Reduced Length Chirp Pilots for Estimation of Linear Time-Varying Communication Channels

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Abstract: In wireless communication multipath fading causes significant performance degradation and necessitates channel estimation. Transmission of two consecutive chirps with different rates as a pilot sequence is a method that has been used in the estimation of linear time-varying (LTV) channel parameters. In this paper, we propose an improvement on the chirp based channel estimation method for LTV model. We show that combination of a chirp with its complex conjugate, in particular a frequency modulated sinusoid, provides us an efficient pilot sequence. Besides reducing the length of the pilot sequence by half, the length and the rate of our proposed pilot sequence can be adjusted to comply with a-priori information on the channel. We implement the proposed method for an orthogonal frequency division multiplexing (OFDM) communication system and compare with conventional two chirps method.

Key–Words: Chirp pilot; time-varying channel; time-frequency methods; OFDM

1 Introduction

Wireless communication channels have undesirable affects on transmitted signals, such as attenuation, distortion, delays and phase shifts. Therefore, estimation of the time-varying response of a communication channel is crucial for improving the performance of a wireless communication system [1, 2, 3]. In this paper, we present a channel estimation method that uses the characteristics of chirps as eigenfunctions of linear time-varying (LTV) channel models, as well as the rate optimality of complex conjugate chirps [4, 5].

The application of chirp signals to parameter estimation of time-varying communications channels is not a new approach. Chirps have also been used in sonar and radar systems [6-11]. Our aim is to generate an efficient channel estimation method than the one achieved by transmitting two consecutive chirps with different rates [12, 4, 13]. By sending two consecutive chirps with different rates as pilots, and assuming the channel does not change during this interval, channel estimation problem can be converted into estimation of harmonic frequencies in noise [12]. Additionally, as shown in [12], two complex conjugate chirps minimize the Crammer-Rao bound. Instead of sending two consecutive chirps, we propose a combination of a chirp and its complex conjugate, in particular a frequency modulated sinusoid as an efficient pilot sequence.

As we showed in [5], using a linear chirp as input to a LTV channel, the linear chirp simplifies the model to that of a complex linear time-invariant (LTI) system with effective time-shifts which are the combinations of the actual time-shifts and Doppler frequency shifts. Using the eigenfunction property and time-frequency analysis, the frequency marginals of the dechirped received signal provides us the information to estimate channel parameters. We perform time-frequency analysis to provide a justification for the use of frequency modulated sinusoids as pilots in the proposed method.

In Section 2, we briefly review the model used for the multi-path communication channel; while in Section 3, we show how to estimate the channel parameters using a frequency modulated (FM) sinusoid pilot. In Section 4, we illustrate the performance of our method for channel estimation in orthogonal frequency division multiplexing (OFDM) [14] and compare our results with those obtained using the two-chirp method with conclusions following.

2 Time-varying Multipath Channel Model

In mobile radio applications, because of the multipaths and relative motion between transmitter and receiver which causes Doppler effects, the communication channel is typically modeled as a linear time-varying (LTV) system. In general, LTV channels are also known as time-frequency (TF) dispersive or doubly dispersive. Their practical relevance, made LTV models gain a lot of interest in the fields of signal processing, communications, information theory, and mathematics [17]. The time-varying frequency response of the channel characterizes the channel in terms of time delays, Doppler frequency shifts and gains, all of which vary randomly in the modeling. An L-path fading channel with Doppler frequency shifts is generally modeled by a discrete-time separable impulse response [4, 19]:

$$\begin{aligned} h(n,m) &= \sum_{\ell=0}^{L-1} h_\ell(n-m) f_\ell(n) \\ &= \sum_{\ell=0}^{L-1} \alpha_\ell \delta(n-N_\ell) e^{-j\phi_\ell n}, \end{aligned} \quad (1)$$

where $h_\ell(n) = \delta(n - N_\ell)$, as the impulse response of the all-pass systems corresponding to time delays $\{N_\ell\}$ and $f_\ell(n) = \alpha_\ell e^{-j\phi_\ell n}$. The Doppler frequency shifts can be represented with $\{\phi_\ell\}$ and gains with $\{\alpha_\ell\}$. Although the model in (1) is assumed to be valid for the duration of a symbol, it changes with time and provides an approximation to the actual channel. Accordingly, the effect of the channel on the transmitted signal $s(n)$ is

$$y(n) = \sum_{\ell=0}^{L-1} \alpha_\ell s(n-N_\ell) e^{-j\phi_\ell n}, \quad (2)$$

where $y(n)$ is the channel output. Both deterministic and stochastic approaches are equally useful in describing a time-varying channel even though they are appealing for different aspects. The stochastic model is better suited for describing global behaviors whereas the deterministic one is more useful to study the transmission through a specific channel realization [18]. As the channel effects cause the transmitted signal disperse in time and frequency, coherent demodulation requires estimating the channel parameters.

3 Proposed Chirp-based Channel Estimation

The channel parameters can be estimated either by transmitting a pilot sequence or by blind estimation. We concentrate on pilot-based channel estimation where we use a chirp signal as our pilot sequence. As it is well known, complex exponentials are the eigenfunctions of linear time-invariant (LTI) systems, as they appear at the output of the system with amplitude and phase changed by the system. In [4, 5], it was shown that chirps are eigenfunctions of the linear time-varying (LTV) channel and chirps can enable us to model a LTV channel as LTI. Let us start with defining the chirp signal $g(n)$ that we are going to use [20],

$$\begin{aligned} g(n) &= e^{-j\frac{\pi}{8}} e^{j\frac{2\pi}{N}\frac{1}{2}n^2} \qquad 0 \le n \le N-1 \\ &= e^{j\theta n^2}, \end{aligned} \quad (3)$$

as input to a LTV channel where the instantaneous frequency $IF(n) = 2\theta n = 2\pi \tan(\beta)\frac{n}{N}$, and the chirp rate $\theta = \pi \tan(\beta)/N$ for $0 < \beta \le \pi/2$. We can obtain the Discrete Fourier Transform (DFT) equivalent of $g(n)$ as $G(k) = e^{j\frac{\pi}{8}} e^{-j\frac{2\pi}{N}\frac{1}{2}k^2}$, $0 \le k \le N-1$. The initial and final instantaneous frequencies are 0 and $2\pi\tan(\beta)$, respectively. These DFT pairs $g(n)$ and $G(k)$ are also related in another way such that $G(k) = g(k)^*$, where $*$ denotes complex conjugate. As we showed in [5], these chirp signals have the following properties :

1. A time delay N_0 on $g(n)$, corresponds to a frequency shift on $g(n)$ and a multiplication by a complex exponential depending on N_0 and the chirp rate,

   $$\textit{proof:} \quad \begin{aligned} g(n-N_0) &= e^{-j\frac{\pi}{8}} e^{j\frac{2\pi}{2N}(n-N_0)^2} \\ &= g(n) e^{-j\frac{2\pi}{N}N_0 n} e^{j\frac{\pi}{N}N_0^2}. \end{aligned}$$

   which is also $g(n-N_0) = g(n)e^{-j2\theta N_0 n} e^{j\theta N_0^2}$. In the equation above, $e^{-j\frac{2\pi}{N}N_0 n}$ corresponds to a Doppler shift of $\phi_0 = 2\pi N_0/N$ and $e^{j(\pi N_0^2/N)}$ is a constant.

2. A frequency shift $\phi_1 = \frac{2\pi N_1}{N}$ on $g(n)$ (i.e., multiplying $g(n)$ by $e^{-j\phi_1 n}$) corresponds to an equivalent time delay $N_1 = 0.5\phi_1/\theta$ on $g(n)$ and a multiplication by a complex exponential depending on N_1 and the chirp rate,

   $$\textit{proof:} \quad \begin{aligned} g(n)e^{-j\phi_1 n} &= e^{-j\frac{\pi}{8}} e^{j\frac{2\pi}{N}\frac{1}{2}n^2} e^{-j\phi_1 n} \\ &= g(n-N_1) e^{-j\theta N_1^2} \end{aligned}$$

   where $g(n - N_1)$ showing delay on $g(n)$ by N_1 samples and $e^{-j(2\pi N_1^2/2N)}$ is a constant.

3. Using the properties obtained above, a time delay and a frequency shift at the same time on $g(n)$, correspond to an equivalent time shift $N_e = N_0 + N_1$ on $g(n)$ and a multiplication by a complex exponential,

   $$\begin{aligned} g(n-N_0)e^{-j\phi_1 n} &= g(n) e^{-j\frac{2\pi(N_0-N_1)n}{N}} e^{j\frac{\pi N_0^2}{N}} \\ &= g(n-N_e) e^{j\frac{\pi(N_1^2-2N_0N_1)}{N}} \end{aligned}$$

Generalizing (1), output of the channel corresponding to the input $s(n) = g(n)$ can be derived as

$$y(n) = \sum_{\ell=0}^{L-1} \alpha_\ell e^{j\theta(N_\ell^2 - N_{e\ell}^2)} g(n - N_{e\ell}), \qquad (4)$$

where $\{N_{e\ell} = N_\ell + \phi_\ell/2\theta\}$ are referred to as equivalent time delays depending on the actual time delays $\{N_\ell\}$ and the Doppler frequency shifts $\{\phi_\ell\}$ in each path.

3.1 Frequency Modulated Sinusoids for Channel Estimation

The channel output corresponding to (4) indicates that it is possible to model the time-varying channel as a complex LTI system. The effect of the channel on the input chirp can be visualized in the time-frequency domain as that of delaying the chirp in time by N_ℓ samples, and then shifting the resulting chirp in the frequency axis by ϕ_ℓ radians, which correspond to an equivalent time shift of $N_{e\ell}$ samples. Accordingly, by dechirping $y(n)$ with $g^*(n)$, we obtain $N_{e\ell}$ values. However, this would not be enough to find the actual time and the Doppler frequency shifts but two chirps with opposite rates θ and $-\theta$ are necessary [12]. In this paper, we use the properties explained in [5] for rewriting the channel output $y(n)$ in (4) and obtain an eigenfunction relation in terms of the chirp $g(n)$. Rewriting (4) according the to properties given above

$$\begin{aligned} y(n) &= \sum_{\ell=0}^{L-1} g(n)\alpha_\ell e^{j\theta N_\ell^2} e^{-j2\theta N_{e\ell} n} \\ &= g(n) \sum_{\ell=0}^{L-1} \alpha_\ell e^{jIF(0.5N_\ell^2)} e^{-jIF(N_{e\ell})n}, \end{aligned} \qquad (5)$$

$g(n)$ acts as the eigenfunction of the LTV model of the channel. The response of the system occurs at instantaneous frequencies $IF(N_{e\ell})$ and $IF(0.5N_\ell^2)$, with $e^{-jIF(N_{e\ell})n}$ corresponding to the equivalent shift in time.

Let us now consider the use of $g(n)$ and $g^*(n)$ as pilots for channel estimation [12, 13] (as suggested in [12] we are taking θ and $-\theta$ as the chirp rates), but instead of sending two consecutive chirps as pilots, we propose to use a combination of these chirps, for instance, a frequency modulated sinusoid:

$$g_T(n) = g(n) + g^*(n) = 2\cos(\theta n^2), \qquad (6)$$

where Wigner-Ville time-frequency representations are as shown in Fig. 1. With this combination as input, the channel output due to linearity of the model

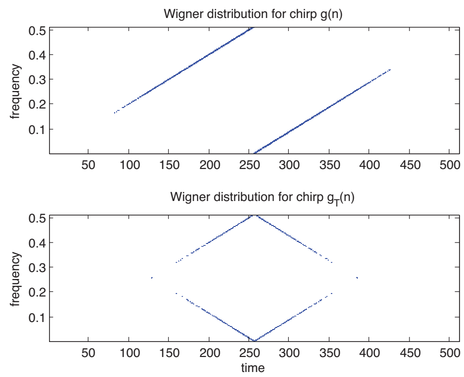

Figure 1: Wigner distributions for chirp and FM sinusoid

is

$$\begin{aligned} y_T(n) &= g(n)\left[\sum_{\ell=0}^{L-1} \alpha_\ell e^{j\theta N_\ell^2} e^{-j2\theta N_{e\ell}^{(1)} n}\right] \\ &\quad + g^*(n)\left[\sum_{\ell=0}^{L-1} \alpha_\ell e^{-j\theta N_\ell^2} e^{j2\theta N_{e\ell}^{(2)} n}\right] \\ &= g(n) f_1(n) + g^*(n) f_2(n), \end{aligned} \qquad (7)$$

where $f_1(n)$ and $f_2(n)$ are the terms in the brackets, and from (4) the equivalent delays are

$$\begin{aligned} N_{e\ell}^{(1)} &= N_\ell + \phi_\ell/2\theta \\ N_{e\ell}^{(2)} &= N_\ell - \phi_\ell/2\theta, \end{aligned} \qquad (8)$$

which can be used to calculate the actual time delays $\{N_\ell\}$ and the Doppler frequency shifts $\{\phi_\ell\}$. Fig. 2 shows the channel output for a 4-path channel with different Doppler frequencies as multiples of $2\pi/N$, and delays of $[40\ 80\ 120\ 160]$ samples. Dechirping the output $y_T(n)$ by $g^*(n)$ and $g(n)$, we get $h_1(n) = y_T(n)g^*(n)$ and $h_2(n) = y_T(n)g(n)$ or

$$\begin{aligned} h_1(n) &= f_1(n) + e^{-j2\theta n^2} f_2(n) \\ h_2(n) &= e^{j2\theta n^2} f_1(n) + f_2(n). \end{aligned} \qquad (9)$$

3.2 Time-Frequency Approach

Using a time-frequency distribution that localizes chirps well, such as the Wigner distribution [21], the corresponding frequency marginals of $h_1(n)$ and $h_2(n)$ (See Fig. 3) will provide the information needed to obtain the channel parameters. Indeed, these marginals are related to the Fourier transform of $h_1(n)$ and $h_2(n)$ as shown in Fig. 4. Thus the connection of the frequencies where the peaks occur in

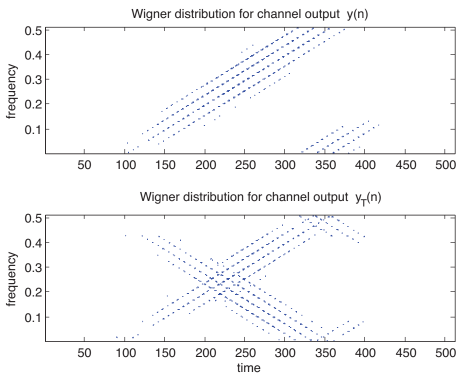

Figure 2: Wigner distribution for 4-path channel output with input g(n) and $g_T(n)$

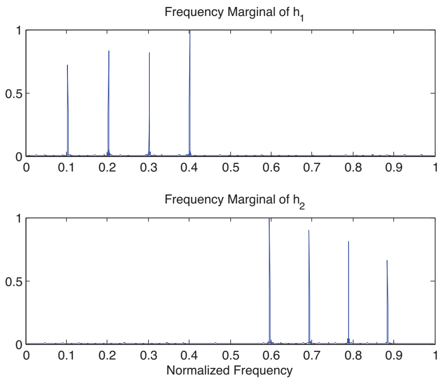

Figure 3: Frequency marginal obtained by Wigner distribution for 4-path channel

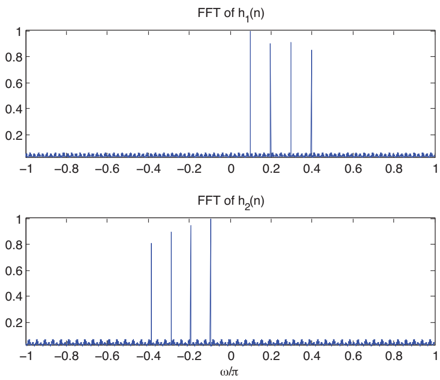

Figure 4: Spectrum for 4-path channel.

Fig. 3 with the parameters can be shown by computing the Fourier transform of $h_1(n)$ and $h_2(n)$ as well. To do so, we consider the Fourier transform of a signal $f(n)$ multiplied by a chirp $c(n)$, with Fourier transform $C(\omega)$. Representing $f(n)$ by its inverse transform we have:

$$\mathcal{F}[f(n)c(n)] = \sum_k F(k)C(\omega - \omega_k), \tag{10}$$

which shows a flat spectrum if the chirp is broadband. The Fourier transforms of $f_1(n)$ and $f_2(n)$ are given by

$$\begin{aligned} F_1(\omega) &= 2\pi \sum_{\ell=0}^{L-1} \alpha_\ell e^{j\theta N_\ell^2} \delta(\omega - 2\theta N_{e\ell}^{(1)}) \\ F_2(\omega) &= 2\pi \sum_{\ell=0}^{L-1} \alpha_\ell^* e^{-j\theta N_\ell^2} \delta(\omega - 2\phi_\ell - 2\theta N_{e\ell}^{(1)}) \end{aligned} \tag{11}$$

On the other hand, the Fourier transforms of $h_1(n)$ and $h_2(n)$ are composed of a wide-band low-amplitude part and impulses at the frequencies $\{\omega_\ell = 2\theta N_{e\ell}^{(1)}\}$ for $F_1(\omega)$ and $\{\omega_\ell = 2\phi_\ell + 2\theta N_{e\ell}^{(1)}\}$ for $F_2(\omega)$. Once we obtain $N_{e\ell}^{(1)}$ from the first equation we will use the second equation to obtain the Doppler shifts ϕ_ℓ, after which we can find the actual time delays N_ℓ using the definition of $N_{e\ell}^{(1)}$ in (8). The number of peaks is an estimate of L and an estimate of the attenuations α_ℓ can be found by looking at the amplitudes of the peaks. Indeed, estimates of α_ℓ can be found from

$$H_1(2\theta N_{e\ell}^{(1)}) \approx 2\pi\hat{\alpha}_\ell e^{j\theta N_\ell^2}. \tag{12}$$

In the above derivations we assumed no noise was present. When using the received signal

$$r(n) = y_T(n) + \eta(n), \tag{13}$$

where $\eta(n)$ is the channel noise, we need to use the dechirped signals $r(n)g^*(n)$ and $r(n)g(n)$. Again the Wigner distribution can be used to find the frequency marginals of the dechirped signals or we can use the periodogram [22] to estimate the frequencies where the peaks of the spectra of $r(n)g^*(n)$ and $r(n)g(n)$ occur.

In the discrete implementation of the method, we need to consider the significant difference in scale between the time and the Doppler frequency shifts. Given as *a-priori* information the possible range of values for the time and frequency delays, the length of the FM sinusoid, N, and the rate θ can be adjusted. After choosing the value of N to represent the range of time delays, one can then choose the chirp rate $\theta = \pi \tan(\beta)/N$ by letting the angle β be so that the instantaneous frequency goes from 0 to ϕ_{max}, the maximum value of the Doppler shift expected. Thus the resolution is set appropriately for both the time and the frequency delays.

4 Simulation Results

We implement the transmission of 1024 symbols in each OFDM block. The data symbols are QPSK modulated. The FM cosine and two-chirp pilots are used for channel estimation, and in each case pilot sequences are sent every 6 symbols. Letting the bandwidth be $BW = B$ kHz, if we wish to transmit M bits/frame the center frequencies of the sub-channels are B/M kHz, $k = 0, \cdots, M-1$.

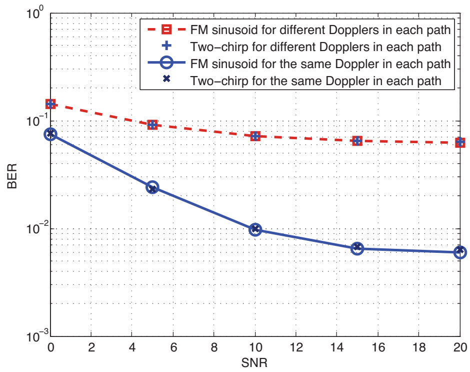

Figure 5: BER vs SNR for chirp channel estimation in OFDM using 1024 symbols

The model we use in our simulation for the communication channel is noisy, frequency-selective fading channel, valid for the duration of an OFDM frame, has $L = 4$ paths, each with attenuation α_ℓ, time and frequency shifts τ_ℓ and Φ_ℓ, respectively. For the discrete-time model it is assumed that the sampling frequency rate F_s is chosen appropriately so that the time shifts are $\tau_\ell = N_\ell T_s$ and likewise the Doppler frequency shifts are $\Phi_\ell = \phi_\ell F_s$ for some integers $\{N_\ell\}$. We assume that the relative velocity between the transmitter and the receiver is between 0 and 150 km/hr so that the Doppler frequency shifts vary between 0 to 150 Hz, and used a sampling frequency $F_s = 2B$ where the available bandwidth $B = 30$ kHz. As expected, in Fig. 5 the BER is larger for the case where each path is assigned a different Doppler frequency shift, than in the case where the Doppler frequency shift is the same for each path. The BERs obtained using two-chirp pilots and the proposed FM sinusoid for channel estimation method are very close to each other using just the half length of the two-chirp pilot sequence.

5 Conclusion

In this paper we proposed an improvement on chirp channel estimation. The pilot signal in our system is an FM sinusoid obtained by combining a linear chirp and its conjugate which allows accurate estimation of the channel parameters needed in developing coherent detectors without the need of sending two consecutive chirps during two symbol time. We are able to estimate channel parameters by sending an FM sinusoid of one symbol duration. Our method is justified using time-frequency analysis and showing that linear chirps have eigenfunction properties. We simulated an OFDM system and compared our method with the two-chirp method which showed a very similar performance to the two-chirp method without sending two-consecutive chirps. We will explore other channel models such as basis expansion model (BEM) using the proposed estimation method as our future work.

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