Sensitivity of FRF's Curvature Difference Method in Detecting Damage

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Abstract: - Structural health monitoring field had been concerned by researcher since it was realized that structural failure might be caused by degradation of structural strength. Some methods related to dynamic analysis, had been developed to establish the damage location. One of them is FRF curvature difference method which can exact locate the damage. From numeric analysis, it can be concluded that variability of damage level and excited of this method is not sensitive to damage location. However, the magnitude of FRF curvature difference is sensitive to them.

Key-Words: - FRF, FRF's Curvature, excited frequency, natural frequencies, mode shapes.

1 Introduction

Structural health monitoring field had been concerned by researcher since it was realized that structural failure might be caused by degradation of structural strength. It can happen to building which either experience extreme load or not [1]. Although building can survive to extreme loading, damage detection need to be conducted to assure how long service-life of building remains [1,2,3].

Some methods related to dynamic analysis, had been developed to establish the damage location [1,2,3,4,5,6,7]. They are classified into modals-parametric method and frequency response (FRF) method. The first method can identify that damage occurs. However, it had limitation in damage localization, especially in modest level of damage. Some numerical evaluation show this method failed to localize more than one damage (crack). So, this method is not recommended for practical purpose.

Next, frequency response method is the advancement of mode shapes curvature difference. One of advantages is that numerical error in curve fitting can be eliminated. Another advantage is that it does not need to obtain modal parameter. Also, this method provides more dynamic behavior of structure.

Beside of the advantages, it is required to check the sensitivity of frequency response method to meet the performance. The reason for this is the variability. Thus, it needs to conduct some conclusion about the sensitivity of this method to damage location, level of damage, and excited frequency.

2. Methodology

This research is based on modeling of 2D simple portal in MATLAB. All of degree of freedoms including axial, shear and bending are considered. Some assumptions are made. First, damage does not reduce mass. Second, damage scenario is based on drop of stiffness by reducing moment of inertia. Last, damping ratio used is 2%. Assembling process of mass and stiffness matrices uses direct stiffness method.

FRF is defined by the amplitude of displacement (X_0) normalized by the amplitude of dynamic force (F_0). As a result, FRF is formulated by:

$$H(\Omega) = \frac{1}{1-\left(\frac{\Omega}{\omega_n}\right)^2 + i2\xi\left(\frac{\Omega}{\omega_n}\right)} \quad (1)$$

Or in complex magnitude, it can be rewritten as:

$$H(\Omega) = \frac{Xk}{F_0} = \frac{X{\omega_n}^2}{f_0} = \frac{1}{\sqrt{(1-r^2)^2+(2\xi r)^2}} \quad (2)$$

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Where:
$H(\Omega)$: FRF of Single Degree of Freedom system

Ω : Output excited frequency of some degree of freedom (radian/sec)
ω_n : Natural frequency (radian/sec)
ξ : Damping ratio
For multi degree of freedom system, FRF between degree of freedom (DOF) r and s is defined by [8]:

$$H_{rs}(\Omega) = \sum_{j=1}^{n} \frac{\phi_{rj}\,\phi_{sj}}{\omega_j{}^2 - \Omega^2 + 2\xi_j \omega_j \Omega i} \tag{4}$$

Where $H_{rs}(\Omega)$ is response of DOF r as a result of single excited frequency at DOF s. FRF is known as *Reacceptance Function* and can be assembled into matrix form, that is

$$[H(\Omega)] = \begin{bmatrix} \begin{bmatrix} H_{11} & \cdots & H_{1n} \\ \vdots & \ddots & \vdots \\ H_{n1} & \cdots & H_{nn} \end{bmatrix} \end{bmatrix} \tag{5}$$

Mode shapes curvature difference of damage structure will cause significant magnitude of curvature difference. Since curvature was element characteristic depending on stiffness, curvature difference can exactly locate damage. As an extension of it, FRF curvature difference method covers not only natural frequency, but also all frequency range determined to solve the limitation of previous method. FRF curvature of frequency Ω is defined by:

$$H"(\Omega)_{i,j} = \frac{H(\Omega)_{i+1,j} - 2H(\Omega)_{i,j} + H(\Omega)_{i-1,j}}{h^2} \tag{6}$$

Where:
$H(\Omega)_{i,j}$: Reacceptance FRF measured at i with input force at j.
h : Distance between two consecutive measurement points

Furthermore, for an applied force at point j, the absolute difference between the FRF curvatures of the damage and undamaged structure at i, in a predetermined frequency range, is defined as:

$$\Delta H"_{i,j} = \sum_{\Omega} \left| H^{"}_{*}(\Omega)_{i,j} - H"(\Omega)_{i,j} \right| \tag{7}$$

Finally, the changing in the FRF curvatures for several locations is added. This leads to parameter S_i for the i^{th} measurement point defined as:

$$S_i = \sum_j \Delta H"_{i,j} \tag{8}$$

Structural modeling and its properties is shown in figure 2 where each elements beam and column is divided into 10 elements. Damage is designed by reducing stiffness and it is located at 5th element of beam and column. Different stiffness reduction from 10% to 50% is conducted to check the sensitivity to level of damage. Also, different excited frequency varied from 5 radian/sec to 40 radian/sec is applied to meet the sensitivity.

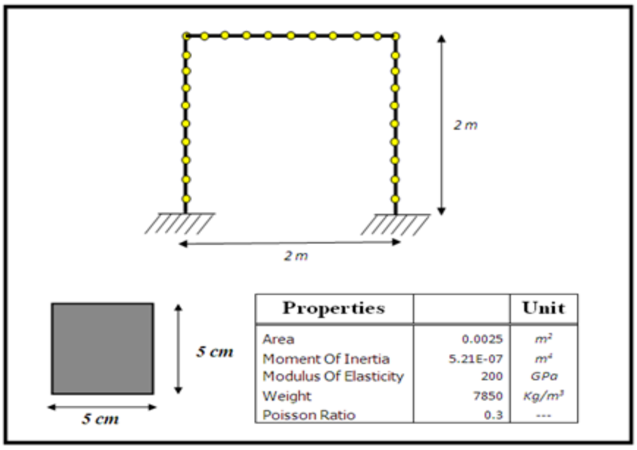

Figure 1.Structural Modelling and Element Properties

3. Result & Discussion

As different level of damage is applied to structure, some results are obtained as shown in figure 2 and 3. They are:

1) Damage location is settled in same location for each level of damage
2) Curvature FRF difference magnitude differs about 87% between 10% and 50% level of damage.

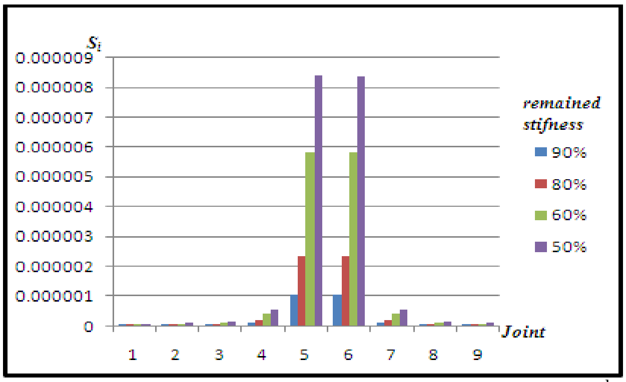

Fig 2. Sensitivity Test: Damage location at 5th element of beam (excited frequency 10 radian/sec)

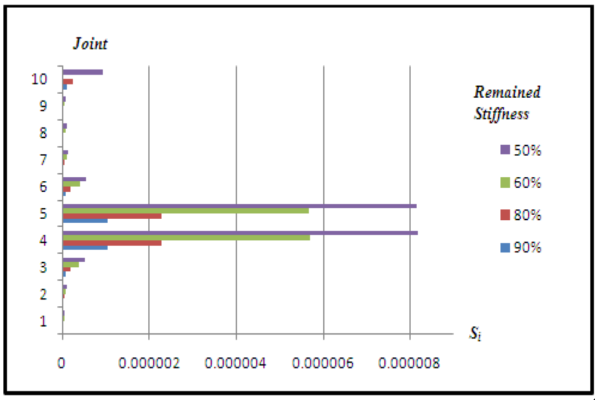

Fig 3. Sensitivity Test: Damage location at 5th element of column (excited frequency 10 radian/sec)

Then as different level of excited frequency is applied to structure, some results are obtained as shown in figure 4 and 5. They are:

1) Damage location is settled in same location for each level of excited frequency.
2) Curvature FRF difference magnitude differs as excited frequency closed to its natural frequency.

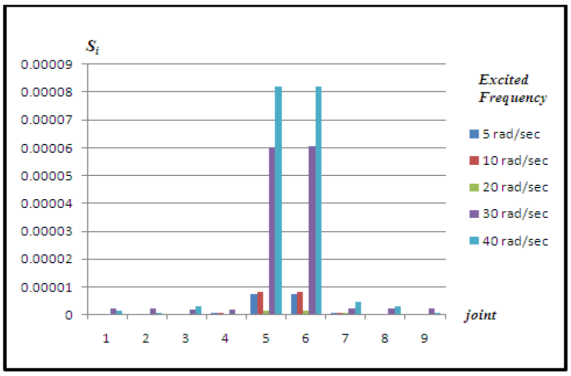

Fig 4.Sensitivity test to excited frequency at 5th element of beam (50% reduction of inertia)

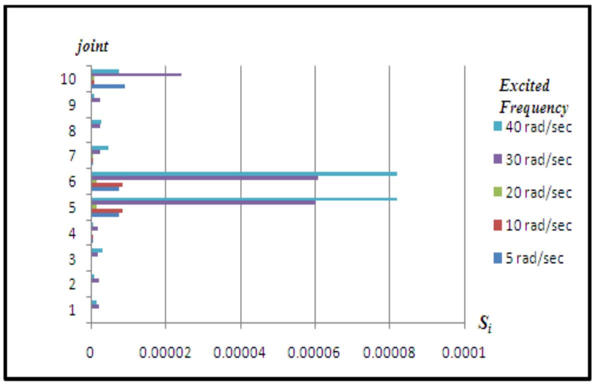

Fig 5.Sensitivity test to excited frequency at 5th element of column (50% reduction of inertia)

4. Conclusion

To conclude, both different level of damage and level of excited frequency are not sensitive to damage location since the location of damage is permanent after applying force. However, the magnitude of curvature FRF difference is sensitive to them which higher for high level of damage and/or applying frequency closed to its natural frequency. Thus, this method can be used for detecting damage, and its magnitude need to be established its generalization to meet the performance required.

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