Mathematical model for heat collector

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Abstract: - Paper describes a mathematical model for the temperature of a heat collector in a system with a solar collector and heat tank. We use first order differential equations to model it. The obtained formula defines the temperature in the time domain and allows us to determine in advance the final temperature of the process. Determination of the temperature at the end of the process enables us to plan the energy consumption and achieve noticeable savings. In this paper we present a mathematical derivation that is a model and example that enables development of other formulas for different processes.

Key-Words: - heat collector; heat-transfer process; energy savings; computer optimisation

1 Introduction

In a previous paper [1] the work was motivated by the need to design a heating system where we can easily determine how efficient is a solar thermal system. We determined what are its characteristics i.e. how quick it can heat a certain heat tank and to which temperature. In this paper the work is continued with the solar collector temperatures in order to predict the reaching of the critical evaporation temperature. When we know the characteristics of the system in the time domain we can select the power of the thermal solar collector. We can also select the appropriate mass of a heat tank so that it meets the desired requirements.

As in the previous paper [1] we will derive the process formula by a first-order differential equation.

The ideal process we will define as heat transfer from a hotter tank of mass m_2, which is a solar collector for heat energy of power P, to a colder heat tank of mass m_1.

The formula for the ideal process is expanded and thus can be very close to the actual process with introduction of additional coefficients.

2 Obtaining the formula for the temperature of a heat storage tank in ideal process

The system consists of colder tank which is a heat tank of mass m1 of a substance of specific heat capacity c1 and a hotter tank which is a thermal solar collector of mass m_2 with a substance of specific heat capacity c_2 and a system of exchangers passing through them within which the liquid flow qm of specific heat capacity c_3.

2.1. Declaring variables:

Index 1 - lower temperature,

index 2 - higher temperature,

ϑ_{1p} - initial temperature of colder tank, [K]

ϑ_{2p} - initial temperature of hotter tank, [K]

ϑ_1 - temperature of colder tank variable in time, [K]

ϑ_2 - temp. of hotter tank variable in time, [K]

c - specific heat capacity of the substance in the system when the same substance is used in all parts of the system, [J/kgK]

c_1 - specific heat capacity of the substance in the colder tank, [J/kgK]

c_2 - specific heat capacity of the substance in the hotter tank, [J/kgK]

c_3 - specific heat capacity of the substance inside the tube and heat exchanger, [J/kgK]

q_m - fluid flow rate in pipes per second, [kg/s]

m_1 - mass of colder tank medium, [kg]

m_2 - mass of hotter tank medium, [kg]

P – power of solar collector, [W]

2.2. Differential equations and substitution of variables

Differential substitution is derived from the differential equation of the system [1][2].

$$q_m \cdot c_3 \cdot \left(\vartheta_2 - \vartheta_1\right) = m_1 \cdot c_1 \cdot \frac{d\vartheta_1}{dt} = -m_2 \cdot c_2 \cdot \frac{d\vartheta_2}{dt} \quad (1)$$

$$q_m \cdot c_3 \cdot \left(\vartheta_2 - \vartheta_1\right) dt = m_1 \cdot c_1 \cdot d\vartheta_1 = -m_2 \cdot c_2 \cdot d\vartheta_2 \quad (2)$$

From the equality (3)

$$m_1 \cdot c_1 \cdot d\vartheta_1 = -m_2 \cdot c_2 \cdot d\vartheta_2 \quad (3)$$

we define temperature change differentials in tanks determined by their masses and specific heat capacity of the substance, the differential for temperature change in a colder tank is equal to:

$$d\vartheta_1 = -\frac{m_2 \cdot c_2}{m_1 \cdot c_1} d\vartheta_2 \quad (4)$$

while the temperature change differential in a hotter tank is defined by:

$$d\vartheta_2 = -\frac{m_1 \cdot c_1}{m_2 \cdot c_2} d\vartheta_1 + \frac{Pdt}{m_2 \cdot c_2} \quad (5)$$

From the differential we get the temperature of the colder tank:

$$\vartheta_1 = \vartheta_{1p} + \int_0^t d\vartheta_1 = \vartheta_{1p} - \frac{m_2 \cdot c_2}{m_1 \cdot c_1} \int_0^t d\vartheta_2 \quad (6)$$

The temperature of the hotter tank is:

$$\vartheta_2 = \vartheta_{2p} + \frac{P}{m_2 \cdot c_2} \cdot \int_0^t dt - \frac{m_1 \cdot c_1}{m_2 \cdot c_2} \int_0^t d\vartheta_1 =$$

$$\vartheta_2 = \vartheta_{2p} + \frac{P}{m_2 \cdot c_2} \cdot t - \frac{m_1 \cdot c_1}{m_2 \cdot c_2} \int_0^t d\vartheta_1$$

$$\vartheta_2 = \vartheta_{2p} + \frac{P}{m_2 \cdot c_2} \cdot t + \int_0^t d\vartheta_2 \quad (7)$$

2.3. Derivation of the formula for a thermal solar collector

We then determine the general formula for the temperature of the thermal solar collector in the ideal process [3][4].

The differential equation for the temperatures of the heat collector is:

$$\frac{d\vartheta_2}{dt} = \frac{q_m \cdot c_3}{m_2 \cdot c_2}\left(\vartheta_1 - \vartheta_2\right) + \frac{P}{c_2 \cdot m_2} =$$

$$\frac{d\vartheta_2}{dt} = -\frac{q_m \cdot c_3}{m_2 \cdot c_2}\left(\vartheta_2 - \vartheta_1\right) + \frac{P}{c_2 \cdot m_2} \quad (8)$$

By substituting the temperatures ϑ_1 and ϑ_2 defined in (6) and (7), we get the differential equation for the temperature of the thermal solar collector expressed only by the variable higher temperature:

$$\frac{d\vartheta_2}{dt} = -\frac{q_m \cdot c_3}{m_2 \cdot c_2}\left[\vartheta_{2p} + \frac{P}{m_2 \cdot c_2} \cdot t + \int d\vartheta_2 - \left(\vartheta_{1p} - \frac{m_2 \cdot c_2}{m_1 \cdot c_1}\int d\vartheta_2\right)\right] + \frac{P}{m_2 \cdot c_2}$$

$$\frac{d\vartheta_2}{dt} = -\frac{q_m \cdot c_3}{m_2 \cdot c_2}\left[\vartheta_{2p} - \vartheta_{1p} + \frac{P}{m_2 \cdot c_2} \cdot t + \left(1 + \frac{m_2 \cdot c_2}{m_1 \cdot c_1}\right)\int d\vartheta_2\right] + \frac{P}{m_2 \cdot c_2}$$

$$\frac{d\vartheta_2}{dt} = -\frac{q_m \cdot c_3}{m_2 \cdot c_2}\left[\vartheta_{2p} - \vartheta_{1p} + \frac{P}{m_2 \cdot c_2} \cdot t + \left(\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\right)\int d\vartheta_2\right] + \frac{P}{m_2 \cdot c_2} \quad (9)$$

When we solve the integral we get:

$$\int d\vartheta_2 = \vartheta_2 + k_1 \quad (10)$$

Let's include it in the equation and get:

$$\boxed{\frac{d\vartheta_2}{dt} = -\frac{q_m \cdot c_3}{m_2 \cdot c_2}\left[\vartheta_{2p} - \vartheta_{1p} + \frac{P}{m_2 \cdot c_2} \cdot t + \left(\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\right)\left(\vartheta_2 + k_1\right)\right] + \frac{P}{m_2 \cdot c_2}}$$

$$\boxed{\begin{aligned}\frac{d\vartheta_2}{dt} &= -\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\right)\left(\vartheta_2 + k_1\right)\\ &-\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\vartheta_{2p}-\vartheta_{1p}\right)-\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\frac{P}{m_2 \cdot c_2}\cdot t+\frac{P}{m_2 \cdot c_2}\end{aligned}}$$

$$\boxed{\begin{aligned}\frac{d\vartheta_2}{dt} + \frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\right)\left(\vartheta_2 + k_1\right) &=\\ =-\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\vartheta_{2p}-\vartheta_{1p}\right)-\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\frac{P}{m_2 \cdot c_2}\cdot t+\frac{P}{m_2 \cdot c_2}\end{aligned}}$$

$$\boxed{\begin{aligned}&\frac{d\vartheta_2}{dt} + \frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\right)\cdot\vartheta_2 =\\ &=-\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\vartheta_{2p}-\vartheta_{1p}+\frac{P}{m_2 \cdot c_2}\cdot t+\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\cdot k_1\right)+\frac{P}{m_2 \cdot c_2}\end{aligned}}$$

$$\boxed{\begin{aligned}&\frac{d\vartheta_2}{dt} + \frac{q_m \cdot c_3 \cdot\left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2}\cdot\vartheta_2 =\\ &=-\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\vartheta_{2p}-\vartheta_{1p}+\frac{P}{m_2 \cdot c_2}\cdot t+\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\cdot k_1\right)+\frac{P}{m_2 \cdot c_2}\end{aligned}} \tag{11}$$

Using equation (12) to solve the 1st order differential equation

$$\frac{dy}{dx}+p(x)\cdot y=q(x) \tag{12}$$

From (11) we define polynomials *p(t)* (13), *q(t)* (14) and a polynomial *μ(t)* (15):+

$$p(t)=\frac{q_m \cdot c_3 \cdot\left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \tag{13}$$

$$q(t)=-\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\vartheta_{2p}-\vartheta_{1p}+\frac{P}{m_2 \cdot c_2}\cdot t+\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\cdot k_1\right)+\frac{P}{m_2 \cdot c_2} \tag{14}$$

$$\mu(t)=e^{\int p(t)dt}=e^{\int\frac{q_m \cdot c_3 \cdot\left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2}dt}= =e^{\frac{q_m \cdot c_3 \cdot\left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2}\cdot t} \tag{15}$$

we get the equation for the temperatures of the colder tank written in the form:

$$\vartheta_2=\frac{1}{\mu(t)}\cdot\left[\int q(t)\cdot\mu(t)\,dt+konst\right] \tag{16}$$

After including all polynomials, the first-order differential equation to be solved takes the form:

$$\boxed{\vartheta_2=\frac{1}{e^{\frac{q_m \cdot c_3 \cdot\left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2}\cdot t}}\cdot\left\{\int\left[-\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\vartheta_{2p}-\vartheta_{1p}+\frac{P}{m_2 \cdot c_2}\cdot t+\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\cdot k_1\right)+\frac{P}{m_2 \cdot c_2}\right]\cdot e^{\frac{q_m \cdot c_3 \cdot\left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2}\cdot t}dt+k_2\right\}}$$

$$\boxed{\vartheta_2=\frac{1}{e^{\frac{q_m \cdot c_3 \cdot\left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2}\cdot t}}\cdot\left\{\begin{aligned}&-\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\frac{P}{m_2 \cdot c_2}\cdot\int t\cdot e^{\frac{q_m \cdot c_3 \cdot\left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2}\cdot t}dt\\ &-\left[\frac{q_m \cdot c_3}{m_2 \cdot c_2}\cdot\left(\vartheta_{2p}-\vartheta_{1p}+\frac{m_1 \cdot c_1 + m_2 \cdot c_2}{m_1 \cdot c_1}\cdot k_1\right)+\frac{P}{m_2 \cdot c_2}\right]\cdot\int e^{\frac{q_m \cdot c_3 \cdot\left(m_1 \cdot c_1 + m_2 \cdot c_2\right)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2}\cdot t}dt+k_2\end{aligned}\right\}} \tag{17}$$

$$\boxed{\int e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}dt=\frac{m_1\cdot c_1\cdot m_2\cdot c_2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}+k_2}$$

One part within a first-order differential equation needs to be solved by partial integration

$$-\frac{q_m\cdot c_3}{m_2\cdot c_2}\cdot\frac{P}{m_2\cdot c_2}\cdot\int t\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}dt \tag{18}$$

Partial integration:

$$\int u(x)\cdot v'(x)dx=u(x)\cdot v(x)-\int v(x)\cdot u'(x)dx \tag{19}$$

We define *u(t) i v(t)*:

$$u=t \quad , \quad dv=e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}dt$$

$$du=dt \ , \quad v=\frac{m_1\cdot c_1\cdot m_2\cdot c_2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}$$

Then we solve the partial integral:

$$-\frac{q_m\cdot c_3}{m_2\cdot c_2}\cdot\frac{P}{m_2\cdot c_2}\cdot\int t\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}dt=$$

$$=-\frac{q_m\cdot c_3}{m_2\cdot c_2}\cdot\frac{P}{m_2\cdot c_2}\cdot\left|\begin{array}{l} t\cdot\dfrac{m_1\cdot c_1\cdot m_2\cdot c_2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t} \\ -\displaystyle\int\frac{m_1\cdot c_1\cdot m_2\cdot c_2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}dt \end{array}\right|=$$

$$=-\frac{q_m\cdot c_3}{m_2\cdot c_2}\cdot\frac{P}{m_2\cdot c_2}\cdot\left|\begin{array}{l} t\cdot\dfrac{m_1\cdot c_1\cdot m_2\cdot c_2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t} \\ -\dfrac{m_1^2\cdot c_1^2\cdot m_2^2\cdot c_2^2}{q_m^2\cdot c_3^2\cdot(m_1\cdot c_1+m_2\cdot c_2)^2}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t} \end{array}\right|=$$

$$=-\frac{P\cdot t\cdot m_1\cdot c_1}{m_2\cdot c_2\cdot(m_1\cdot c_1+m_2\cdot c_2)}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}+\frac{P\cdot m_1^2\cdot c_1^2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)^2}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t} \tag{20}$$

then we include the solution of partial integration in the equation and get:

$$\boxed{\vartheta_2=\frac{1}{e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}}\cdot\left|\begin{array}{l} -\dfrac{P\cdot t\cdot m_1\cdot c_1}{m_2\cdot c_2\cdot(m_1\cdot c_1+m_2\cdot c_2)}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}+\dfrac{P\cdot m_1^2\cdot c_1^2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)^2}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t} \\ -\left[\dfrac{q_m\cdot c_3}{m_2\cdot c_2}\cdot\left(\vartheta_{2p}-\vartheta_{1p}+\dfrac{m_1\cdot c_1+m_2\cdot c_2}{m_1\cdot c_1}\cdot k_1\right)+\dfrac{P}{m_2\cdot c_2}\right]\cdot\dfrac{m_1\cdot c_1\cdot m_2\cdot c_2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}\cdot e^{\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}+k_2 \end{array}\right|}$$

The solution of the differential equation for a heat collector finally has the form:

$$\boxed{\begin{array}{l}\vartheta_2=-\dfrac{P\cdot t\cdot m_1\cdot c_1}{m_2\cdot c_2\cdot(m_1\cdot c_1+m_2\cdot c_2)}+\dfrac{P\cdot m_1^2\cdot c_1^2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)^2} \\ \quad -\left[\dfrac{q_m\cdot c_3}{m_2\cdot c_2}\cdot\left(\vartheta_{2p}-\vartheta_{1p}+\dfrac{m_1\cdot c_1+m_2\cdot c_2}{m_1\cdot c_1}\cdot k_1\right)+\dfrac{P}{m_2\cdot c_2}\right]\cdot\dfrac{m_1\cdot c_1\cdot m_2\cdot c_2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}+k_2\cdot e^{-\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}\end{array}}$$

$$\boxed{\begin{array}{l}\vartheta_2=-\dfrac{P\cdot t\cdot m_1\cdot c_1}{m_2\cdot c_2\cdot(m_1\cdot c_1+m_2\cdot c_2)}+\dfrac{P\cdot m_1^2\cdot c_1^2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)^2}-\dfrac{P\cdot m_1\cdot c_1}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)} \\ \quad -\left(\vartheta_{2p}-\vartheta_{1p}+\dfrac{m_1\cdot c_1+m_2\cdot c_2}{m_1\cdot c_1}\cdot k_1\right)\cdot\dfrac{m_1\cdot c_1}{m_1\cdot c_1+m_2\cdot c_2}+k_2\cdot e^{-\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}\end{array}}$$

$$\boxed{\begin{array}{l}\vartheta_2=-\dfrac{P\cdot t\cdot m_1\cdot c_1}{m_2\cdot c_2\cdot(m_1\cdot c_1+m_2\cdot c_2)}+\dfrac{P\cdot m_1^2\cdot c_1^2}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)^2}-\dfrac{P\cdot m_1\cdot c_1}{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)} \\ \quad -\dfrac{m_1\cdot c_1\cdot(\vartheta_{2p}-\vartheta_{1p})}{m_1\cdot c_1+m_2\cdot c_2}-k_1+k_2\cdot e^{-\frac{q_m\cdot c_3\cdot(m_1\cdot c_1+m_2\cdot c_2)}{m_1\cdot c_1\cdot m_2\cdot c_2}\cdot t}\end{array}} \tag{21}$$

3 Calculation of the coefficient k_1 and k_2 using boundary conditions

It is necessary to determine the coefficients k_1 and k_2, using boundary conditions.

The first boundary condition is defined at the initial moment:

$$\text{for } t = 0 \Rightarrow \vartheta_2(0) = \vartheta_{2p} \tag{22}$$

The parts containing t are equal to zero, the temperature has an initial value ϑ_{2p}, and the exponential part containing t is equal to 1, after including that t = 0 in equation (22) we get the equation of the first boundary condition:

$$\boxed{\vartheta_{2p} = \frac{P \cdot m_1^2 \cdot c_1^2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} - \frac{P \cdot m_1 \cdot c_1}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} - \frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2} - k_1 + k_2} \tag{23}$$

If we single out the coefficients on the left side, then the equation has the form:

$$\boxed{-k_1 + k_2 = \vartheta_{2p} - \frac{P \cdot m_1^2 \cdot c_1^2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} + \frac{P \cdot m_1 \cdot c_1}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} + \frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2}} \tag{24}$$

If we take the finite boundary condition for $t = \infty$, we will define it so that we have a known solution and not an infinite temperature. We will do this using a special case so that the power of the solar collector is equal to P = 0, so we will know the final temperature in the ideal process equal to the temperature of the mixture which will be equal to:

$$\vartheta_2(\infty) = \frac{\vartheta_{1p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} \tag{25}$$

In the solution of the differential equation (22) when we include that $t = \infty$, the term containing $P=0$ is equal to 0, the exponential part is equal to zero, and with it the coefficient $k_2 = 0$:

$$\vartheta_2(\infty) = -\frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2} - k_1 \tag{26}$$

We include (25) on the left side of the equation, so the equation of the second boundary condition takes the form:

$$\frac{\vartheta_{1p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2} = -\frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2} - k_1 \tag{27}$$

Also, when the coefficient is separated to the left side we get:

$$k_1 = -\frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2} - \frac{\vartheta_{1p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2}$$

$$k_1 = -\frac{-\vartheta_{1p} \cdot m_1 \cdot c_1 + \vartheta_{1p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2}$$

$$k_1 = -\frac{\vartheta_{2p} \cdot m_1 \cdot c_1 + \vartheta_{2p} \cdot m_2 \cdot c_2}{m_1 \cdot c_1 + m_2 \cdot c_2}$$

$$k_1 = -\frac{\vartheta_{2p} \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 + m_2 \cdot c_2}$$

$$\boxed{k_1 = -\vartheta_{2p}} \tag{28}$$

As expected, the coefficient k_1 is equal to the initial temperature in the hotter tank, ϑ_{2p} and the negative sign is because it has $-k_1$ in the formula

From the two boundary conditions we have a system of two equations with two unknown variables:

$$-k_1 + k_2 = \vartheta_{2p} - \frac{P \cdot m_1^2 \cdot c_1^2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} + \frac{P \cdot m_1 \cdot c_1}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} + \frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2}$$

$$k_1 = -\vartheta_{2p}$$

When we solve the equation system we get the coefficients:

$$k_1 = -\vartheta_{2p} \tag{29}$$

$$k_2 = -\frac{P \cdot m_1^2 \cdot c_1^2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} + \frac{P \cdot m_1 \cdot c_1}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} + \frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2} \tag{30}$$

By including the coefficients, we obtain the final form of the formula for the temperature of the heat collector in the ideal process of heat transfer from a thermal solar collector of mass m_2 of specific heat c_2 to a heat tank of mass m_1 of specific heat capacity c_1, via heat exchanger with specific heat capacity c_3 and flow q_m:

$$\vartheta_2 = \vartheta_{2p} - \frac{P \cdot t \cdot m_1 \cdot c_1}{m_2 \cdot c_2 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} + \frac{P \cdot m_1^2 \cdot c_1^2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} - \frac{P \cdot m_1 \cdot c_1}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} - \frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2} + \left[- \frac{P \cdot m_1^2 \cdot c_1^2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} + \frac{P \cdot m_1 \cdot c_1}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} + \frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2} \right] \cdot e^{-\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t}$$

or

$$\vartheta_2 = \vartheta_{2p} - \frac{P \cdot t \cdot m_1 \cdot c_1}{m_2 \cdot c_2 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} + \frac{P \cdot m_1^2 \cdot c_1^2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} \cdot \left(1 - e^{-\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t}\right) - \frac{P \cdot m_1 \cdot c_1}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} \cdot \left(1 - e^{-\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t}\right) - \frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2} \cdot \left(1 - e^{-\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t}\right)$$

or

$$\vartheta_2 = \vartheta_{2p} - \frac{P \cdot m_1 \cdot c_1}{m_2 \cdot c_2 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} \cdot t + \left[\frac{P \cdot m_1^2 \cdot c_1^2}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)^2} - \frac{P \cdot m_1 \cdot c_1}{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)} - \frac{m_1 \cdot c_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 \cdot c_1 + m_2 \cdot c_2} \right] \cdot \left(1 - e^{-\frac{q_m \cdot c_3 \cdot (m_1 \cdot c_1 + m_2 \cdot c_2)}{m_1 \cdot c_1 \cdot m_2 \cdot c_2} \cdot t}\right) \quad (31)$$

If we assume that each part of the system has the same type of liquid, and thus the same specific heat capacity, we obtain:

$$\vartheta_2 = \vartheta_{2p} - \frac{P \cdot m_1}{m_2 \cdot c \cdot (m_1 + m_2)} \cdot t + \left[\frac{P \cdot m_1^2}{q_m \cdot c \cdot (m_1 + m_2)^2} - \frac{P \cdot m_1}{q_m \cdot c \cdot (m_1 + m_2)} - \frac{m_1 \cdot (\vartheta_{2p} - \vartheta_{1p})}{m_1 + m_2} \right] \cdot \left(1 - e^{-\frac{q_m \cdot (m_1 + m_2)}{m_1 \cdot m_2} \cdot t}\right) \quad (32)$$

The time constant of the system in which each part of the system may be of another substance is:

$$\tau_0 = \frac{c_1 m_1 c_2 m_2}{q_m c_3 (c_1 m_1 + c_2 m_2)} \quad (33)$$

And the time constant of a system made of the same heat medium in each part of the system, ie. equal specific heat capacities $c = c_1 = c_2 = c_3$ is equal to:

$$\tau_0 = \frac{m_1 m_2}{q_m (m_1 + m_2)} \quad (34)$$

4 Conclusion

With introducing real coefficients to derived formula in the time domain for the temperature of the thermal solar collector of ideal process we can describe the losses in the system. With this addition the derived formula describes the actual real process. This formula makes it possible to predict the temperature of a thermal solar collector and allows us to predict the moment when the medium in the thermal solar collector will reach the evaporation temperature.

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