Fuzzy Analysis of the Serviceability Limit State of a Truss Cantilever

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Abstract: - The presented article deals with the analysis of the reliability of a steel truss structure. The principal design criteria of structural analysis are described. Conventional and unconventional methods of reliability analysis are discussed. The serviceability limit state is studied in the light of fuzzy uncertainty of the geometric parameters of load bearing members of the truss structure. The deformation of the studied truss structure is evaluated as the output fuzzy number using the general extension principle. Material and geometric linear and material linear and geometric non-linear FEM methods are used to determine the deformation of the truss structure. The application of advanced computational methods of reliability analysis in operational research is discussed.

Key-Words: - Fuzzy, set, serviceability, limit state, structure, reliability, truss, cantilever

1 Introduction

In civil engineering the term load bearing structure refers to the skeleton of the entire construction. The main advantage of load bearing steel structures is the speed of construction, which together with the cost of transportation and assembly could contribute not only to reducing the time between the commencement of construction and it being put into operation, but also the economy [1]. Truss structures are suitable whenever there is a need to realize inlets in structures. The members of steel truss structures are slender with low mass. This requires that increased attention is paid to the elements, which are loaded in compression.

A small deformation in members under compression could significantly influence the stress and decrease the load carrying capacity [2], [3]. Reliability of steel structures can be formally differentiated according to the type and slenderness of the load bearing members [4-6]. In slender compressed elements the deformations are not directly proportional to the load [7]. The influence of initial imperfections on the limit states is studied using sensitivity analysis [8], [9]. The reliability of wall supporting elements may be very sensitive to the initial local imperfections of slender walls [10]. The resistance of beam supporting elements is influenced by the so-called bow imperfections of columns [11-13], which in interaction with the initial out of plumb imperfections [14] influence the reliability of frame structures [15].

The limit state of slender steel frames can be divided into the elastic limit state and plastic limit state. Cost savings achieved via the low weight of steel structures are clearly apparent when optimization methods based on classical mathematical formulations of the shakedown theory [16] are applied. New practical applications of the shakedown theory, which replaces the exhausting conventional iterative analysis, was presented in [17].

Analysis of the reliability and limit states of load bearing structures is generally based on the theory of mathematical statistics and probability [18] or the theory of fuzzy logic [19-21]. Generally, reliability analysis of building structures is an important part of operational research, whose sub-disciplines include the analysis of decision-making problems based on multiple criteria [22-24]. Good and proper division of complex problems in view of multiple criteria explicitly leads to more informed and better decision-making [25], [26]. Various mathematical and computational methods in science and engineering, and numerous specialized decision-making software have been developed in a number of fields lately [27]. The increasing number of publications on the use of fuzzy logic in light of MCDM applications in engineering during the past two decades is documented in article [22].

The presented article is aimed at the analysis of the reliability of a truss cantilever using Zadeh's theory of fuzzy sets [28].

2 Structural analysis

The focus of the presented study is the static response of a planar truss cantilever depicted in Fig.1. The members are joined by hinges at the joints. The truss structure is loaded on the upper joint at the end of the cantilever. Since the truss is loaded only at the joints, the members are stressed only by tension or compression. The diagonal members and lower left horizontal member are in compression, so it was necessary to solve problems associated with their stability.
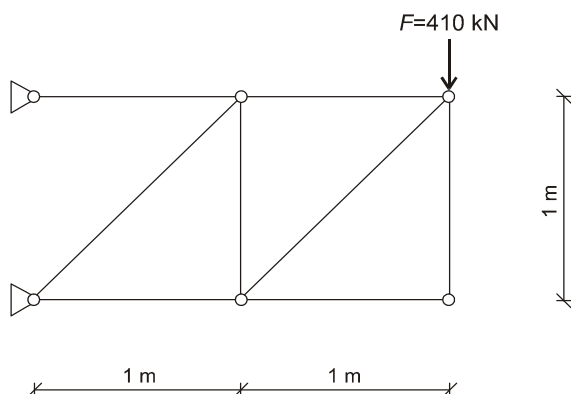

Fig.1: Cantilever geometry

2.1 Cross-section parameters

The members of the truss cantilever are made up of hot-rolled hollow steel cross-sections, see Fig.2. These cross-sections are classified in the Eurocode 3 standard as Class 1 cross-sections. Class 1 cross-sections are those that can form a plastic hinge with the rotation capacity required for plastic hinges.
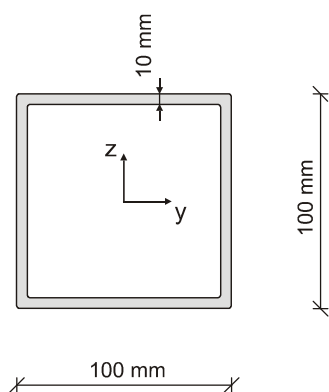

Fig.2: Rolled hollow cross-section

The cross-section depicted in Fig.2 is a Class 1 cross-section. This can be verified using equation (1) listed in the Eurocode 3 standard.

$$b - 3t_f \leq 42\varepsilon \qquad (1)$$

where $\varepsilon = 1$ for yield strength f_y=235 MPa and steel grade S235. Local stability phenomena such as buckling of slender compressed walls can be neglected for Class 1 cross-sections. Otherwise, we would have to introduce the shapes of initial imperfections of slender walls and analyse the loss of stability of members in interaction with local buckling of slender walls [29].

2.2 Linear finite element analysis

The static analysis of the truss cantilever was performed using the beam finite element method. The primary quantity that was monitored was the deformation, see Fig.3. Linear analysis of the dependence of the deformation on the load was performed in the first step. The conditions of equilibrium are defined on the initial undeformed structure. This type of analysis is, in most cases, sufficient for the design and verification of reliability of structures according to valid standards.
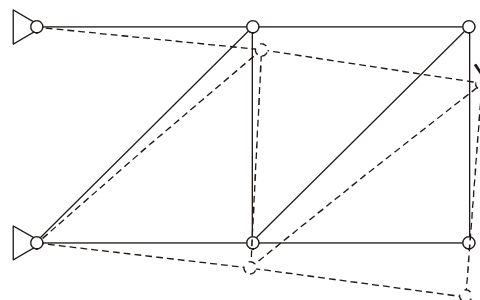

Fig.3: Deformation of the truss cantilever

The largest axial stress of 234.96 MPa was obtained in the upper left horizontal member. The other members are stressed less.

2.3 Stability analysis

The truss cantilever has three members in compression. These members have a tendency to loose stability by buckling. Stability analysis determines the magnitude of load at which loss of stability occurs. A particularity of the truss cantilever depicted in Fig.1 is that buckling of the diagonal members occurs simultaneously. Numerical examination of this phenomenon requires computer analysis of the stability with slightly different stiffness parameters of the diagonal members. Stiffness parameters of the diagonal members must vary only to the extent that the values of buckling loads F_{cr1} and F_{cr2} differ. The difference between F_{cr1} and F_{cr2} of 5 N is sufficient

for numerical analysis. If the truss girder is loaded by buckling load F_{cr1} then the left diagonal member buckles, whilst the right diagonal member buckles under load F_{cr2}. Since the difference in stiffness parameters was introduced in order to enable numerical analysis, it has to hold for the actual buckling forces that $F_{cr1}=F_{cr2}\approx 3385.4$ kN, see Fig.4.
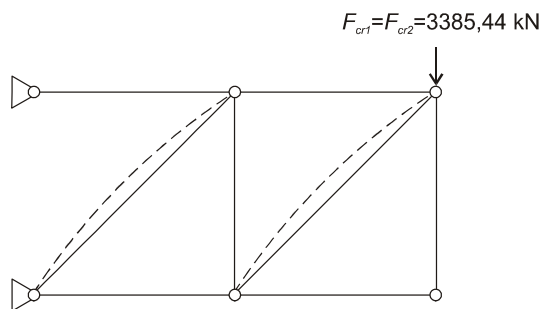
$F_{cr1}=F_{cr2}=3385,44$ kN

Fig.4: First and second buckling modes

The third buckling load F_{cr3} is obtained using the standard stability solution. Loading the truss cantilever with buckling load F_{cr3} leads to the buckling of the lower left member, see Fig.5.
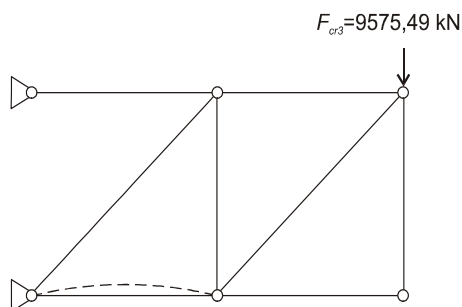
$F_{cr3}=9575,49$ kN

Fig.5: The third buckling mode

Buckling of the diagonal members is more important than buckling of the lower horizontal member in the stability analysis. The buckling length of each diagonal member is 1.414 m. The slenderness of each diagonal member is 38.869, which is relatively small. Thus, we can conclude that the effect of stability phenomena on the behaviour of the truss cantilever is relatively small. It is clear from the comparison of the values F_{cr1} and F_{cr2} with F=410 kN that buckling of the diagonal members occur when the load is more than eight times greater than load F.

2.4 Nonlinear finite element analysis
The geometrically non-linear elastic analysis may yield different dependencies of load vs. deformation from the geometrically linear elastic analysis. The

equilibrium conditions are defined on the deformed structure. Analysis was performed using the geometrically non-linear solution, which was published in [30]. This solution has been used in many reliability studies of theoretical and applied mechanics [14]. The basis of the geometrically non-linear solution is the Newton Raphson incremental method. The load was divided into fifty incremental steps.

3 Limit states
The article is aimed at the analysis of the serviceability limit state of a truss cantilever as a function of the geometrical characteristics of the member cross-section. The applicability or inapplicability of the truss is assessed according to the size of the deformation. The maximum permissible deformation is a subjectively selected value, which can be found in design standards. Whilst random numbers are more relevant as input for the analysis of the ultimate limit state, fuzzy numbers are more relevant as input for the serviceability limit state. Fuzzy analysis offers a wide range of possibilities on how to interpret and integrate the results of fuzzy analysis into global reliability analysis of structures.

Deformation is studied using the finite element method based on the linear and geometrically non-linear solution. The input data were fuzzified to reflect their uncertainty, which is relevant for the serviceability limit state.

3.1 Fuzzy logic
The notion of fuzzy logic was first mentioned in 1965 in the article [28] by professor Lotfi A. Zadeh. The basic concept of fuzzy logic, i.e. fuzzy sets, was then defined. The word fuzzy means unclear, blurred, hazy, uncertain, vague. This is reflected in what the fuzzy theory deals with, i.e. it tries to describe reality with its imprecisions and uncertainties.

Steel bars are produced by people, who are also the authors of definitions and standards associated with its production. The influence of human factor on the quality of metallurgical products is indisputable, however, methods for the description of uncertainties of the product parameters are questionable. The presence of human factor urges us to use stochastic methods with great caution. Only humans define what is and is not a quality product, therefore, the application of the fuzzy set theory for the study of the reliability of structures is minimally

as relevant as the application of other methods to reflect uncertainty in structural parameters. Both types of uncertainty may be considered in advanced problems according to their relevance to the studied problem [31].

3.2 Fuzzy input quantities

Fuzzy uncertainties of the cross-sectional areas of the members of the truss cantilever were considered for the analysis. The cross-sectional area A_i of the i-th bar was introduced as a fuzzy number, see Fig.6.
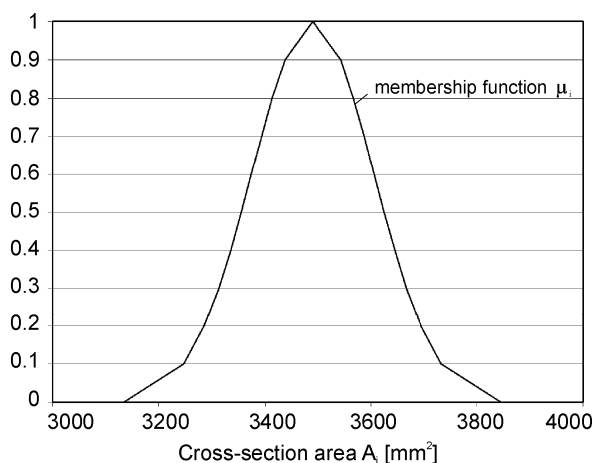

Fig.6: Fuzzy number of cross-section area A_i

The kernel of the fuzzy number in Fig.6 is the nominal value of the cross-sectional area 3490 mm^2, which is specified by the manufacturer. The degree of membership of the fuzzy number shown in Fig.6 defines the degree of truth with which the cross section is the hot-rolled hollow cross-section depicted in Fig.2. If the cross-sectional area is less than 3136 mm^2 or on the other hand greater than 3841 mm^2 then it is not the cross-section depicted in Fig.2. If the cross-sectional area is in the interval 3136 mm^2 to 3841 mm^2 then it is with a greater or lesser degree of truth the cross-section depicted in Fig.2. Fuzzification in Fig.6 was performed with consideration to deviations of the hot-rolled cross-section from experimental research [32].

3.3 Fuzzy analysis

Fuzzy analysis was performed using the general extension principle [33]. The general extension principle is one of the fundamental ideas prof. L. Zadeh used to enrich the theory [28]. It enables the application of operations and functions originally defined as functions of real variables in fuzzy sets. Let us consider μ_i the membership function of fuzzy

numbers A_i. Then μ_w is the membership function of fuzzy number w defined by the relation

$$\mu_w = \bigvee_w \left(\mu_1(A_1) \wedge \ldots \wedge \mu_i(A_i) \wedge \ldots \wedge \mu_n(A_n) \right) \quad (2)$$

The symbol $\bigvee$ in equation (2) corresponds to the supremum. Supremum in (2) is introduced as an alternative to the concept of the greatest element, however, compared to the greatest element, it can be found in more sets, for e.g. open intervals of real numbers do not contain a greatest element but have a supremum. If a set of real numbers has a maximum, then it has a supremum, for which it holds that the maximum is equal to the supremum. The supremum is a generalisation of the concept of the greatest element not only on the set of real numbers, but generally in all sets.

Cross-sectional areas of the members were considered as input fuzzy numbers, see Fig.6. Displacement w of the lower right hinge of the truss cantilever was considered as the output variable, see Fig.7. Equation (2) was evaluated using the so-called α–cuts. Practically, the conditions of maximum and minimum of the realizations of the output displacement from all possible combinations of input values are performed on each α–cuts. With the exception of the cross-sectional areas all material and geometric characteristics were considered in the computational model as the nominal and characteristic values, which are listed in the Eurocode 3 and other related standards.
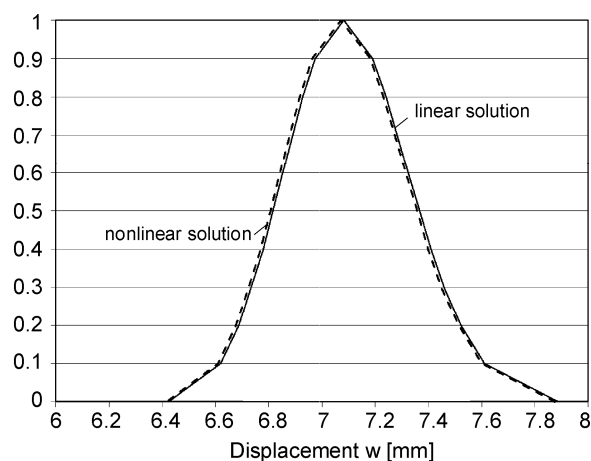

Fig.7: Fuzzy number of maximum displacement w

Since the cross-sectional areas are fuzzy numbers the displacement is also a fuzzy number. The displacement w was solved in two variants. The displacement was analysed using the linear solution in the first variant. The displacement was analysed

using the geometrically non-linear solution in the second variant. Initial imperfections of the members were taken into consideration in the geometrically non-linear solution. Initial curvatures of the compressed members were introduced in the shape of a half-wave of the sine function acc. to Fig.4 and Fig.5. The amplitudes of the initial curvatures of member axes were considered as one-thousandth of the member length, which is a common approach applied, for e.g., in [34], [35]. Deformation of the compressed member increases with increased loading of the members.

It is apparent from Fig.7. that the membership functions obtained from the linear and non-linear solution differ very little. The two membership functions are similar in shape to the membership function in Fig.6.

4 Conclusion

The article deals with the fundamental methods of design, assessment and analysis of the reliability of steel trusses. Design criteria that must be taken into consideration when planning and designing reliable and economical steel structures are mentioned. The computational model of a truss cantilever with fuzzy uncertain input parameters was described. The presented model can be adapted and used to seek better solutions of practical problems that arise in the field of fuzzy optimisation. Fuzzy analysis, which can be used to analyse the serviceability limit state of steel structures, was performed on a specific example. The maximum deformation of the truss cantilever was evaluated as the output variable. Analyses were performed using linear and non-linear computational models based on the finite element method. Fuzzy numbers of the cross-sectional areas were considered as the input parameters of the model. The output fuzzy number is the deformation.

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