Studies regarding the ways of estimation the reliability of cutting tools and handling stress constraints

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Abstract: The present paper underlines the necessity of the cutting tools producers warranty about tools reliability and propose also the ways of estimation for such of indicators. In the paper is also proposed to complete the existing standards with requirements about the reliability testing conditions and the methods of testing data proceeding are presented.

Key-Words: cutting tools, reliability, testing conditions, quality standard.

1 Introduction

The present cutting tools quality standards are requiring tests about the tool behavior under prescribed work conditions as well as prescribed cutting trail. After the cutting test, the tools must agree with the technical quality conditions previously required by the standards and they have to preserve their former cutting qualities.

Such a testing does not permit to point out the measure in which the tools maintain their cutting capacities in time and therefore does not offer an objective indicator able to compare tools from different purveyors. The use of the *tools reliability notion* could be a solution, because it represents the property of tools to keep their capabilities in time, under prescribed cutting conditions.

2 Standard reliability requirements for cutting tools

By definition, the cutting tool reliability is the probability than the cutting duration till a prescribed criterion of failure would be reached be greater than a prescribed time, t:

$$R(t) = \Pr ob\{T > t\} \quad (1)$$

Using for cutting tools the same standard about reliability indicators as for industrial products, the tools reliability level may be expressed by following indicators:

- reliability function R(t);
- failure intensity z(t);
- probability density f(t);
- mean time between failures MTBF.

Between these indicators there exist the following relationships:

$$f(t) = -\frac{dR(t)}{dt}, \quad (2)$$

$$z(t) = \frac{f(t)}{R(t)},$$

$$R(t) = \exp\left[-\int_0^\infty z(t)dt\right], \quad (4)$$

$$MTBF = \int_0^\infty tf(t)dt. \quad (5)$$

and their estimation must be done on basis of cutting time till the failure criterion will be reached.

A very suggestive indicator of the tools reliability may be the *durability,* which is the cutting time under prescribed conditions, till the failure criterion will be reached within a prescribed probability. In order to raise the degree of certainty of the supplied value of the durability, one can impose a greater value for the probability, approaching the unity, let us say 0.95.

So, for an imposed R(t)=0.95, we can determine from relation (4) the value T of durability which is concordant with the condition (1). In other words, the tool will have durability $T_{ef} \geq T$, with a probability of 0.95.

In order to warrant the tools reliability, we must add to the present standards requirements about the necessary reliability testing made on batches of tools. With every tool from the tested batches the cutting will be proceeded until the failure criterion will be reached; in such a way will be determined the TBF for every tested sample.

3 Reliability indicators estimation for cutting tools

The testing data obtained for every tool batch can be processed, in order to get reliability indicator estimation, using parametrical and non-parametrical methods.

The local reliability indicators estimation using non-parametrical methods is made on the basis of following relations:

$$\hat{f}(t, t+\Delta t) = \frac{n(t) - n(t+\Delta t)}{\Delta t \cdot n}, \qquad (6)$$

$$\hat{R}(t) = \frac{n(t) - r}{n(t)}, \qquad (7)$$

$$\hat{z}(t, t+\Delta t) = \frac{n(t) - n(t+\Delta t)}{\Delta t \cdot n(t)}, \qquad (8)$$

$$MTBF = \frac{1}{n} \cdot \sum_{i=1}^{n} t_i. \qquad (9)$$

where: n is the number of samples in the batch; n(t) is the number of tools in good state at the moment t; r is number of observed failures; t_i is the moment of appearance of the “i” failure; Δt is a convenient time interval.

The estimated indicators obtained with relations (6)...(9) are characterizing the tested “population”, but cannot be extended over time intervals grater than the testing duration. In order to overcome this trouble, we must use the parametrical methods, which need a presumed repartition of TBF.

In some previous papers [1],[2],[3] we showed that bi-parametrical Weibull repartition is quite suitable to describe the HSS tools reliability. So, the testing data processing may be done presuming for the studied batch a Weibull behavior.

In consequence, the probability density function will be:

$$f(t) = \frac{\beta}{\eta^{\beta}} t^{\beta-1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right],$$

and the reliability function will be:

$$R(t) = \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]. \qquad (11)$$

where the parameters β and η can be found using analytical methods (maximum likelihood method, least square method or moments method).

Among them, the moment’s method is the most rapid. It is supposing to equalize the theoretical moments of “1” and “2’ order with the analogous moments deduced from the testing data basis. In this aim [4], we shall determine for the tested batch the mean:

$$\bar{t} = \frac{1}{n} \sum_{i=1}^{n} t_i,$$

and the standard deviation:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (t_i - \bar{t})^2}. \qquad (13)$$

With these values, we calculate the variation coefficient:

$$c_v = \frac{s}{\bar{t}}$$

which allows to find the parameter β (see table 2.2 in [4]) and the η parameter will be then:

$$\eta = \frac{\bar{t}}{\Gamma\left(\frac{1}{\beta} + 1\right)}. \qquad (15)$$

Now, having settled these parameters, the reliability for any time may be estimated with relation (1) and mean time between failures will be:

$$MTBF = \eta \Gamma\left(1 + \frac{1}{\beta}\right). \qquad (16)$$

In order to confirm the method proposed here and to prove that requirements about reliability certification of cutting tools may be introduced in the quality standard of the last one, we present in that follows our results obtained with HSS cylindrical cutters of 63 mm diameter.

A batch of 12 cutters STAS 578-76 was tested, machining test-pieces of OLC45 with 190HB hardness. The machining conditions were: s_d= 0.05 mm/teeth, depth t=6mm, cutting speed v=30 m/min, regular cooling emulsion. The test was stopped when the flank wear reached 0.5 mm.

The obtained data for TBF (in minutes) were: 80, 110, 112, 120, 130, .140, .155,.162, 180, 190, 200, 234. After that, we get from (12)... (14):

c_v= 0.29397,

and from [4]

β=3.8; η=167

In consequence, the probability density has the form:

$$f(t) = 1{,}36 \cdot 10^{-8} \cdot t^{2,8} \exp\left[-\left(\frac{t}{167}\right)^{3,8}\right].$$

and mean value of TBF was 150 minutes.

4 Optimization problem

The most common structural design problem requires finding the optimal topology of a structure, such that it can support the applied loads using a minimum amount of material. For a linear elastic structure built of a material with ultimate strength σ_l, the material distribution problem can be written as [4]

$$\left.\begin{array}{l} \min\limits_{0\le\rho(x)\le 1} \int_\Omega \rho(x)dx \\ \\ \text{subjectto}: \sigma_{VM}(\rho(x),x) \le \sigma_l \;\text{ if }\; \rho(x) > 0 \end{array}\right\} \quad (1)$$

where ρ *(x)* is the material density at point *x* and σ_{VM} *(*ρ *(x), x)* is the von Misses stress at point *x*. The design problem is usually solved by discrediting the design domain with a number N of finite elements, using the shape density function as design variable, as in compliance design based optimization problems. Due to the uneven contours and convergence problems induced by the classical 0/ 1 integer problem, authors used several methods to transform it in a continuous one Bendsoe and Kikuchi (homogenization method) [1], Allaire and Kohn (penalty exponential function). This allows the variable ρ to take intermediate values and use of sensitivity analysis and numerical methods to solve the problem.

5 The numerical model

The modelling of material properties in elements with intermediate densities is based on the power-law approach (SIMP method), which introduce a penalty exponential factor (η > 1) for the basic physical and mechanical properties of the material quantified with the design variable ρ

$$E^* = \rho^\eta E^0, \quad (2)$$

where "*" denotes an effective or overall value and "O" indicates the value for solid material. The use of η allows obtaining more accurate solution in terms of void/ fully dense (solid) material. The most efficient value for η is 3 [1], [4].

For the stress, a model of the strength properties is given by a similar power law with the exponent η. This allows reformulating the failure criterion [4] (17)

$$\frac{\sigma^*_{VM}}{\rho^\eta} = \sigma_1. \quad (3)$$

6 Stress criteria

Stress constraints induce the "singularity phenomenon" [3]. Low density regions sometimes remain highly strained. When the density decreases to zero in these regions, the limit of the stress state in the microstructure tends to a value, higher than the stress limit. The procedure cannot remove this region, despite the fact that, removing totally the material, the stress constraint would not be active. To circumvent this situation, Cheng and Guo replaced the solution of the singular problem with a sequence of perturbed non-singular problems which can be solved with usual optimization algorithms: the ε relaxation method. This method, initially developed for truss optimization problems, does not cover the continuum type topology optimization problems. The reduction of the perturbation parameter ε leads to constraint violations and slows down the convergence of the procedure. Duysinix and Sigmund [4] proposed a set of perturbed constraints, similar to the original relaxation technique

$$\left.\begin{aligned} &\rho\left(\frac{\sigma^*_{VM}}{\rho^\eta \sigma_1}-1\right) \le \varepsilon - \varepsilon\rho \\ &1 \ge \rho \ge \varepsilon^2 \end{aligned}\right\}. \tag{4}$$

The perturbation vanishes for $\rho = 1$ so that the solution remains feasible when ε is reduced. The permissible stress is increased for low densities, as shown by the rewritten form of equation (4)

$$\frac{\sigma^*_{VM}}{\rho^\eta} \le \sigma_1 (1-\varepsilon+\frac{\varepsilon}{\rho}). \tag{5}$$

The permissible stress is increased for low densities which give the possibility to create or remove holes without violating the stress constraint.

To control the local stress state, the stress is treated as a local constraint, for every finite element, as Duysinix and Bendsoe did. Due to the significant increase of the optimization problem induced by this approach, Duysinix and Sigmund proposed to include both the ε – relaxation technique that alleviates the singularity phenomena and the use of effective stress criterion into the global stress constraint. There are two global measures of the relaxed distributed stress criterion (5).

The first global measure is the "p-norm" of the relaxed stress criterion

$$\left[\sum_{e=1}^{N}\left(\max\left\{0, \frac{\sigma^*_{VM,e}}{\rho_e^\eta \sigma_1}+\varepsilon-\frac{\varepsilon}{\rho_e}\right\}\right)^p\right]^{\frac{1}{p}} \le 1. \tag{6}$$

The second global stress constraint is the "p- mean" of the relaxed stress criterion

$$\left[\frac{1}{N}\sum_{e=1}^{N}\left(\max\left\{0, \frac{\sigma^*_{VM,e}}{\rho_e^\eta \sigma_1}+\varepsilon-\frac{\varepsilon}{\rho_e}\right\}\right)^p\right]^{\frac{1}{p}} \le 1. \tag{7}$$

For a given *p*, the maximum stress value is always bounded from above by the p-norm and from below by the p-mean. Negative values of the relaxed criterion only appear for low stressed elements which can be truncated without influencing the global constraint, which remains continuous up to the *p-1* derivative – smooth enough. From the numerical experiments of Duysinix and Sigmund, the choice of parameter *p* must result from a compromise of the high values for the control the maximum value of the stress criterion and the p-norm and p-mean ill-conditioning when *p* increases. Good results were obtained with *p* = 4 [4].

5 Conclusions

The use of integrated equivalent stress constraints proposed by Duysinix and Sigmund proves an alternative to the use of local stress constraints for continuum-type structures. A theoretical study of the two integrated constraints shows that they bound the maximum value of the criterion, which is the limit value of the two p-norms and p-means as p grows to infinity. The p-mean function converges by lower value towards the infinite mean while the p-norm provides upper bounds to the maximum stress criterion. There is no relation identified yet between the p-norms and p-means to the maximum value and the number of elements considered in the constraint. With only one integrated constraint function is difficult to control the large number of elements which are close to the admissible stress. The convergence process becomes oscillatory and the constraint violation increases. For practical applications the parameter *p* was taken to 4 in order to consider a large influence of all active local stress criteria in the global constraints.

The producer guaranty about the reliability indicators of the cutting tools raises the customers' reliance and offers an advantage against the concurrence. Therefore, the setting of a unitary method for reliability indicators estimation is a very actual and timely attempt.

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