A Simple Solution for Randomized Failure Modeling with the SPH Method

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Abstract: Utilizing the Smoothed Particle Hydrodynamics (SPH) method to simulate structural strength material failures under high-speed loading currently constitutes, due to a variety of practical reasons, a common and favored procedure. In this technique, the absence of a physical computational mesh prevents numerical instabilities, and the comparatively simple code facilitates the creation of elementary add-ons that enable the approach to be employed within almost any suitable discipline. Considering the ever-increasing popularity of theories of randomness, which are related to, for example, the merely problematically predictable failures of heterogeneous materials, it is not surprising that the requirement for their implementation into actively used numerical methods (SPH being no exception) is gaining in intensity. The paper describes a simple drop test analyzing the failure of a concrete disc colliding with a rigid surface. The disc is simulated with the SPH method, into which the heterogeneity of concrete is introduced via a very simple algorithm; the heterogeneity stems from the structure of concrete, which is based on combining a concrete additive, a cement binder, and water. The results indicate that the applied process can be successfully practiced if several conditions also specified within the paper are observed.

Key-Words: Smoothed particle hydrodynamics; failure; numerical fracture; randomness; concrete; impact.

1 Introduction

The original idea in the very foundations of structure designing and related domains consisted in creating a simple and robust concept. With the progress of time, simple outlines gradually became more complicated, eventually reaching a stage where a mere pencil with a sheet of paper had ceased to suffice. Interestingly, at present we are often faced with efforts to design structures as simple as reasonably possible; after being completed, however, such structures may not necessarily be the simplest ones in terms of their behavior and the presumptions of structural mechanics [1-3].

In concrete structures, the material heterogeneity offers a new dimension of complexity. Concrete, as the mixture of a concrete additive, a cement binder, and water, does not constitute a regular structure from any perspective or at any scale; with respect to this fact, not even the behaviour of the given structures can be assumed to remain identical in all cases. The introduced randomness is further multiplied by the type of load, which is invariably comprised in structural mechanics analyses [4-6].

In real-world conditions, countless loading tests can be carried out, and the rates of occurrence of an effect enable us to presume the most probable result of the series of tests to follow. However, numerical analyses and simulations, using common computational techniques such as the Finite Element Method (FEM), Finite Difference Method (FDM), or Smoothed Particle Hydrodynamics method (SPH), will not provide divergent results even with an infinite number of computations unless special add-ons are applied.

In simulations where concrete-based materials are involved, we always encounter the problem of what material model is to be used [7,8]. The results obtained from the varied models and their modified versions can differ fundamentally. Yet it is not advisable to introduce the heterogeneity directly into the description of the material model, mainly due to the increasing complexity of the operations and the lack of knowledge concerning the sensitivities of the model's individual parameters before running the actual computation [9,10]. For example, in the case of the FEM, a material model would have to be generated for each finite element to ensure that the material heterogeneity is contained in the numerical model.

A hitherto scarcely employed option consists in implementing the heterogeneity directly into the source code of the numerical method; however, there still remain the questions of whether this approach can be used to define the random behavior of the monitored area, or the heterogeneity, and whether such numerical treatment can be further interconnected and explained using a real concrete structure. Numerical attenuation is utilized in fracture mechanics simulations, where the FEM often finds effective application. As the material model is the same in the entire monitored area, the finite element geometry can be modified such that any failure is immediately localizable at a pre-selected point (for example, via notching on a concrete beam). In high-speed loading, however, the FEM does not offer a suitable solution [11,12].

Conversely, the attractivity of the SPH approach intensifies with the increasing loading speed; yet we also need to note that, in the case of the SPH, there is no physical mesh to interconnect the individual particles, and a mere change in the configuration of the particles may not provide the desired results (in fact, this type of action may even cause adverse effects, including the formation of numerical cracks) [13,14].

The paper presents a procedure to introduce the heterogeneity into the source code of the SPH method in such a manner that the individual particles appear like concrete additive, binder, or water particles to create a random material structure, which then generates diverse simulation results. The contribution also defines the restrictions to be observed. Further, a simple fracture mechanics experiment where a concrete disc strikes a rigid surface and is deformed is simulated.

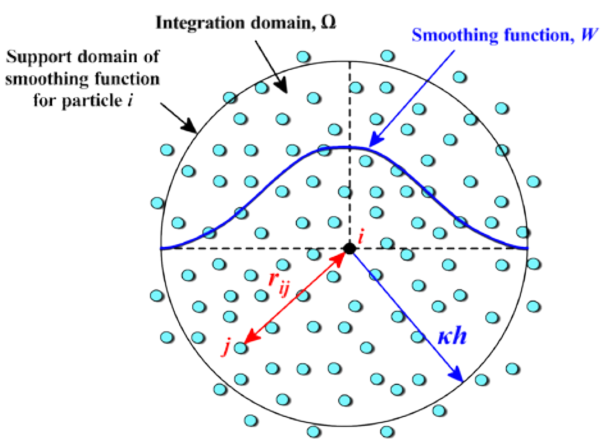

Fig. 1. The particle approximations using particles within the support domain of the smoothing function *W* for particle *i*.

2 Essential formulation of the SPH

The formulation of the SPH method is often divided into two key steps. The first step is the *integral representation* of field functions, and the second one is *particle approximation*. Assuming that the finite volume ΔV_j is assigned to the SPH particle *j*, the following relationship applies:

$$m_j = \Delta V_j \rho_j; \tag{1}$$

where m_j and ρ_j are the mass and density of the particle *j*. The value of the monitored quantity $f(\boldsymbol{x}_i)$, which is the product of integral representation and particle approximation operations, can thus be written as:

$$f(\boldsymbol{x}_i) \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\boldsymbol{x}_j) W(\boldsymbol{x}_i - \boldsymbol{x}_j, h); \tag{2}$$

where *W* is the so-called smoothing function and *h* denotes the smoothing length defining the influence area of the smoothing function *W*. Equation (2) states that the value of a function at particle *i* is approximated using the average of those values of the function at all the particles in the support domain of the particle *i* weighted by the smoothing function shown in Fig. 1.

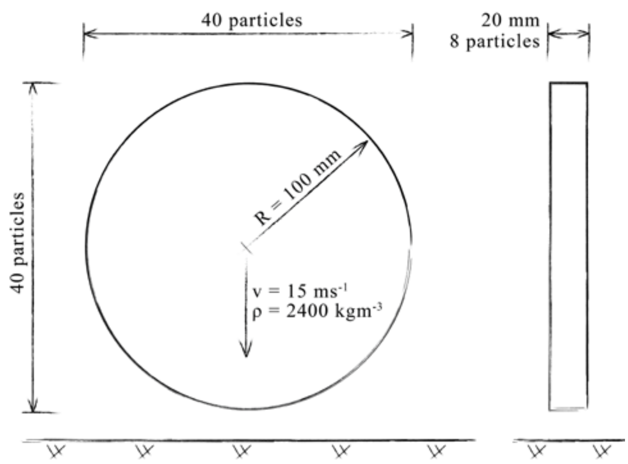

Fig. 2. A diagram of the experiment.

3 Experiment set up and restrictions

Before characterizing the procedure applied to introduce the material heterogeneity into the numerical model, a brief description of the simulated experiment, whose diagram is presented in Fig. 2, is provided. The scheme shows a cylindrical concrete body striking a rigid surface at the speed of 15 ms^{-1}. As one of the preconditions for the functionality of the introduced algorithm consists in regular initial particle distribution, the cylinder was discretized with 8 particles in the depth and 40 particles in the height and width.

The particles were arranged to form a regular, grid-based (and therefore not a radial) field, as also indicated in Fig. 5. The simulations involved 10,112 SPH elements in total and were performed via the LS-DYNA program [15]. The Continuous Surface Cap Model (CSCM) was chosen as the material model of concrete to be used [16,17]. Table 1 shows the parameters employed in the simulations.

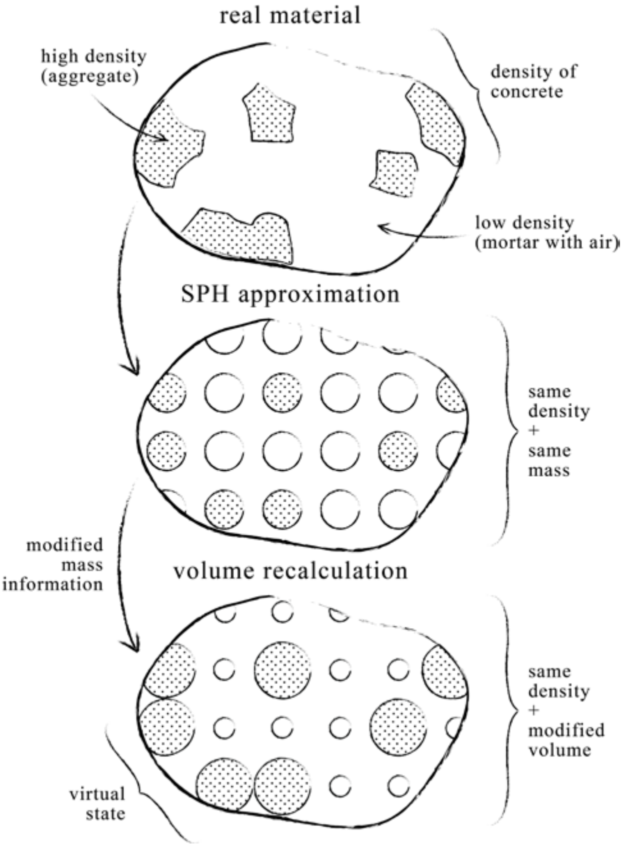

Fig. 3. Including the numerical heterogeneity in the SPH model.

3.1 Essential restrictions

A drawback of the SPH method lies in the effect of the initial particle distribution. The imperfect regularity of the particles, such as that observed in the presence of particle clusters with interparticle distances markedly different from the rest of the model, may result in the formation of numerical (false) cracks. Even though the problem can be tackled through introducing the smoothing function defined in the material coordinates (a Langrangian kernel), such a procedure would cause the analyzed model to lose the capability of solving excessive deformations. When, however, using the smoothing function defined in the spatial coordinates (an Eulerian kernel), we need to suitably change the size of the smoothing length to ensure that the number of particles in the support domain ideally remains constant during the entire simulation [14].

Table 1

The material parameters for the CSCM model.

Mass density, ρ_c (kgm^{-3})	2400
Compressive strength, f_c (MPa)	43.10
Initial shear modulus, G (GPa)	12.92
Initial bulk modulus, K (GPa)	14.15
Poisson's ratio, v_c	0.18
Fracture energy, G_F (Jm^{-2})	83.25
Maximum aggregate size, a_g (mm)	8

As the simulated experiment was a high-speed one and the discretization of the monitored body via the SPH particles was regular, we selected the smoothing function defined in the spatial coordinates, namely, an Eulerian kernel.

4 Numerical heterogeneity

The implementation of the material heterogeneity was performed via modifying the weight value function in the individual SPH particles; in other words, the particle mass m_j included in (2) was modified. Further, the same density value was preserved in all the particles, meaning that – with respect to the validity of (1) – the volume assigned to the individual particles was virtually modified (enlarged/reduced).

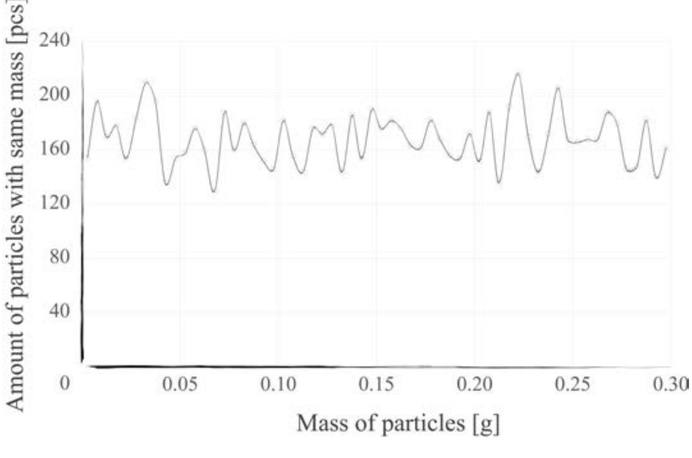

Fig. 4. The mass distribution function of specimen #1.

However, due to the necessity to maintain the best possible particle distribution regularity (an identical smoothing length for all the particles), the initial particle distribution was generated for the state where all the particles are assigned the same volume ΔV_j. This operation is graphically represented in Fig. 3.

4.1 Mass distribution

As the individual SPH particles had to be assigned various masses, it was necessary to build the entire distribution algorithm upon the precondition of a constant weight of the resulting body. The algorithm can nevertheless satisfy the conditions of any distribution function (including, for example, the normal, uniform, or Poisson distribution variants). For the actual testing, we selected the uniform distribution of the occurrence of masses, and the relevant occurrence rate resembled that shown in Fig. 4.

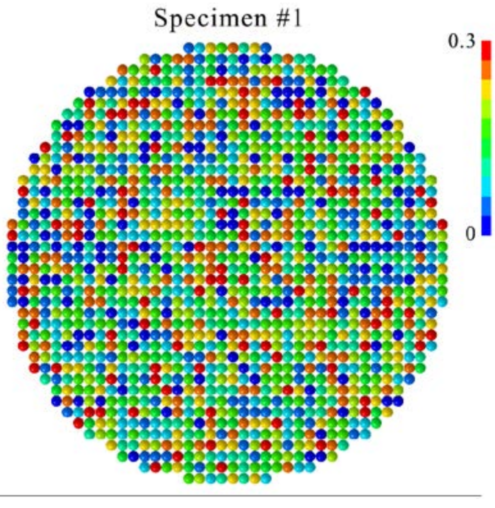

Fig. 5. The mass distribution of specimen #1 (units in g).

The reason for choosing uniform distribution consisted in testing the computational stability. The properties of the smoothing function enable the SPH method to suppress or highlight the heterogeneities contained in a computational procedure [13]. By another definition, the sensitivity of the computational stability in the presence of particles with almost zero mass (such as air pores in concrete) was tested.

Figure 5 demonstrates one of the tested samples with color representation of the masses. Before running a computation, the pattern of Fig. 4 was re-generated to create a modified model according to the algorithm in Fig. 3, with a result similar to that shown in Fig. 5.

5 Results

This section compares the behavior of the modified models with that of the homogeneous one (no heterogeneity introduced; all particles exhibit the same mass and density). The blue-marked portions of the images invariably represent crackless material, whereas the red sectors stand for failures. All the results were captured at the same instant of time.

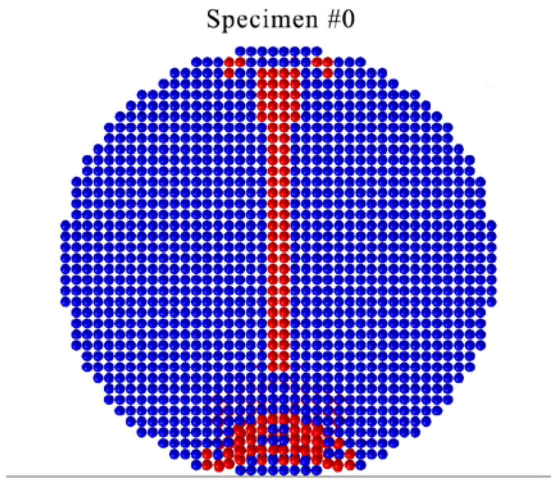

Fig. 6. The deformation of the homogeneous model.

The homogeneous model, Fig. 6, clearly exhibits a crushed region at the zone of contact with the rigid surface. The localized marginal crack symmetrically passes through the center of the model, and it does not tend to open any further during the simulation. With respect to the material homogeneity, the concrete cylinder shows very high resistance to complete failure.

Figure 7 presents disrupted models with the introduced material heterogeneity. A unique mass distribution pattern was generated for each specimen, as is obvious from Fig. 4. Importantly, Fig. 7 indicates that the introduction of the heterogeneity renders the models more sensitive to not only accidental disruption but also failure in general. In this context, another finding consists in the identification of multiple marginal cracks developing simultaneously.

6 Conclusion

The paper discusses a simple procedure for introducing numerical heterogeneity into SPH-based models. Interestingly, such numerical heterogeneity stems from a real property of concrete-based materials.

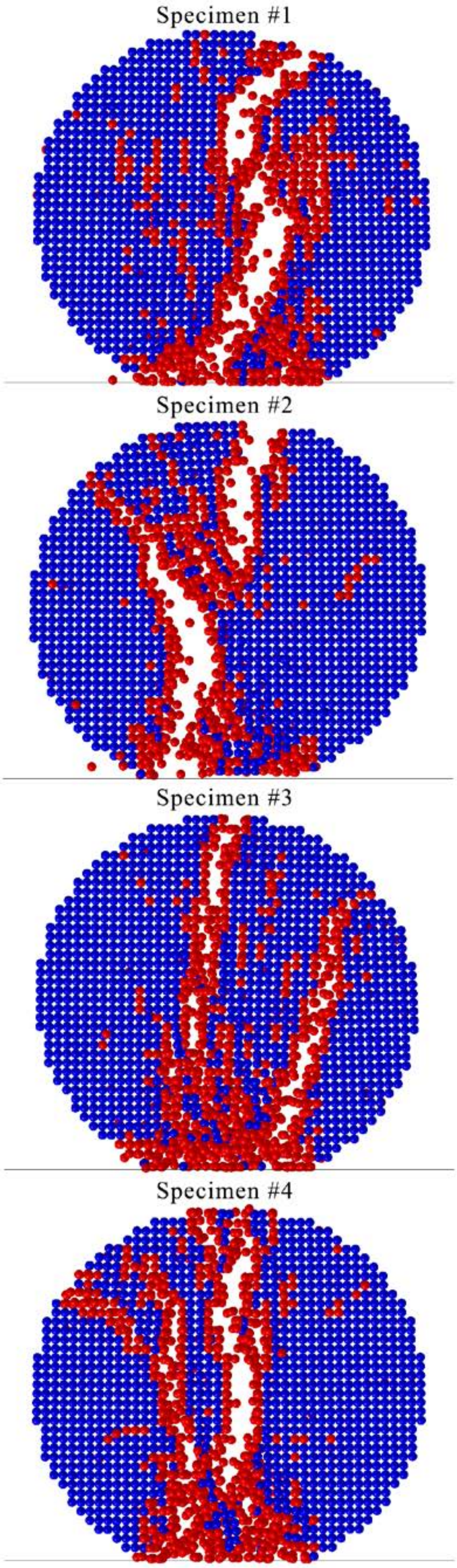

Fig. 7. The deformation of the heterogeneous models.

The principles of the relevant algorithm and the conditions that, if satisfied, enable to avoid numerical cracks, which are unrelated to the numerical heterogeneity are also characterized.

The entire principle is presented using a simple fracture mechanics test where a concrete cylinder strikes a rigid surface; the results clearly point to the functionality of the algorithm.

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