The Second Order Performance of Macrodiversity Reception in the Presence of Weibull Fading, Gamma Fading and α-κ-μ Co-channel Interference

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Abstract: - In this paper, the wireless system consisting of macrodiversity selection combining (SC) receiver and two microdiversity SC receivers under the influence of small scale fading and large scale fading, as well as co-channel interference is observed. Small scale fading has Weibull distribution. Correlated large scale fading is described by Gamma distribution. Co-channel interference is disturbed by α-κ-μ fading and Gamma large scale fading. Probability density function (PDF) and cumulative distribution function (CDF) of the ratio of Weibull random variable and α-κ-μ random variable are given. The formula for CDF of macrodiversity SC receiver output signal to interference ratio (SIR) is also presented. Level crossing rates at the outputs of microdiversity SC receivers are determined. Then, the level crossing rate (LCR) of wireless system output signal to interference ratio is derived and shown in some figures. Based on them, the influence of Weibull fading nonlinearity parameter, α-κ-μ fading severity parameter, α-κ-μ fading nonlinearity parameter, Rician factor, Gamma long term fading severity parameter and Gamma long term fading correlation coefficient is studied.

Key-Words: - macrodiversity receiver; microdiversity receiver; selection combining (SC); Gamma fading; Weibull fading; α-κ-μ fading; level crossing rate

1 Introduction

The characteristics of the first and the second orders of the wireless system are impaired owing to the influence of small scale fading, large scale fading fading and co-channel interference. The second order performances are level crossing rate (LCR) and average fade duration (AFD) of wireless mobile communication system [1]. To mitigate the small scale fading, large scale fading fading and co-channel interference effects on the level crossing rate it is necessary to use a macrodiversity system.

The macrodiversity system consists of one macrodiversity receiver and two or more microdiversity receivers [2]. The macrodiversity receiver is the most often of selection combining (SC) type. Its SC receiver selects microdiversity receiver with higher signal envelope (or signal to interference ratio) average power at inputs, resulting in Gamma large scale fading reduction. The microdiversity receivers could be maximal ratio combining (MRC), equal gain combining (EGC), or selection combining receivers [3].

The simplest is SC receiver which chooses the branch with the highest signal, or signal to interference ratio, which implies the small scale fading effects reduction and co-channel interference effects reduction [1].

Many different distributions describe signal envelope in fading environments such as Rayleigh, Rician, Nakagami, Weibull distribution, or general distributions like α-κ-μ distribution [4]. Weibull distribution describes small scale signal envelope variation in nonlinear, non line of sight (NLoS) fading surroundings [5]. It has parameter α, called nonlinearity parameter. Weibull distribution is general distribution because for $\alpha=2$, Weibull distribution reduces to Rayleigh distribution. For α tends to infinity Weibull channel becomes no fading channel.

On the other side, the α-κ-μ distribution is characterized by three parameters, where parameter α is nonlinearity parameter, κ is Rician factor and μ is small scale fading severity parameter [6]-[9]. Rician factor is the ratio of dominant component power and scattering components powers.

The α-κ-μ distribution is also general distribution. The other few distributions could be obtained from α-κ-μ distribution as a special case. If α=2, this distribution reduces to κ-μ distribution; if κ=0, the α-κ-μ becomes α-μ distribution; if κ=0 and μ=1, the α-κ-μ distribution comes down to Weibull distribution; if α=2 and μ=1, the α-κ-μ distribution reduces to Rician distribution; if α=2 and κ=0, the α-κ-μ is Nakagami-*m* distribution, and if α=2, κ=0 and μ=1, the Rayleigh distribution occurs.

The analysis of wireless systems in α-k-μ fading environment subjected to shadow effect is presented in [8]. The second order performance of the α-κ-μ envelope is derived in [9]. Many articles given in literature review consider macrodiversity system performance in the presence of large scale fading, small scale fading and co-channel interference. For example, the wireless system with macrodiversity SC receiver and two microdiversity SC receivers in Gamma shadowed Weibull multipath fading environment, under Weibull co-channel interference influence, is considered in [10]. The level crossing rate is calculated.

Macrodiversity system with macrodiversity SC receiver and two microdiversity SC receivers where desired signal is under Weibull short term fading, correlated Gamma long term fading and co-channel interference, which experiences α-κ-μ short term fading and correlated Gamma long term fading is analyzed in [11]. Here, the outage probability is evaluated from cumulative distribution function.

In this article, the wireless system with macrodiversity SC receiver, used to mitigate fading and co-channel interference effects on the system performance of the second order, is analyzed. Useful signal suffers Weibull fading and Gamma fading, and co-channel interference is under α-κ-μ fading and Gamma large scale fading. Level crossing rate of the ratio of Weibull random variable and α-κ-μ random variable is derived and used for calculation the LCR of microdiversity and macrodiversity SC receivers.

2 PDF of the Ratio of Weibull Random Variable and α-κ-μ Random Variable

The ratio of Weibull random variable and α-κ-μ random variable is:

$$z_1 = \frac{x_1}{y_1} = \frac{x^{\frac{2}{\alpha}}}{y^{\frac{2}{\alpha}}}, \quad z_1^{\frac{\alpha}{2}} = \frac{x}{y},$$

$$x_1 = z_1 y_1, \; x = z_1^{\frac{\alpha}{2}} \cdot y \tag{1}$$

where x follows Rayleigh distribution:

$$p_x(x) = \frac{2x}{\Omega_1} \cdot e^{-\frac{x^2}{\Omega_1}}, \; x \ge 0; \tag{2}$$

Ω_1 is average power of x. x_1 follows Weibull distribution [5]:

$$p_{x_1}(x_1) = \frac{\alpha}{\Omega_1} \cdot x_1^{\alpha-1} e^{-\frac{1}{\Omega_1} x_1^{\alpha}}, \; x_1 \ge 0; \tag{3}$$

α is Weibull short term fading nonlinearity parameter.

The random variable y follows κ-μ distribution [12] [13]:

$$p_y(y) = \frac{2\mu (k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu \frac{\sqrt{k(k+1)}}{\Omega_2} \right)^{2i_2+\mu-1} \frac{1}{i_2! \Gamma(i_2+\mu)}, \; y^{2i_2+2\mu-1} e^{-\frac{\mu(k+1)}{\Omega_2} y^2}, \; y \ge 0 \tag{4}$$

and random variable y_1 has α-κ-μ distribution [6]:

$$p_{y_1}(y_1) = \frac{\alpha\mu (k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_1=0}^{\infty} \left(\mu \frac{\sqrt{k(k+1)}}{\Omega_2} \right)^{2i_1+\mu-1} \frac{1}{i_1! \Gamma(i_1+\mu)} y_1^{\alpha i_1 + \alpha\mu - 1} \cdot e^{-\frac{\mu(k+1)}{\Omega_2} y_1^{\alpha}}, \; y_1 \ge 0 \tag{5}$$

where κ is Rician factor; α is nonlinearity parameter; Ω_2 is average power of y_1; Γ(.) is incomplete gamma function [14].

The first derivative of z_1 is:

$$\frac{\alpha}{2} z_1^{\frac{\alpha}{2}-1} \dot{z}_1 = \frac{\dot{x}}{y} - \frac{x}{y^2} \dot{y}$$

$$\dot{z}_1 = \frac{2}{\alpha z_1^{\alpha/2}} \left(\frac{\dot{x}}{y} - \frac{x}{y^2} \dot{y} \right) \tag{6}$$

The first derivative of Rayleigh random variable has Gaussian distribution and the first derivative of κ-μ random variable has also Gaussian distribution. Therefore, $\dot{x}$ and $\dot{y}$ are Gaussian variables. Since the linear transformation of Gaussian random variables has also Gaussian distribution, $\dot{z}_1$ follows Gaussian distribution. The mean of $\dot{z}_1$ is:

$$\overline{\dot{z}_1} = \frac{2}{\alpha z_1^{\alpha/2}}\left(\frac{\overline{\dot{x}}}{y} - \frac{x}{y^2}\overline{\dot{y}}\right) = 0 \quad (7)$$

because $\overline{\dot{x}} = \overline{\dot{y}} = 0$.

The variance of $\dot{z}_1$ is:

$$\sigma_{\dot{z}_1} = \frac{4}{\alpha^2 z_1^{\alpha-1}}\left(\frac{1}{y^2}\sigma_{\dot{x}} + \frac{x^2}{y^4}\sigma_{\dot{y}^2}\right) =$$

$$= \frac{4}{\alpha^2 z_1^{\alpha-1} y^2}\left(\sigma_{\dot{x}} + z_1^{\alpha}\sigma_{\dot{y}^2}\right) \quad (8)$$

where

$$\sigma_{\dot{x}} = \pi^2 f_m^2 \Omega_1 \quad (9)$$

$$\sigma_{\dot{y}} = \pi^2 f_m^2 \frac{\Omega_2}{\mu(k+1)} \quad (10)$$

After substituting, the expression for variance of $\dot{z}_1$ becomes:

$$\sigma_{\dot{z}_1} = \frac{4\pi^2 f_m^2}{\alpha^2 z_1^{\alpha-1} y^2 \mu(k+1)}\left(\Omega_1 \mu(k+1) + z_1^{\alpha}\Omega_2\right). \quad (11)$$

The joint probability density function of z_1, $\dot{z}_1$ and y is:

$$p_{z_1\dot{z}_1 y}(z_1\dot{z}_1 y) = p_{\dot{z}_1}(\dot{z}_1 / z_1 y)\cdot p_{z_1 y}(z_1 y) =$$

$$= p_{\dot{z}_1}(\dot{z}_1 / z_1 y)\cdot p_y(y)\, p_{z_1}(z_1 / y) \quad (12)$$

The $p_{z_1}(z_1 / y)$ is:

$$p_{z_1}(z_1 / y) = \left|\frac{dx}{dz_1}\right| p_x\left(y \cdot z_1^{\frac{\alpha}{2}}\right) \quad (13)$$

with:

$$\frac{dx}{dz_1} = y \cdot \frac{\alpha}{2} \cdot z_1^{\frac{\alpha}{2}-1}. \quad (14)$$

After the next substituting, the expression for $p_{z_1\dot{z}_1 y}(z_1\dot{z}_1 y)$ is:

$$p_{z_1\dot{z}_1 y}(z_1\dot{z}_1 y) = y \cdot \frac{\alpha}{2} \cdot z_1^{\frac{\alpha}{2}-1} p_x\left(y z_1^{\alpha/2}\right) p_y(y)\, p_{\dot{z}_1}(\dot{z}_1 / z_1 y) \quad (15)$$

The joint probability density function of z_1 and $\dot{z}_1$ is:

$$p_{z_1\dot{z}_1}(z_1\dot{z}_1) = \int_0^{\infty} dy\, p_{z_1\dot{z}_1 y}(z_1\dot{z}_1 y) =$$

$$= \frac{\alpha}{2} z_1^{\frac{\alpha}{2}-1}\int_0^{\infty} dy\, p_x\left(y z_1^{\alpha/2}\right) p_y(y)\, p_{\dot{z}_1}(\dot{z}_1 / z_1 y) \quad (16)$$

Level crossing rate of random process z_1 is:

$$N_{z_1} = \int_0^{\infty} d\dot{z}_1\, \dot{z}_1\, p_{z_1\dot{z}_1}(z_1\dot{z}_1) =$$

$$= \frac{\alpha}{2} z_1^{\frac{\alpha}{2}-1}\int_0^{\infty} dy\, y\, p_x\left(y z_1^{\alpha/2}\right) p_y(y)\int_0^{\infty} d\dot{z}_1\, \dot{z}_1\, p_{\dot{z}_1}(\dot{z}_1 / z_1 y) =$$

$$= \frac{\alpha}{2} z_1^{\frac{\alpha}{2}-1}\int_0^{\infty} dy\, y\, p_x\left(y z_1^{\alpha/2}\right) p_y(y)\frac{1}{\sqrt{2\pi}}\sigma_{\dot{z}_1} =$$

$$= \frac{\alpha}{2} z_1^{\frac{\alpha}{2}-1}\int_0^{\infty} dy\, y\, p_x\left(y z_1^{\alpha/2}\right) p_y(y)\frac{1}{\sqrt{2\pi}}\sigma_{\dot{z}_1} =$$

$$= \frac{\alpha}{2} z_1^{\frac{\alpha}{2}-1}\frac{2\pi f_m}{\alpha z_1^{\alpha/2-1}\mu^{1/2}(k+1)^{1/2}}$$

$$\left(\Omega_1\mu(k+1) + z_1^{\alpha}\Omega_2\right)^{1/2}\cdot z_1^{\alpha/2}\cdot\frac{2}{\Omega_1}\cdot\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}}$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\mu\frac{\sqrt{k(k+1)}}{\Omega_2}\right)^{2i_1+\mu-1}\frac{1}{i_1!\,\Gamma(i_1+\mu)},$$

$$\int_0^{\infty} dy\, y^{2i_1+2\mu+1} e^{-\frac{1}{\Omega_1}\cdot y^2 z_1^{\alpha} - \frac{\mu(k+1)}{\Omega_2}y^2} = \frac{\pi f_m}{\mu^{1/2}(k+1)^{1/2}}$$

$$\left(\Omega_1\mu(k+1) + z_1^{\alpha}\Omega_2\right)^{1/2}\cdot z_1^{\alpha/2}\cdot\frac{2}{\Omega_1}\cdot\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}}$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\mu\frac{\sqrt{k(k+1)}}{\Omega_2}\right)^{2i_1+\mu-1}\frac{1}{i_1!\,\Gamma(i_1+\mu)}$$

$$\frac{1}{2}\left(\Omega_1\Omega_2\right)^{i_1+\mu+1/2}\frac{1}{\left(\Omega_1\mu(k+1) + z_1^{\alpha}\Omega_2\right)^{i_1+\mu+1/2}}. \quad (17)$$

3 Level Crossing Rate of Macrodiversity System Output SIR

The model of macrodiversity system observed in this paper is presented in Fig. 1.

Probability density function of x_{ij}, i=1,2; j=1,2 is Weibull (3).

$$p_{x_{ij}}(x_{ij}) = \frac{\alpha x_{ij}}{\Omega_i}\cdot e^{-\frac{x_{ij}^2}{\Omega_i}}, \qquad x_{ij} \ge 0,\ i = 1,2,\ j = 1,2.$$

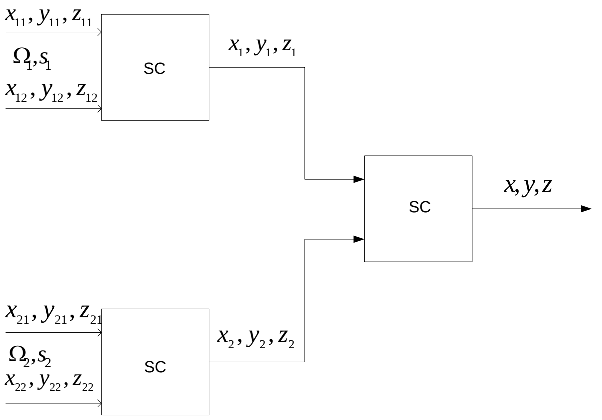

Fig.1. System model

Random variables y_{ij} follows α-κ-μ distribution (5):

$$p_{y_{ij}}\left(y_{ij}\right)=\frac{\alpha\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}s_i^{\frac{\mu+1}{2}}}$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\mu\frac{\sqrt{k\left(k+1\right)}}{\Omega_2}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma\left(i_1+\mu\right)}$$

$$y_{ij}^{\alpha i_1+\alpha\mu-1}\cdot e^{-\frac{\mu(k+1)}{s_i}y_{ij}^{\alpha}},\ y_{ij}\geq 0\text{.`}$$

Joint probability density function (JPDF) of Ω_1 and Ω_2 is [1]:

$$p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)=\frac{2}{\Gamma\left(c\right)\left(1-\rho^2\right)\rho^{c-1}\Omega_0^{c+1}}\cdot$$

$$\cdot\sum_{i_3=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_3+c-1}\cdot\frac{1}{i_3!\Gamma\left(i_3+c\right)}\cdot$$

$$\Omega_1^{i_3+c-1}\Omega_2^{i_3+c-1}\cdot e^{-\frac{\Omega_1+\Omega_2}{\Omega_0\left(1-\rho^2\right)}},\ \Omega_1\geq 0,\Omega_2\geq 0 \quad (18)$$

where c is Gamma long term fading severity parameter, ρ is correlation coefficient, and Ω_0 is average value of Ω_1 and Ω_2.

The joint probability density function (JPDF) of s_1 and s_2 is [1]:

$$p_{s_1s_2}\left(s_1s_2\right)=\frac{1}{\Gamma\left(c_1\right)\beta^{c_1}}\cdot s_1^{c_1-1}e^{-\frac{1}{\beta}s_1}\cdot$$

$$\cdot\frac{1}{\Gamma\left(c_1\right)\beta^{c_1}}\cdot s_2^{c_1-1}e^{-\frac{1}{\beta}s_2},\ s_1\geq 0,\ s_2\geq 0 \quad (19)$$

where c_1 is Gamma long term fading severity parameter of interference, β is average value of s_1 and s_2.

The CDF of macrodiversity SC receiver output signal to interference ratio is [11, eq. (13)]:

$$F_z\left(z\right)=\int_0^{\infty}ds_1\,p_{s_1}\left(s_1\right)\int_0^{\infty}d\Omega_1\int_0^{\Omega_1}F_{z_1}\left(z/\Omega_1,s_1\right)p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)d\Omega_2+$$

$$+\int_0^{\infty}ds_2\,p_{s_2}\left(s_2\right)\int_0^{\infty}d\Omega_2\int_0^{\Omega_2}d\Omega_1F_{z_2}\left(z/\Omega_2,s_2\right)p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)=$$

$$=2\int_0^{\infty}ds_1\,p_{s_1}\left(s_1\right)\int_0^{\infty}d\Omega_1\int_0^{\Omega_1}F_{z_1}\left(z/\Omega_1,s_1\right)p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)d\Omega_2=$$

$$=\beta^{\alpha}\Gamma\left(\alpha\right)\cdot\frac{1}{\left(\mu\left(k+1\right)\right)^{i_1+i_2+2\mu}}$$

$$\left(\Omega_0\left(1-\rho^2\right)\right)^{2i_3+2c+j_1}\Gamma\left(2i_3+2c+j_1\right)-$$

$$-\left(\frac{1}{\left(\mu\left(k+1\right)\right)^{i_1+\mu}}\left(\mu\left(k+1\right)\right)^{\alpha-i_2-\mu}\frac{1}{z^{2\alpha}}\right.$$

$$\cdot\Gamma\left(2i_3+2c+j_1+\mu+i_2\right)\left(\Omega_0\left(1-\rho^2\right)\right)^{2i_3+2c+j_1+\alpha}$$

$$\frac{\Gamma\left(2i_3+2c+j_1+\alpha\right)\Gamma\left(\alpha\right)}{\Gamma\left(2i_3+2c+j_1+\mu+i_2+\alpha\right)}$$

$$\left.{}_2F_1\left(2i_3+2c+j_1+\alpha,\alpha,2i_3+2c+j_1+\mu+i_2+\alpha;1-\frac{\Omega_0\left(1-\rho^2\right)\mu\left(k+1\right)}{\beta z^2}\right)\right)$$

$$-\left(\frac{1}{\left(\mu\left(k+1\right)\right)^{i_2+\mu}}\left(\mu\left(k+1\right)\right)^{\alpha-i_1-\mu}\frac{1}{z^{2\alpha}}\right.$$

$$\cdot\Gamma\left(2i_3+2c+j_1+\mu+i_1\right)\left(\Omega_0\left(1-\rho^2\right)\right)^{2i_3+2c+j_1+\alpha}$$

$$\frac{\Gamma\left(2i_3+2c+j_1+\alpha\right)\Gamma\left(\alpha\right)}{\Gamma\left(2i_3+2c+j_1+\mu+i_1+\alpha\right)}$$

$$\left.{}_2F_1\left(2i_3+2c+j_1+\alpha,\alpha,2i_3+2c+j_1+\mu+i_1+\alpha;1-\frac{\Omega_0\left(1-\rho^2\right)\mu\left(k+1\right)}{\beta z^2}\right)\right)$$

$$+\left(\left(\mu\left(k+1\right)\right)^{\alpha-i_1-i_2-2\mu}\frac{1}{z^{2\alpha}}\right.$$

$$\cdot\Gamma\left(2i_3+2c+j_1+i_1+i_2+2\mu\right)\left(\Omega_0\left(1-\rho^2\right)\right)^{2i_3+2c+j_1+\alpha}$$

$$\frac{\Gamma\left(2i_3+2c+j_1+\alpha\right)\Gamma\left(\alpha\right)}{\Gamma\left(2i_3+2c+j_1+i_1+i_2+2\mu+\alpha\right)}$$

$$ {}_2F_1\left(2i_3+2c+j_1+\alpha,\alpha,2i_3+2c+j_1+i_1+i_2+2\mu+\alpha;1-\frac{\Omega_0\left(1-\rho^2\right)\mu\left(k+1\right)}{\beta z^2}\right)\Bigg) \quad (20) $$

Macrodiversity system chooses microdiversity SC receiver with higher signal envelope average power or signal to interference ratio from its inputs.

The level crossing rate at the SC receiver output is:

$$ N_{z_1}=2F_{z_{11}}N_{z_{11}}=\frac{\alpha}{\Omega_1}\cdot\frac{\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}}\cdot $$

$$ \cdot\sum_{i_2=0}^{\infty}\left(\mu\frac{\sqrt{k\left(k+1\right)}}{\Omega_2}\right)^{2i_2+\mu-1}\frac{1}{i_2!\Gamma\left(i_2+\mu\right)}\Gamma\left(\mu+i_2+1\right) $$

$$ \frac{1}{2}\left(\Omega_1\Omega_2\right)^{\mu+i_2+1}\frac{1}{\alpha\Omega_2}\frac{1}{\mu+i_2} $$

$$ \left(\frac{1}{\left(\Omega_1\mu\left(k+1\right)\right)^{\mu+i_2}}-\frac{1}{\left(\Omega_1\mu\left(k+1\right)+z_1^{\alpha}\Omega_2\right)^{\mu+i_2}}\right) $$

$$ =\frac{\pi f_m}{\mu^{1/2}\left(k+1\right)^{1/2}}z_1^{\alpha/2}\cdot\frac{2}{\Omega_1}\cdot\frac{2\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}} $$

$$ \cdot\sum_{i_1=0}^{\infty}\left(\mu\frac{\sqrt{k\left(k+1\right)}}{\Omega_2}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma\left(i_1+\mu\right)}\frac{1}{2}\left(\Omega_1\Omega_2\right)^{i_1+\mu+1/2} $$

$$ \frac{1}{\left(\Omega_1\mu\left(k+1\right)+z_1^{\alpha}\Omega_2\right)^{i_1+\mu}} \quad (21) $$

The level crossing rate of macrodiversity SC receiver output signal to interference ratio process is:

$$ N_z\left(z\right)=\int_0^{\infty}ds_1\,p_{s_1}\left(s_1\right)\int_0^{\infty}d\Omega_1\int_0^{\Omega_1}d\Omega_2N_{z_1}\left(z/\Omega_1s_1\right)p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)+ $$

$$ +\int_0^{\infty}ds_2\,p_{s_2}\left(s_2\right)\int_0^{\infty}d\Omega_2\int_0^{\Omega_2}d\Omega_1N_{z_2}\left(z/\Omega_2s_2\right)p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)= $$

$$ =2\int_0^{\infty}ds_1\,p_{s_1}\left(s_1\right)\int_0^{\infty}d\Omega_1\int_0^{\Omega_1}d\Omega_2N_{z_1}\left(z/\Omega_1s_1\right)p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)= $$

$$ =2\frac{1}{\Gamma\left(c_1\right)\beta^{c_1}}\cdot\frac{1}{\Gamma\left(c\right)\left(1-\rho^2\right)\rho^{c-1}\Omega_0^{c+1}}\cdot $$

$$ \cdot\sum_{i_3=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_3+c-1}\cdot\frac{1}{i_3!\Gamma\left(i_3+c\right)}\cdot\alpha\cdot\frac{\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}} $$

$$ \cdot\sum_{i_2=0}^{\infty}\left(\mu\sqrt{k\left(k+1\right)}\right)\frac{1}{i_2!\Gamma\left(i_2+\mu\right)}\Gamma\left(\mu+i_2+1\right)\frac{1}{\alpha}\frac{1}{\mu+i_2} $$

$$ \cdot\frac{\pi f_m}{\mu^{1/2}\left(k+1\right)^{1/2}}z^{\alpha/2}2\cdot\frac{2\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}} $$

$$ \cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{k\left(k+1\right)}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma\left(i_1+\mu\right)}\frac{1}{2} $$

$$ \int_0^{\infty}ds_1s_1^{c_1-1-\frac{\mu}{2}-\frac{1}{2}-i_2-\frac{\mu}{2}+\frac{1}{2}+\mu+i_2+1-1-\frac{\mu}{2}-\frac{1}{2}-i_1-\frac{\mu}{2}+\frac{1}{2}-i_1-\mu-1/2}e^{-\frac{1}{\beta_1}s_1} $$

$$ \int_0^{\infty}d\Omega_1\Omega_1^{i_3+c-1-1+\mu+i_2+1+i_1+\mu+1/2}e^{-\frac{\Omega_1}{\Omega_0\left(1-\rho^2\right)}} $$

$$ \left(\frac{1}{\left(\Omega_1\mu\left(k+1\right)\right)^{\mu+i_2}}-\frac{1}{\left(\Omega_1\mu\left(k+1\right)+z^{\alpha}s_1\right)^{\mu+i_2}}\right)\cdot $$

$$ \frac{1}{\left(\Omega_1\mu\left(k+1\right)+z^{\alpha}s_1\right)^{\mu+i_1}}\int_0^{\Omega_1}d\Omega_2\Omega_2^{i_3+c-1}e^{-\frac{\Omega_2}{\Omega_0\left(1-\rho^2\right)}}= $$

$$ =\frac{2}{\Gamma\left(c_1\right)\beta^{c_1}}\cdot\frac{1}{\Gamma\left(c\right)\left(1-\rho^2\right)\rho^{c-1}\Omega_0^{c+1}}\cdot $$

$$ \cdot\sum_{i_3=0}^{\infty}\left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_3+c-1}\cdot\frac{1}{i_3!\Gamma\left(i_3+c\right)}\cdot\alpha\cdot\frac{\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}} $$

$$ \cdot\sum_{i_2=0}^{\infty}\left(\mu\sqrt{k\left(k+1\right)}\right)\frac{1}{i_2!\Gamma\left(i_2+\mu\right)}\Gamma\left(\mu+i_2+1\right)\frac{1}{\alpha}\frac{1}{\mu+i_2} $$

$$ \frac{\pi f_m}{\mu^{1/2}\left(k+1\right)^{1/2}}z^{\alpha/2}\cdot2\cdot\frac{2\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}} $$

$$ \cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{k\left(k+1\right)}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma\left(i_1+\mu\right)}\frac{1}{2} $$

$$ \cdot\frac{1}{i_3+c}\sum_{j_1=0}^{\infty}\frac{1}{\left(i_3+c+1\right)\left(j_1\right)}\left(\frac{1}{\Omega_0\left(1-\rho^2\right)}\right)^{j_1} $$

$$ \int_0^{\infty}ds_1s_1^{c_1-2\mu-2i_1-1/2-1}e^{-\frac{1}{\beta_1}s_1} $$

$$ \int_0^{\infty}d\Omega_1\Omega_1^{i_3+c+i_1+i_2+2\mu-1+1/2+i_3+c+j_1}e^{-\frac{2\Omega_1}{\Omega_0\left(1-\rho^2\right)}} $$

$$\left(\frac{1}{\left(\Omega_1\mu\left(k+1\right)\right)^{\mu+i_2}}-\frac{1}{\left(\Omega_1\mu\left(k+1\right)+z^{\alpha}s_1\right)^{\mu+i_2}}\right)\cdot$$

$$\frac{1}{\left(\Omega_1\mu\left(k+1\right)+z^{\alpha}s_1\right)^{\mu+i_1}} \tag{22}$$

In the previous expression, two-fold integral can be written in the form [15] [16]:

$$J=\int_0^\infty ds_1 s_1^{c_1-2\mu-2i_1-1/2-1}e^{-\frac{1}{\beta_1}s_1}$$

$$\int_0^\infty d\Omega_1\Omega_1^{i_3+c+i_1+i_2+2\mu-1+1/2+i_3+c+j_1}e^{-\frac{2\Omega_1}{\Omega_0\left(1-\rho^2\right)}}$$

$$\left(\frac{1}{\left(\Omega_1\mu\left(k+1\right)\right)^{\mu+i_2}}-\frac{1}{\left(\Omega_1\mu\left(k+1\right)+z^{\alpha}s_1\right)^{\mu+i_2}}\right)\cdot$$

$$\frac{1}{\left(\Omega_1\mu\left(k+1\right)+z^{\alpha}s_1\right)^{\mu+i_1}}=J_1-J_2 \tag{23}$$

The integral J_1 is:

$$J_1=\frac{1}{\left(\mu\left(k+1\right)\right)^{\mu+i_2}}\int_0^\infty ds_1 s_1^{c_1-2\mu-2i_1-1/2-1}e^{-\frac{1}{\beta_1}s_1}$$

$$\int_0^\infty d\Omega_1\Omega_1^{2c+2i_3+i_1+i_2+2\mu-1+1/2+j_1-\mu-i_2}e^{-\frac{2\Omega_1}{\Omega_0\left(1-\rho^2\right)}}$$

$$\frac{1}{\left(\Omega_1\mu\left(k+1\right)+z^{\alpha}s_1\right)^{\mu+i_1}} \tag{24}$$

The integral can be solved by using the formulae:

$$\int_0^\infty ds s^{p_1-1}e^{-\alpha_1 s}\int_0^\infty d\Omega\Omega^{p_2-1}e^{-\alpha_2\Omega}\frac{1}{\left(a\Omega+bs\right)^n}=$$

$$=\frac{a^{p_1-n}}{b^{p_1}}\Gamma\left(p_2\right)\frac{1}{\alpha_2^{p_1+p_2-n}}\frac{\Gamma\left(p_1+p_2-n\right)\Gamma\left(p_1\right)}{\Gamma\left(p_1+p_2\right)}$$

$${}_2F_1\left(p_1+p_2-n,p_1;p_1+p_2;1-\frac{\alpha_1 a}{\alpha_2 b}\right) \tag{25}$$

Here, ${}_2F_1(\alpha,\beta;\gamma;x)$ is a Gauss's hypergeometric function (Gauss 1812, Barnes 1908). In general, it arises the most frequently from physical problems derived by Gauss and Barnes in [17] and [18]:

$$\frac{\alpha\beta}{1!\gamma}x+\frac{\alpha\left(\alpha+1\right)\beta\left(\beta+1\right)}{2!\gamma\left(\gamma+1\right)}x^2+$$

$$+\frac{\alpha\left(\alpha+1\right)\left(\alpha+2\right)\beta\left(\beta+1\right)\left(\beta+2\right)}{3!\gamma\left(\gamma+1\right)\left(\gamma+2\right)}x^3+... \tag{26}$$

This is so-called regular solution, denoted by:

$${}_2F_1\left(\alpha,\beta;\gamma;x\right)=1+\frac{\alpha\beta}{1!\gamma}x+\frac{\alpha\left(\alpha+1\right)\beta\left(\beta+1\right)}{2!\gamma\left(\gamma+1\right)}x^2+...=$$

$$=\sum_{n=0}^{\infty}\frac{\left(\alpha\right)_n\left(\beta\right)_n}{\left(\gamma\right)_n}\frac{x^n}{n!}. \tag{27}$$

It converges if c is not a negative integer for all of $|z|<1$ and is on the unit circle $|z|=1$ if $R[c-a-b]>0$. Here, $(a)_n$ is a Pochhammer symbol.

For solution of the integral J_1, the parameters are:

$$p_1=c-2\mu-2i_1-1/2$$

$$\alpha_1=\frac{1}{\beta_1}$$

$$p_2=2c+2i_3+i_1+\mu+1/2+j_1$$

$$\alpha_2=\frac{2}{\Omega_0\left(1-\rho^2\right)}$$

$$a=\mu\left(k+1\right)$$

$$b=z^{\alpha}$$

$$n=\mu+i_1$$

$$p_1-n=c-3\mu-3i_1-1/2$$

$$p_1+p_2=3c-\mu-i_1+2i_3+j_1$$

$$p_1+p_2-n=3c-2\mu-2i_1+2i_3+j_1$$

After introducing these parameters in (25), the integral J_1 becomes:

$$J_1=\left(\mu\left(k+1\right)\right)^{c-3\mu-3i_1-1/2}\frac{1}{z^{\alpha\left(c-2\mu-2i_1-1/2\right)}}$$

$$\Gamma\left(2c+2i_3+i_1+\mu+1/2+j_1\right)\left(\frac{\Omega_0\left(1-\rho^2\right)}{2}\right)^{3c-2\mu-2i_1+2i_3+j_1}$$

$$\frac{\Gamma\left(3c-2\mu-2i_1+2i_3+j_1\right)\Gamma\left(c-2\mu-2i_1-1/2\right)}{\Gamma\left(3c-\mu-i_1+2i_3+j_1\right)}$$

$${}_2F_1\left(3c-2\mu-2i_1+2i_3+j_1,c-2\mu-2i_1-1/2;3c-\mu-i_1+2i_3+j_1;1-\frac{\Omega_0\left(1-\rho^2\right)\mu\left(k+1\right)}{2\beta_1 z^{\alpha}}\right) \tag{28}$$

The integral J_2 is:

$$J_2=\int_0^\infty ds_1 s_1^{c_1-2\mu-2i_1-1/2-1}e^{-\frac{1}{\beta_1}s_1}$$

$$\int_0^\infty d\Omega_1 \Omega_1^{2c+2\mu+i_1+i_2+2i_3+1/2+j_1-1} e^{-\frac{2\Omega_1}{\Omega_0\left(1-\rho^2\right)}}$$

$$\frac{1}{\left(\mu\left(k+1\right)\Omega_1+z^\alpha s_1\right)^{2\mu+i_1+i_2}} \tag{29}$$

It could be solved in the same way, with the next group of parameters which will be put into (25):

$$p_1 = c-2\mu-2i_1-1/2$$

$$\alpha_1 = \frac{1}{\beta_1}$$

$$p_2 = 2c+2\mu+i_1+i_2+2i_3+1/2+j_1$$

$$\alpha_2 = \frac{2}{\Omega_0\left(1-\rho^2\right)}$$

$$a = \mu\left(k+1\right)$$

$$b = z^\alpha$$

$$n = 2\mu+i_1+i_2$$

$$p_1-n = c-4\mu-3i_1-i_2-1/2$$

$$p_1+p_2 = 3c-i_1+i_2+2i_3+j_1$$

$$p_1+p_2-n = 3c-2\mu-2i_1+2i_3+j_1$$

The integral J_2 is:

$$J_2 = \left(\mu\left(k+1\right)\right)^{c-4\mu-3i_1-i_2-1/2} \frac{1}{z^{\alpha\left(c-2\mu-2i_1-1/2\right)}}$$

$$\Gamma\left(2c+2\mu+2i_3+i_1+i_2+1/2+j_1\right)\left(\frac{\Omega_0\left(1-\rho^2\right)}{2}\right)^{3c-2\mu-2i_1+2i_3+j_1}$$

$$\frac{\Gamma\left(3c-2\mu-2i_1+2i_3+j_1\right)\Gamma\left(c-2\mu-2i_1-1/2\right)}{\Gamma\left(3c-i_1+i_2+2i_3+j_1\right)}$$

$${}_2F_1\left(3c-2\mu-2i_1+2i_3+j_1, c-2\mu-2i_1-1/2; 3c-i_1+i_2+2i_3+j_1; 1-\frac{\Omega_0\left(1-\rho^2\right)\mu\left(k+1\right)}{2\beta_1 z^\alpha}\right) \tag{30}$$

Finally, after substitution J_1 from (28) and J_2 from (30) into (23) and (22), the level crossing rate of macrodiversity SC receiver output signal to interference ratio becomes:

$$N_z\left(z\right) == \frac{2}{\Gamma\left(c_1\right)\beta^{c_1}} \cdot \frac{1}{\Gamma\left(c\right)\left(1-\rho^2\right)\rho^{c-1}\Omega_0^{c+1}} \cdot$$

$$\cdot \sum_{i_3=0}^\infty \left(\frac{\rho}{\Omega_0\left(1-\rho^2\right)}\right)^{2i_3+c-1} \cdot \frac{1}{i_3!\Gamma\left(i_3+c\right)} \cdot \alpha \cdot \frac{\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}}$$

$$\cdot \sum_{i_2=0}^\infty \left(\mu\sqrt{k\left(k+1\right)}\right) \frac{1}{i_2!\Gamma\left(i_2+\mu\right)} \Gamma\left(\mu+i_2+1\right) \frac{1}{\alpha} \frac{1}{\mu+i_2}$$

$$\frac{\pi f_m}{\mu^{1/2}\left(k+1\right)^{1/2}} z^{\alpha/2} \cdot 2 \cdot \frac{2\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}}$$

$$\cdot \sum_{i_1=0}^\infty \left(\mu\sqrt{k\left(k+1\right)}\right)^{2i_1+\mu-1} \frac{1}{i_1!\Gamma\left(i_1+\mu\right)} \frac{1}{2}$$

$$\cdot \frac{1}{i_3+c} \sum_{j_1=0}^\infty \frac{1}{\left(i_3+c+1\right)\left(j_1\right)} \left(\frac{1}{\Omega_0\left(1-\rho^2\right)}\right)^{j_1}$$

$$\cdot \left[\left(\mu\left(k+1\right)\right)^{c-3\mu-3i_1-1/2} \frac{1}{z^{\alpha\left(c-2\mu-2i_1-1/2\right)}}\right.$$

$$\Gamma\left(2c+2i_3+i_1+\mu+1/2+j_1\right)\left(\frac{\Omega_0\left(1-\rho^2\right)}{2}\right)^{3c-2\mu-2i_1+2i_3+j_1}$$

$$\frac{\Gamma\left(3c-2\mu-2i_1+2i_3+j_1\right)\Gamma\left(c-2\mu-2i_1-1/2\right)}{\Gamma\left(3c-\mu-i_1+2i_3+j_1\right)}$$

$${}_2F_1\left(3c-2\mu-2i_1+2i_3+j_1, c-2\mu-2i_1-1/2; 3c-\mu-i_1+2i_3+j_1; 1-\frac{\Omega_0\left(1-\rho^2\right)\mu\left(k+1\right)}{2\beta_1 z^\alpha}\right) -$$

$$-\left(\mu\left(k+1\right)\right)^{c-4\mu-3i_1-i_2-1/2} \frac{1}{z^{\alpha\left(c-2\mu-2i_1-1/2\right)}}$$

$$\Gamma\left(2c+2\mu+2i_3+i_1+i_2+1/2+j_1\right)\left(\frac{\Omega_0\left(1-\rho^2\right)}{2}\right)^{3c-2\mu-2i_1+2i_3+j_1}$$

$$\frac{\Gamma\left(3c-2\mu-2i_1+2i_3+j_1\right)\Gamma\left(c-2\mu-2i_1-1/2\right)}{\Gamma\left(3c-i_1+i_2+2i_3+j_1\right)}$$

$$\left.{}_2F_1\left(3c-2\mu-2i_1+2i_3+j_1, c-2\mu-2i_1-1/2; 3c-i_1+i_2+2i_3+j_1; 1-\frac{\Omega_0\left(1-\rho^2\right)\mu\left(k+1\right)}{2\beta_1 z^\alpha}\right)\right] \tag{31}$$

4 Numerical results

Fig. 2 to 4. present average level crossing rate of macrodiversity system for several values of Gamma long term fading severity parameter *c*, Weibull short term fading nonlinearity parameter α, α-κ-μ short term fading nonlinearity parameter α, α-κ-μ short term fading Rician factor κ and α-κ-μ short term fading severity parameter μ.

In Fig. 2, the average LCR is plotted versus output signal to interference ratio *z*.

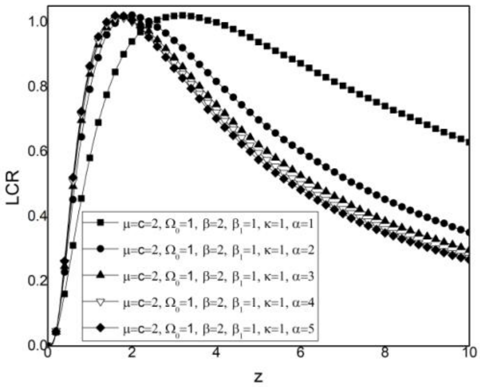

Fig. 2. Level crossing rate of macrodiversity system versus output signal to interference ratio *z*

The average LCR increases for small values of SC combiner output signal to interference ratio *z*, achieves the maximum, than declines. The influence of SC combiner output signal to interference ratio on average LCR is bigger for higher values of Weibull multipath fading severity parameters. Average LCR has lower values for bigger values of Weibull multipath fading severity parameter α and then system performance is better.

Average LCR versus output SIR *z* is shown in Fig. 3 for variable average power of desired signal. The other parameters are: $\mu=c=\beta=2$, $\beta_1=\kappa=\alpha=1$. One can notice from this figure that when average power Ω_0 increases, LCR increases too. This increase is more pronounced for higher values of *z*.

Also, average LCR of macrodiversity SC receiver output signal versus the SC receiver output signal to interference ratio *z* is presented in Fig. 4.

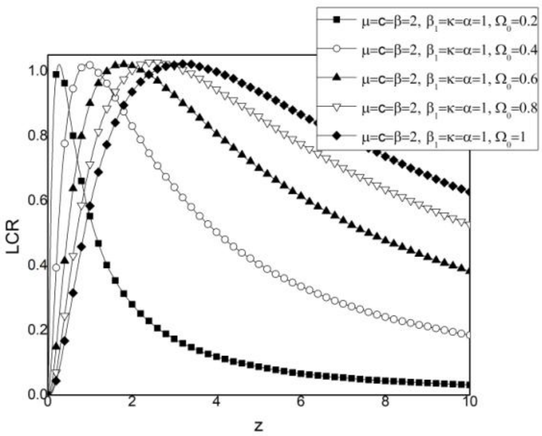

Fig. 3. LCR of macrodiversity system versus output SIR

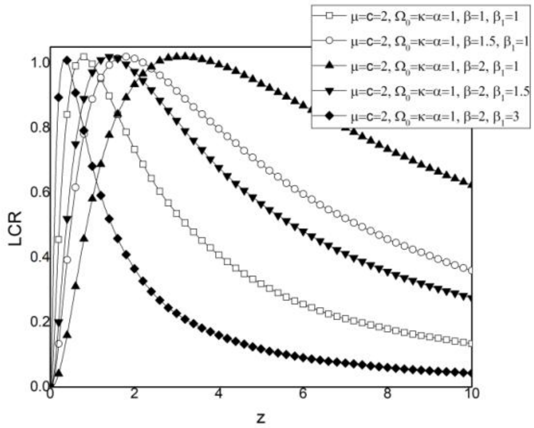

Fig. 4. LCR of macrodiversity system versus SIR *z*

Here, average power of Gamma fading is variable and other parameters are constant. LCR has lower values for greater values of average power of Gamma variable *β* and SC combiner output signal to interference ratio *z*.

System performances are better for lower values of the average LCR. These figures for the level crossing rate versus signal envelope are drown to show the impact of fading parameters to the LCR and for choosing optimal parameters.

5 Conclusion

In this paper, macrodiversity system with macrodiversity SC receiver and two microdiversity SC receivers in the presence of small scale fading and large scale fading is observed. Desired signal is hindered by Weibull multipath fading and correlated Gamma large scale fading, and interference is under the affect of α-κ-μ small scale fading and Gamma large scale fading. Macrodiversity system mitigates all of them, small and large scale fading effects, and co-channel interference effects on the level crossing rate.

Macrodiversity SC receiver works so that chooses microdiversity with higher signal to interference ratio. Microdiversity receivers combine SIRs from multiple antennas from base stations and select the branch with higher SIR, and reduces small term fading and co-channel interference.

In the article, PDF and CDF of the ratio of Weibull and α-κ-μ random variables are derived. Then, the expressions for cumulative distribution functions of SIR at outputs of microdiversity SC receivers and macrodiversity SC receiver output SIR are given, and finally, the LCR of macrodiversity system in the presence of Weibull small scale fading, Gamma large scale fading and α-κ-μ co-channel interference is performed. The influence of Weibull small scale fading nonlinearity parameter, Gamma large scale fading severity parameter, α-κ-μ small scale fading Rician factor, α-κ-μ small scale fading nonlinearity parameter and α-κ-μ small scale fading severity parameter on LCR is analyzed.

LCR decreases when Gamma long term fading severity parameter increases, Weibull short term fading severity parameter and α-κ-μ short term fading severity parameters increase and average power decreases.

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