An Idea Calculus for Artificial and Natural Intelligence

IOACHIM DRUGUS
Institute of Mathematics and Computer Science
Moldova State University
Alexei Mateevici Str. 60, Md-2009
REPUBLIC OF MOLDOVA

Abstract: - The author of the current paper proposes an axiomatic *idea calculus* (IC) for complex ideas in the sense of philosopher John Locke. The IC can help both in modeling the workings of the human mind for research purposes and in implementing in software the mechanism of mind. In machine learning the use of IC can shift the paradigm towards object-orientation. The IC is based on the *idea theory* introduced earlier by the author.

Key-Words: - idea theory, association, aggregation, atomification, set theory, calculus

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1 Introduction

Current research started with a paper [1] describing a virtual machine capable of modeling the mental content by treating it as built by three fundamental mental operations: *association*, *aggregation, atomification.* The approach to data prescribing to treat any unit of data as built by these operations obtained the name "A3 approach" and later (as in [2]) the structures built in this manner obtained the name "deep structures" to invoke the idea that these are *refined* mathematical structures and can be used in both deep learning and in linguistics. In [2], the author of this paper outlined a *theory of ideas*, or *idea theory* by pattern 'set theory', building upon philosopher John Locke's vision on ideas [3]. The idea theory presents to a certain extent rigorously the "*objects* of thought" named *ideas* by John Locke.

Whereas thinking is a *continuous* physical underlying process, John Locke proposed for a higher level of mind, a *discrete* approach which can be described as *object-oriented* treatment. He describes *understanding* as a level higher than reflection and treats ideas as forming at the level of understanding. From a linguistic perspective an idea is what is *expressed* by a name or a description.

Object orientation is a paradigm where the objects are usually classified, i.e. attributed to different classes. However, even though machine learning as an engineering discipline is preoccupied exactly by classification, this discipline cannot be described as object oriented (to use John Locke's term, currently machine learning algorithms work without understanding*)*. A way to raise machine learning to the level of object orientation is by treating the *patterns* of machine learning as ideas treated here.

2 Problem Formulation

Whereas idea theory of [2] offers a more precise treatment of ideas than any informal approach, this theory has no formalization and axiomatization convenient for both discourse and computing. An idea theory which would fit best for computing should be one in the form of a calculus. Specifying an axiomatic calculus of ideas is the aim of the current paper.

3 Problem Solution

An *idea calculus* (IC) sought in the problem will be introduced in this section in the three subsections coming next.

3.1 The mathematical structure of an idea

An idea is a mental object $\boldsymbol{i}$ which represents other objects. The multitude of these *other* objects is treated here as class $\boldsymbol{E}$, a mathematical structure originating in set theory and referenced here as *extension* of the idea $\boldsymbol{i}$*.* The ordered pair $\boldsymbol{I} = (\boldsymbol{i}, \boldsymbol{E})$ is treated here as the mathematical structure of idea $\boldsymbol{i}$. An idea $\boldsymbol{i}$ represents in mind at a certain moment exactly one object $\boldsymbol{e} \in \boldsymbol{E}$, and the extension $\boldsymbol{E}$ is the class of objects which can be represented by $\boldsymbol{i}$. It sounds natural to map ideas to classes and treat this mapping as an operator e***xt****, so* in the context of notations above, ext (***i***) = ***E***. This operator reflects how the ideas relate to classes.

The extension of an idea is treated as a class, and not as a set, since there are no reasons for claiming this multitude to be a set. A set has certain cardinality, whereas a proper class has no cardinality. However, a class and a set do not differ from each other structurally. Thus, a weak set theory focused on structure rather than cardinality is sufficiently rich for a rigorous treatment of ideas.

Different ideas may have the same extension and as an example of this can serve two ideas expressed by two names, *morning star* and *evening star*, both of which have exactly one thing in their extension, the planet *Venus*. One can say that any idea theory refutes the properly formulated is an idea specific language the set-theoretic extensionality axiom:

$$\forall z(z \in A \leftrightarrow z \in B) \to (A = B).$$

Thus, an axiomatic idea theory can be obtained by discarding the extensionality axiom from the list of axioms of an axiomatic set theory. The axiomatic set theory in algebraic form [4] *without* the extensionality axiom is proposed in this paper as the *idea calculus* sought in the problem.

3.2 About an idea-theoretic language

The symbolic notations of [4] are symbols of operations in the singature of a universal algebra, which is a generalized Boolean algebra (GBA) equipped with two unary operators:

(a) the operator '°' ("grade") which maps any object $\boldsymbol{x}$ onto the one-element set $\{x\}$, singleton;

(b) the unary operator '⋃' ("big union") which is intended to correspond to the set theoretic operation defined like this:

$$\bigcup x = \{y \mid \exists z\, (y \in z)\}.$$

The GBAs are Boolean algebras with an optional top element, and these can be axiomatized in various manners by using different operations. Two binary operations, denoted as '\' ("set minus') and '∪' ("union"), have been used in [4] for this purpose.

Here are several symbols defined through symbols in the signature:

$$x \subseteq y \leftrightarrow x \cup y = y,$$
$$x \in y \leftrightarrow x^{\circ} \subseteq y,$$
$$x \cap y = x \setminus (x \setminus y) = y \setminus (y \setminus x).$$

The first two defined symbols are used in the axioms below to make the meaning of the axioms clearer.

The signature used in [4] is proposed here to be taken over to the idea calculus, which is a formal system. However, it is not easy to find the informal names to refer to the application of these operations and the attempt to resolve this linguistic issue has not been made here.

3.2 Axioms of the idea calculus

The axioms of the IC break into three groups: axioms (1 - 9) axiomatize the class of GBAs, axiom (10) characterizes singletons, (11-12) characterizes big unions as least upper bounds.

$$a \cup (b \cup c)) = (a \cup b) \cup c, \tag{1}$$
$$a \cup b = b \cup a, \tag{2}$$
$$a \cup a = a, \tag{3}$$
$$(a \cup b) \setminus c = (a \setminus c) \cup (b \setminus c), \tag{4}$$
$$a \setminus (b \cup c) = (a \setminus b) \setminus c, \tag{5}$$
$$a \cup (b \setminus a) = a \cup b \tag{6}$$
$$a \cup (a \setminus b) = a \tag{7}$$
$$(a \setminus b) \setminus c = (a \setminus c) \setminus (b \setminus c) \tag{8}$$
$$(a \setminus b) \setminus c = (a \setminus b) \cup (a(a \setminus c)) \tag{9}$$
$$a^{\circ} = b^{\circ} \to a = b, \tag{10}$$
$$b \in a \to b \subseteq \bigcup a \tag{11}$$
$$(\forall b \in a)(b \subseteq c) \to \bigcup a \subseteq c. \tag{12}$$

4 Conclusion

The IC is a weak theory of ideas, possibly the weakest among possible, but since IC is built in the image of a set theory calculus, and set theory is a foundation for mathematics, there are chances that IC can serve as a useful framework for discourse and formal generation of ideas.

References:

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