Blood Flow in Catheterized Elastic Artery with Stenosis

D. N. RIAHI
School of Mathematical & Statistical Sciences, University of Texas Rio Grande Valley,
One West University Boulevard, Brownsville, Texas 78520-4933, USA
(Email:daniel.riahi@utrgv.edu)(Web address: http://www.utrgv.edu/math/)

Abstract: We consider unsteady two-phase blood flow in an elastic artery and in the presence of stenosis, which is based on the available experimental data. The stenosis is a condition where an artery wall thickens as a result of fatty materials such as cholesterol. The investigated artery system is considered in the presence of a catheter. Catheter is a standard tool that is used for diagnosis and treatment in patients whose artery flow is affected by the presence of stenosis. The governing unsteady equations are solved subjected to reasonable approximations and under restriction of low blood frequency. Blood flow quantities such as blood speed, blood pressure force, wall shear stress and impedance are computed for different values of the parameters and the hematocrit due to percentage level of the red cells in the plasma.

Key-Words: arterial flow, blood flow, stenosis, elastic artery, two-phase flow, blood cells, blood plasma

1. Introduction

Diseases that occur in the blood vessels and in the heart are the major causes of mortality worldwide. The underlying cause for these events is the formation of lesions, known as stenosis. These lesions can grow and occlude the artery and hence prevent blood supply to the distal bed, which could lead to heart attacks or stroke. Srivastava et al. [5] studied arterial blood flow through an overlapping stenosis and in the absence of a catheter. They calculated impedance and shear stress for different stenosis height. There have also been studies of the blood flow systems in the catheterized arteries such as the one due to Srivastava and Rastogi [6].

With the exception of [2, 3], all the other studies that have been carried out so far to investigate arterial blood flow systems, the shape of stenosis were based and assumed to be analytical due to some forms of certain mathematical functions. Ledesma et al. [2] and more recently Riahi [3] applied an experimentally based shape [1] for the stenosis shape in the artery, where the ratio of the artery radius to the axial extent of the stenosis is small, and in the presence of a catheter, where the blood was represented by a two-phase macroscopic model. In the present study we extend the work in [3] for catheterized elastic arterial blood flow in the presence of stenosis but under restriction of low frequency of the blood flow oscillation. In Figure 1 we provide in plane (*r*, *z*) non-dimensional shape function *R*(*z*) in the radial direction *r* of the internal surface of arterial system in the presence of stenosis versus non-dimensional axial variable *z* of an actual data calculated from the experimentally values of the cross-sectional area of the artery of a human [1]. Figure 2 presents flow geometry in a segment of the artery with stenosis.

2. Formulation & Analysis

We consider the problem of blood flow in catheterized elastic artery in the form of a circular cylindrical annulus tube with the outer radius R_0 (radius of the artery) and the inner radius r_1 (radius of the catheter) and in the presence of stenosis whose shape (figure 1) is determined from the experimentally collected data [1]. The artery length is assumed to be sufficiently large in comparison to its radius so that the end effects can be neglected.

The two-phase flow system in a catheterized artery is based on the original governing equations for the mass conservation and momentum [7] for both fluid plasma and the suspended particles (red cells) as their unsteady axisymmetric form in cylindrical coordinate system with axial direction along the co-axial direction of the catheterized artery are given by [6]

$$(1-C)\rho_f(\partial u_f/\partial t+u_f\partial u_f/\partial z+v_f\partial u_f/\partial r)=-(1-C)\partial P/\partial z+(1-C)\mu_s\nabla^2 u_f+CS'(u_p-u_f), \quad (1a)$$

$$(1-C)\rho_f(\partial v_f/\partial t+u_f\partial v_f/\partial z+v_f\partial v_f/\partial r)=-(1-C)\partial P/\partial r+(1-C)\mu_s(\nabla^2-1/r^2)v_f+CS'(v_p-v_f), \quad (1b)$$

$$(1/r)\partial/\partial r[r(1-C)v_f]+(\partial/\partial z)[(1-C)u_f]=0, \quad (1c)$$

$$C\rho_p[\partial u_p/\partial t+u_p\partial u_p/\partial z+v_p\partial u_p/\partial r]=-C\partial P/\partial z+CS'(u_f-u_p), \quad (1d)$$

$$C\rho_p[\partial v_p/\partial t+u_p\partial v_p/\partial z+v_p\partial v_p/\partial r]=-C\partial P/\partial r+CS'(v_f-v_p), \quad (1e)$$

$$(1/r)(\partial/\partial r)(Cr\, v_p)+(\partial/\partial z)(Cu_p)=0. \quad (1f)$$

Here $\nabla^2\equiv[(1/r)(\partial/\partial r)(r\partial/\partial r)+\partial^2/\partial z^2]$ is the Laplacian operator. Here r ($r_1\leq r\leq R_0$) and z are the cylindrical coordinates with axial variable z along the tube axis and radial variable r along the direction perpendicular to the tube axis, subscripts " f " and "p" refer to fluid (plasma) and particle (erythrocyte) quantities, respectively, u and v are the axial and radial velocity components, respectively, ρ is density, P is pressure, C is the volume fraction density of the particles, refers here as the hematocrit percentage in the blood, and the expressions for the viscosity of suspension μ_s and the drag coefficient of interaction S' have been chosen to be [4]

$$\mu_s=\mu_0/(1-mC), m=0.07exp[2.49C+(1107/T)\, exp(-1.69C)], \quad (1g)$$

$$S'=4.5(\mu_0/a_0^2)\{[4+3(8C-3C^2)^{0.5}+3C]/(2-3C)^2, \quad (1h)$$

where μ_0 is the plasma viscosity, a_0 is the radius of a red cell and T is absolute temperature measured in Kelvin.

We consider a catheter in the form of a tube with small radius but along the axis of artery is placed in the artery. The inside boundary of the artery is partially structured along a distance L_0 due to the presence of an atherosclerosis (Figure 2). In the figure 2, where the flow system and the geometry is shown in the cylindrical annulus, the catheterized arterial tube is given over a distance $L=2d+L_0$ in the axial direction, δ is the maximum height of the atherosclerosis into the lumen, which appears at particular location in the axial direction.

In the present paper, we consider low frequency of the oscillatory blood flow system. We introduce a slow time scale $t_s=t\varepsilon$, where $\varepsilon<<1$ is a small quantity. To the lowest order in ε, we determine the corresponding system for the blood flow, which turned out to be not affected by the displacement motion of the elastic artery. Since we present the result for this system alone in this paper, there is to need to provide here the equations for elastic artery's displacements.

We non non-dimensionalize the governing equations (1a-h) using U, L_0, R_0, δ, L_0/U and $\mu_0 U L_0/\delta^2$ as scales for velocity, axial length, radial length, rate of radial change, time and pressure, respectively, where U is the maximum velocity for the unidirectional flow in a cylindrical annulus [7]. Next, we simplify the dimensionless forms of the governing equations (1a-h) under the reasonable conditions for mild stenosis with $\delta/R_0<<1$, unidirectional flow assumption [7], simple oscillatory type motion in time and subjected to the assumptions that the inertial terms in the equations (1a-b, d-e) are small and $R_e(\delta/L_0)<<1$. Under these conditions and assumptions [5], the pressure is only a function of z and t and (1a-h) lead to the simpler equations, which are given below using the same symbols for the variables as their dimensional ones for simplicity of notations

$(1\text{-}C)\partial P/\partial z=[(1\text{-}C)/(1\text{-}mC)][(1/r)(\partial/\partial r)(r\partial u_f/\partial r)]+CS\,\beta^2(u_p-u_f),$ (2a)

$\partial P/\partial z=S\beta^2(u_f-u_p), S=4.5[4+3(8C\text{-}3C^2)^{0.5}+3C]/(2\text{-}3C)^2,$ (2b)

where $\beta=\delta/a_0$. The equations (2a-b) are subjected to the following no slip boundary conditions

$u_f=0$ on $r=r_1$ and $u_f=0$ on $r=R(z)$, (2c)

Using (2b) for (u_f-u_p) in (2a) and integrating twice with respect to r and making use of the boundary conditions given in (2c), we find

$u_f=(-1/4)[(1\text{-}mC)/(1\text{-}C)](\partial P/\partial z)\{(R^2\text{-}r^2)+[(R^2\text{-}r_1^2)\,l_n(r/R)]/l_n(R/r_1)\}.$ (3)

The expression for the axial velocity for the red cells is then found from (2b) in terms of the axial velocity for the plasma. Since both expressions for the axial velocity of plasma and red cells are in terms of the unknown $\partial P/\partial z$, we obtain an expression for the pressure gradient by assuming a prescribed value of volume flow rate factor Q_0, where the volume flow rate in the annulus given by

$Q=2\pi\int^{R}_{r1} r[(1\text{-}C)u_f+C\,u_p]dr\equiv Q_0 sin(\omega t_s).$ (4)

Using (2b) and (3) in (4) and evaluate the integral analytically, it can then lead to the expression for the pressure gradient. Next, we determine the blood flow resistanceλ, which is referred to as impedance, by integrating the pressure gradient over the artery's axial section and dividing by the constant Q. We also determine the wall shear stress by evaluating the negative of the radial derivative of u_f at the artery wall. All the blood flow quantities are assumed to be proportional to $\sin(\omega t_s)$, where ω is the frequency of the pulse oscillation. Such consideration satisfies all the results (2)-(4).

3. Results and Discussion

We carried out numerical calculations of several blood flow quantities for several different values of C, t_s and z and for fixed values of $Q_0=1$, b=0.5, ε=0.001 and $\beta=\delta/0.004$, where δ=1-minimum value of R. Our first calculated results are for dP/dz (axial rate change of the blood pressure in the catheterized artery) different values of the axial variable, catheter radius, time and the hematocrit parameter. We find that the blood pressure gradient is oscillatory in time, the magnitude of oscillation increases in the stenosis zone. The blood pressure force does not vary with respect to the axial variable at the axial locations outside the stenosis zones. However, the magnitude of the blood pressure force increases with the stenosis effect in the stenosis zone. The magnitude of the pressure force oscillations increase with the hematocrit effect. These results are physically and bio-medically reasonable since higher percentage of the blood cells amount in the plasma as well as more severity of the stenosis can intensify the blood pressure force in the artery. For given value of time, our results for values of the pressure gradient versus the axial variable but for different values of the catheter radius indicate that the magnitude of blood pressure force oscillation in the artery increases with the catheter radius, which is again reasonable physically since higher catheter radius implies smaller annulus gap leading to higher value of such force. Our calculated results for impedance (flow resistance) indicate that the impedance is oscillatory in time, and the magnitude of the oscillation increases with either catheter radius or hematocrit effect.

Next, we calculated the axial velocity of the blood plasma versus time or axial variable and for several values of the hematocrit parameter, the radial variable and catheter radius. We find that the plasma velocity is oscillatory in time, the magnitude of the oscillation increases with the catheter radius, but magnitude of such oscillations decrease with increasing the hematocrit parameter. The plasma velocity does not vary with respect to z outside the stenosis zones, but it varies with respect to z in the stenosis zone and its magnitude increases with the stenosis effect. This result is reasonable since higher stenosis effect increases the strength of the pressure force leading to higher blood plasma speed. The magnitude of the plasma velocity oscillation is also higher in the presence of catheter, which is reasonable since presence of catheter decreases the annulus gap leading to higher plasma velocity. Our results for the axial velocity of the red cells indicated that velocity of the red cells is oscillatory, but the magnitude of such oscillation is slightly higher than the corresponding one for the plasma velocity, and the dependence of the red cell velocity with respect to the parameters C and r_1 is similar to those for the plasma velocity. Our calculated results for wall shear stress indicate that it is oscillatory in time, and the magnitude of its oscillation increases with catheter radius and the hematocrit parameter. The wall shear stress varies notably with respect to z inside the stenosis zone, while is independent of the axial variable outside such zone.

4. Conclusion

We investigated the two-phase arterial blood flow with a catheter and in the presence of an experimentally determined multi stenosis for human, where the ratio of the artery radius to the axial extent of the stenosis is small, and under the restriction of low frequency of oscillation. We found, in particular, that the blood pressure force and the velocities of the plasma and the red cells are oscillatory, non-uniform in the stenosis zones, and the magnitude of their oscillations increase with the catheter radius. The magnitude of the oscillation of the blood pressure force increases with the hematocrit effect while the magnitudes of the oscillations of the blood speed decreases with increasing the hematocrit effect. The magnitudes of the oscillations of both impedance and wall shear stress increase with the catheter radius and hematocrit effect.

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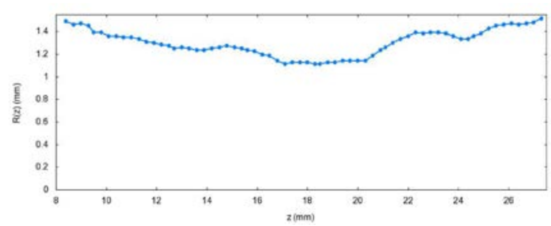

Figure 1. Experimental form of stenosis

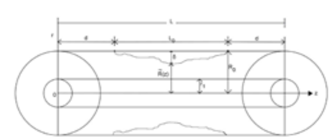

Figure2.Flow geometry in an artery