1000	0.1000	0.1000	0.1010	0.1589	0.2116	0.1156	0.1006
8	2.9731	3.5650	3.1042	2.9683	3.1080	3.1210	3.1072	3.0666	3.0903	3.1133	3.1105
9	0.1309	0.3470	0.1000	0.1000	0.1000	0.1000	0.1010	0.1002	0.1031	0.1006	0.1007
10	4.1831	4.8050	4.1042	3.9456	4.1060	4.1730	4.1062	4.0418	4.0903	4.1100	4.1138
11	0.3967	0.4400	0.4034	0.3742	0.4090	0.4010	0.4049	0.4142	0.4502	0.4165	0.4102

12	0.4416	0.4400	0.1912	0.4501	0.1910	0.1810	0.1944	0.4852	0.1007	0.1843	0.1571
13	5.1873	5.9520	5.4284	4.9603	5.4280	5.4230	5.4299	5.4196	5.4798	5.4567	5.4243
14	0.1912	0.3470	0.1000	1.0738	0.1000	0.1000	0.1010	0.1000	0.1011	0.1000	0.1000
15	6.2410	6.5720	6.4284	5.9785	6.4270	6.4220	6.4299	6.3749	6.4798	6.4559	6.4332
16	0.6994	0.9540	0.5734	0.7863	0.5810	0.5710	0.5755	0.6813	0.5329	0.5800	0.5759
17	0.1158	0.3470	0.1327	0.7374	0.1510	0.1560	0.1349	0.1576	0.1325	0.1547	0.2940
18	7.7643	8.5250	7.9717	7.3809	7.9730	7.9580	7.9747	8.1447	7.9445	8.0132	7.9988
19	0.1000	0.1000	0.1000	0.6674	0.1000	0.1000	0.1010	0.1000	0.1005	0.1000	0.1000
20	8.8279	9.3000	8.9717	8.3000	8.9740	8.9580	8.9747	9.0920	8.9444	9.0135	9.0063
21	0.6986	0.9540	0.7049	1.1967	0.7190	0.7200	0.7065	0.7462	0.7011	0.7391	0.8194
22	1.5563	1.7639	0.4196	1.0000	0.4220	0.4780	0.4225	0.2114	1.3777	0.7870	0.4748
23	10.9806	13.3006	10.8636	10.8262	10.8920	10.8970	10.8685	10.9587	11.2394	11.1795	11.1442
24	0.1317	0.3470	0.1000	0.1000	0.1000	0.1000	0.1010	0.1000	0.2287	0.1462	0.1279
25	12.1492	13.3006	11.8606	11.6976	11.8870	11.8970	11.8684	11.9832	12.2394	12.1799	12.1455
26	1.6373	2.1421	1.0339	1.3880	1.0400	1.0800	1.0360	0.9241	1.6849	1.3424	1.1763
27	5.0032	4.8050	6.6818	4.9523	6.6460	6.4620	6.6859	6.7676	4.9136	5.4844	5.9177
28	9.3545	9.3000	10.8113	8.8000	10.8040	10.7990	10.8111	10.9639	9.7190	10.1372	10.3697
29	15.0919	17.1740	13.8404	14.6645	13.8700	13.9220	13.8465	13.8186	15.0219	14.5262	14.2756
Best Weight (lb)	25447.1	28544.0	25445.6	25156.5	25491.9	25488.2	25467.9	25698.9	25664.0	25521.8	25542.98
Number of structural analyses	48000	-	9650	9875	19670	28059	31700	14406	-	10685	Duration 550.99 s

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