Properties of Q-Intuitionistic L-Fuzzy Lower$Q$-Level Subset of ℓ-Subsemiring of an ℓ-Semiring

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Abstract: - In this paper, the idea of a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring is introduced. We made an effort to learn more about the algebraic nature of ℓ-semiring. In addition, several findings about the characteristics of lower Q-level subsets of a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring are developed. We also explored the fundamental theorem for homomorphism and anti-homomorphism.

Key-Words: -Fuzzy subset,(Q, L)-fuzzy subset, (Q, L)-fuzzy ℓ-subsemiring, Q-intuitionistic L-fuzzy subset, Q-intuitionistic L-fuzzy ℓ-subsemiring, Q-intuitionistic L-fuzzy relation, strongest Q-intuitionistic L-fuzzy relation, Product of Q-intuitionistic L-fuzzy subsets, Lower Q-level subset Q-intuitionistic L-fuzzy subsets.

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1 Introduction

Numerous researchers looked into the concept of fuzzy sets' generalisation after L.A. Zadeh presented it [30]. The term "lattice" was first used by Dedekind in 1897, and it was later expanded upon by Birkhofft, G. [8,9]. A special kind of lattice known as a Boolean ring with identity was similar to the Boolean algebra that Boole invented. This relationship led to the establishment of the link between lattice theory and contemporary algebra. K.T. Atanassov [5,6] developed the idea of intuitionistic fuzzy subset as a generalisation of the idea of fuzzy set. Q-fuzzy subgroups is a brand-new algebraic structure that was developed and defined by A. Solairaju and R. Nagarajan [26,27]. In this article, we discuss some of the characteristics of lower Q-level subsets of a Q-intuitionistic L-fuzzy ℓ-semiring.

The heading of each section should be printed in small, 14pt, left justified, bold, Times New Roman. You must use numbers 1, 2, 3, … for the sections' numbering and not Latin numbering (I, II, III, …)

2 Preliminaries

Definition 2.1. [31]Let X be a collection of set that isn't empty. A function$\mu_A: X \to [0, 1]$ a **fuzzy subset** μ_A of X.

Definition 2.2.[27, 28]Let X be a non-empty set, $L = (L, \leq)$ is a lattice with least member 0 and largest element 1, and Q is a non-empty collection. A (Q, L)**-fuzzy subset** μ_A of a function$\mu_A: X \times Q \to L$.

Definition 1.3.[20,21]Let R be a ℓ-semiring and Q to be a set that isn't empty. A (Q, L)-fuzzy subset A of R is referred to as a (Q, L)**-fuzzy ℓ-subsemiring (QLFLSSR)** of R if it meets the following criteria:

(i) $\mu_A (x + y, q) \geq \mu_A (x, q) \wedge \mu_A (y, q)$,
(ii) $\mu_A (xy, q) \geq \mu_A (x, q) \wedge \mu_A (y, q)$,
(iii) $\mu_A (x \vee y, q) \geq \mu_A (x, q) \wedge \mu_A (y, q)$,
(iv) $\mu_A (x \wedge y, q) \geq \mu_A (x, q) \wedge \mu_A (y, q)$, for every x and y in R and q in Q.

Example 2.1. Let $(Z , + , \square, \vee, \wedge)$ be a ℓ-semiring and $Q = \{p\}$, Then the (Q, L)-fuzzy set A of Z is defined by

$$A(x, q) = \begin{cases} 1 & if \quad x = 0 \\ 0.33 & if \quad x \epsilon < 2 > -0 \\ 0 & if \quad x \epsilon\, Z - < 2 > \end{cases}$$

A is unmistakably a (Q, L)-Fuzzy ℓ-subsemiring of a ℓ-semiring.

Definition 2.4.[5,6]An **intuitionistic fuzzy subset (IFS)**A in X is defined as an object of the form $A = \{< x, A_\mu(x), A_\vartheta(x) >/x \in X\}$, where $A_\mu: X \to [0,1]$ and $A_\vartheta: X \to [0,1]$ define the degree of membership and the degree of non-membership of the element

$x \in X$ respectively and for every $x \in X$ satisfying $0 \leq A_\mu(x) + A_\vartheta(x) \leq 1$.

Definition2.5.[24]Let $(L, \leq)$ be a complete lattice with an involutive order reversing operation $N : L \to L$ and Q be a set that isn't empty. A Q-**intuitionistic L-fuzzy subset (QILFS)**A in X is defined as an object of the form $A = \{< (x,q), A_\mu(x,q), A_\vartheta(x,q) > / x$ in X and q in Q$\}$, where $A_\mu: X \times Q \to L$ and $A_\vartheta: X \times Q \to L$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $A_\mu(x) \leq N(A_\vartheta(x))$.

Definition 2.6.[22]Let R be a ℓ-semiring. A Q-intuitionistic L-fuzzy subset A of R is referred to as a Q-**intuitionistic L-fuzzy ℓ-subsemiring (QILFLSSR)** of R if it meets the following criteria:

(i) $A_\mu(x + y, q) \geq A_\mu(x, q) \wedge A_\mu(y, q)$,
(ii) $A_\mu(xy, q) \geq A_\mu(x, q) \wedge A_\mu(y, q)$,
(iii) $A_\mu(x \vee y, q) \geq A_\mu(x, q) \wedge A_\mu(y, q)$,
(iv) $A_\mu(x \wedge y, q) \geq A_\mu(x, q) \wedge A_\mu(y, q)$,
(v) $A_\vartheta(x + y, q) \leq A_\vartheta(x, q) \vee A_\vartheta(y, q)$,
(vi) $A_\vartheta(xy, q) \leq A_\vartheta(x, q) \vee A_\vartheta(y, q)$,
(vii) $A_\vartheta(x \vee y, q) \leq A_\vartheta(x, q) \vee A_\vartheta(y, q)$,
(viii) $A_\vartheta(x \wedge y, q) \leq A_\vartheta(x, q) \vee A_\vartheta(y, q)$, for every x and $y \in R$and $q \in Q$.

Example 2.2. Let $(Z, +, ⊡, \vee, \wedge)$ be a ℓ-semiring and Q $= \{p\}$, Then Q-intuitionistic L-Fuzzy subset $A = \{< (x,q), A_\mu(x,q), A_\vartheta(x,q) > / x$ in Z and q in $Q\}$ of Z is defined by

$$A_\mu(x,q) = \begin{cases} 0.6 \; if \; x \in <2> \\ 0.3 \; otherwise \end{cases}$$

and

$$A_\vartheta(x,q) = \begin{cases} 0.4 \; if \; x \in <2> \\ 0.7 \; otherwise \end{cases}$$

A is unmistakably a Q -intuitionistic L-Fuzzy ℓ-subsemiring of a ℓ-semiring.

Definition 2.7. Let A and B represent any two Q–intuitionistic ℓ-subsemiring of a ℓ-semiring G and H, respectively. The product of A and B, designated by $A \times B$, is defined as $A \times B = \{< (x,y), q), (A \times B)_\mu((x,y), q), (A \times B)_\vartheta((x,y), q) > /$ for every x in G and y in $H$and q in $Q\}$, where$(A \times B)_\mu((x,y), q) = A_\mu(x,q) \wedge B_\mu(y,q)$ and $(A \times B)_\vartheta((x,y), q) = A_\vartheta(x,q) \vee B_\vartheta(y,q)$.

Definition 2.8. Let A be a Q-intuitionistic L-fuzzy subset in a set Ş, and the strongest Q-intuitionistic L-fuzzy relation on Ş, that is a Q-intuitionistic L-fuzzy relation on A is V which is provided by $V_\mu((x,y), q) = A_\mu(x,q) \wedge A_\mu(y,q)$ and $V_\vartheta((x,y), q) = A_\vartheta(x,q) \vee A_\vartheta(y,q)$, for every x and y in Ş and q in Q.

Definition 2.9. Let $(R, +, ⊡, \vee, \wedge)$ and $(R', +, ⊡, \vee, \wedge)$ to be any two semirings. Let $f: R \to R'$be any function and A be a Q-intuitionistic L-fuzzy ℓ-subsemiring in R, V be an Q-intuitionistic L-fuzzy ℓ-subsemiring in $f(R) = R'$, defined by $V_\mu(y,q) = \sup_{x \in f^{-1}(y)} A_\mu(x,q)$ and $V_\vartheta(y,q) = \inf_{x \in f^{-1}(y)} A_\vartheta(x,q)$, for every x in R and y in R' and q in the Q. Then A is known as a V preimage under f and is indicated by $f^{-1}(V)$.

Definition2.10. Let A and B be any two Q-intuitionistic L-fuzzy subsets of a set X. We define the following operations:

(i) $A \cap B = \{< x, A_\mu(x,q) \wedge B_\mu(x,q), A_\vartheta(x,q) \vee B_\vartheta(x,q) >\}$, for all $x \in X$and q in Q.
(ii) $A \cup B = \{< x, A_\mu(x,q) \vee B_\mu(x,q), A_\vartheta(x,q) \wedge B_\vartheta(x,q) >\}$, for all $x \in X$and q in Q.
(iii) $\square A = \{< x, A_\mu(x,q), 1 - A_\mu(x,q) > / x \in X\}$, for all x in $X$and q in Q.
(iv) $\lozenge A = \{< x, 1 - A_\vartheta(x,q), A_\vartheta(x,q) > / x \in X\}$, for all x in $X$and q in Q.

Definition 2.11. Let A be an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring $(R, +, ⊡, \vee, \wedge)$ and a in R. Then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is defined by $\left((aA_\mu)^p\right)(x,q) = p(a)A_\mu(x,q)$ and $((aA_\vartheta)^p)(x,q) = p(a)A_\vartheta(x,q)$, for every x in R and for some p in P and q in Q.

Example 2.3. Let $(Z_5, +, ⊡, \vee, \wedge)$ to be a ℓ-semiring. Then Q-intuitionistic L-fuzzy subset $A = \{((x,q), A_\mu(x,q), A_\vartheta(x,q))/x \in Z_5$ and q in $Q\}$ of Z_5, where

$$A_\mu(x) = \begin{cases} 0.59 & \text{if } x = 0 \\ 0.28 & \text{if } x = 1,2,3,4 \end{cases}$$

and

$$A_\vartheta(x) = \begin{cases} 0.24 & \text{if } x = 0 \\ 0.38 & \text{if } x = 1,2,3,4 \end{cases}$$

without a doubt, A is a Q-Intuitionistic L-fuzzy ℓ-subsemiring.

Take $p(a) = 0.2$ for each a in Z_5.

Then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is defined by

$$A_\mu(x) = \begin{cases} 0.12 & \text{if } x = 0 \\ 0.01 & \text{if } x = 1,2,3,4 \end{cases}$$

and

$$A_\vartheta(x) = \begin{cases} 0.5 & \text{if } x = 0 \\ 0.08 & \text{if } x = 1,2,3,4 \end{cases}$$

without a doubt, $(aA)^p$ is a Q-Intuitionistic L-fuzzy ℓ-subsemiring.

Definition 2.12. Assume A is a Q-intuitionistic L-fuzzy subset of X. For α, β in L, the lower Q-level subset of A is the set $A_{(\alpha,\beta)} = \{x \in X: A_\mu(x,q) \geq \alpha, A_\vartheta(x,q) \leq \beta\}$.

Definition 2.13. Let say R be a ℓ-semiring. A Q-intuitionistic L-fuzzy subset A of Ʀ is referred to as

a Q-**intuitionistic** L-**fuzzy normal** ℓ-**subsemiring (QILFNLSSR)** of R if it meets the following criteria:
(i) $A_\mu(x+y,q) = A_\mu(y+x,q)$,
(ii) $A_\mu(xy,q) = A_\mu(yx,q)$,
(iii) $A_\mu(x \vee y,q) = A_\mu(y \vee x,q)$,
(iv) $A_\mu(x \wedge y,q) = A_\mu(y \wedge x,q)$,
(v) $A_\vartheta(x+y,q) = A_\vartheta(y+x,q)$,
(vi) $A_\vartheta(xy,q) = A_\vartheta(yx,q)$,
(vii) $A_\vartheta(x \vee y,q) = A_\vartheta(y \vee x,q)$,
(viii) $A_\vartheta(x \wedge y,q) = A_\vartheta(y \wedge x,q)$, for every x and $y \in R$ and $q \in Q$.

Example 2.4. Let L say be the complete lattice and $A: Z \to L$ be an Q-intuitionistic L-fuzzy subset $A = \{< (x,q), A_\mu(x,q), A_\vartheta(x,q) > / \, x \in X, q$ in $Q\}$defined as

$$A_\mu(x,q) = \begin{cases} 0.79 & if \quad x =< 4 > \\ 0.31 & if \quad x \in < 2 > - < 4 > \\ 1 & if \quad otherwise \end{cases}$$

and

$$A_\vartheta(x,q) = \begin{cases} 0.27 & if \quad x =< 4 > \\ 0.75 & if \quad x \in < 2 > - < 4 > \\ 1 & if \quad otherwise \end{cases}$$

A is unmistakably a Q-intuitionistic L-fuzzy normalℓ-subsemiring.

3 The Properties of Q-Intuitionistic L-Fuzzy ℓ-Subsemiring of a ℓ-Semiring in Homomorphism and Anti-Homomorphism

Theorem3.1. Let $(R,+,\cdot,\vee,\wedge)$ and $(R',+,\cdot,\vee,\wedge)$ to be any two ℓ-semirings, and Q to be a non-empty set. The homomorphic image of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.

Proof: Let $(R,+,\cdot,\vee,\wedge)$ and $(R',+,\cdot,\vee,\wedge)$ to be any two ℓ-semirings Q to be a non-empty set. Let $f: R \to R'$ be a ℓ-semiring homomorphism. Then
(i) $f(x+y) = f(x) + f(y)$,
(ii) $f(xy) = f(x)f(y)$,
(iii) $f(x \vee y) = f(x) \vee f(y)$ and
(iv) $f(x \wedge y) = f(x) \wedge f(y)$, for all x and y in R.

Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of R. We have to prove that V is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.

Now, for $f(x), f(y)$ in $R'$and q in Q,
(i) $V_\mu(f(x)+f(y),q) = V_\mu(f(x+y),q) \geq A_\mu(x+y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$ which implies that $V_\mu(f(x)+f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q))$.
(ii) $V_\mu(f(x)f(y),q) = V_\mu(f(xy),q) \geq A_\mu(xy,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$ which implies that $V_\mu(f(x)f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q))$.
(iii) $V_\mu(f(x) \vee f(y),q) = V_\mu(f(x \vee y),q) \geq A_\mu(x \vee y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$ which implies that $V_\mu(f(x) \vee f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q))$.
(iv) $V_\mu(f(x) \wedge f(y),q) = V_\mu(f(x \wedge y),q) \geq A_\mu(x \wedge y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$ which implies that $V_\mu(f(x) \wedge f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q))$.Now, for $f(x), f(y)$ in R' and q in Q.
(v) $V_\vartheta(f(x)+f(y),q) = V_\vartheta(f(x+y),q) \leq A_\vartheta(x+y,q) \leq A_\vartheta(x,q) \vee A_\vartheta(y,q)$ which implies that $V_\vartheta(f(x)+f(y),q) \leq V_\vartheta(f(x),q) \vee V_\vartheta(f(y,q))$.
(vi) $V_\vartheta(f(x)f(y),q) = V_\vartheta(f(xy),q) \leq A_\vartheta(xy,q) \leq A_\vartheta(x,q) \vee A_\vartheta(y,q)$ which implies that $V_\vartheta(f(x)f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q))$.
(vii) $V_\vartheta(f(x) \vee f(y),q) = V_\vartheta(f(x \vee y),q) \leq A_\vartheta(x \vee y,q) \leq A_\vartheta(x,q) \vee A_\vartheta(y,q)$ which implies that $V_\vartheta(f(x) \vee f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q))$.
(viii) $V_\vartheta(f(x) \wedge f(y),q) = V_\vartheta(f(x \wedge y),q) \leq A_\vartheta(x \wedge y,q) \leq A_\vartheta(x,q) \vee A_\vartheta(y,q)$ which implies that $V_\vartheta(f(x) \wedge f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q))$, for all $f(x)$ and $f(y)$ in R' and q in Q.

As a result, V is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.

Theorem 3.2. Let $(R,+,\cdot,\vee,\wedge)$ and $(R',+,\cdot,\vee,\wedge)$ to be any two ℓ-semirings and Q to be a non-empty set. The homomorphic preimage of a Q-intuitionistic L-fuzzy ℓ-subsemiring of $f(R) = R'$ is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let $(R,+,\cdot,\vee,\wedge)$ and $(R',+,\cdot,\vee,\wedge)$ to be any two ℓ-semirings Q to be a non-empty set. Let $f: R \to R'$ be a ℓ-semiring homomorphism. Then
(i) $f(x+y) = f(x) + f(y)$,
(ii) $f(xy) = f(x)f(y)$,
(iii) $f(x \vee y) = f(x) \vee f(y)$ and
(iv) $f(x \wedge y) = f(x) \wedge f(y)$, for all x and y in R.

Let V be a Q-intuitionistic L-fuzzy ℓ-subsemiring of $f(R) = R'$. We have to prove that A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R. Let x and y in R and q in Q, Then,
(i) $A_\mu(x+y,q) = V_\mu(f(x+y),q)$, since $V_\mu(f(x,q)) = A_\mu(x,q) = V_\mu(f(x)+f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q)) =$

$A_\mu(x,q) \wedge A_\mu(y,q)$, since $V_\mu(f(x,q)) = A_\mu(x,q)$ which implies that $A_\mu(x+y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$.

(ii) $A_\mu(xy,q) = V_\mu(f(xy),q)$, since $V_\mu(f(x)) = A_\mu(x,q) = V_\mu(f(x)f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q)) = A_\mu(x,q) \wedge A_\mu(y,q)$, since $V_\mu(f(x,q)) = A_\mu(x,q)$ which implies that $A_\mu(xy,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$.

(iii) $A_\mu(x \vee y,q) = V_\mu(f(x \vee y),q)$, since $V_\mu(f(x)) = A_\mu(x,q) = V_\mu(f(x) \vee f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q)) = A_\mu(x,q) \wedge A_\mu(y,q)$, since $V_\mu(f(x,q)) = A_\mu(x,q)$ which implies that $A_\mu(x \vee y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$.

(iv) $A_\mu(x \wedge y,q) = V_\mu(f(x \wedge y),q)$, since $V_\mu(f(x)) = A_\mu(x,q) = V_\mu(f(x) \wedge f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q)) = A_\mu(x,q) \wedge A_\mu(y,q)$, since $V_\mu(f(x,q)) = A_\mu(x,q)$ which implies that $A_\mu(x \wedge y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$. Let x and y in R and q in Q. Then,

(v) $A_\vartheta(x+y,q) = V_\vartheta(f(x+y),q)$, since $V_\vartheta(f(x,q)) = A_\vartheta(x,q) = V_\vartheta(f(x) + f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q)) = A_\vartheta(x,q) \vee A_\vartheta(y,q)$, since $V_\vartheta(f(x,q)) = A_\vartheta(x,q)$ which implies that $V_\vartheta(x+y,q) \leq V_\vartheta(x,q) \vee V_\vartheta(y,q)$.

(vi) $A_\vartheta(xy,q) = V_\vartheta(f(xy,q))$, since $V_\vartheta(f(x,q)) = A_\vartheta(x,q) = V_\vartheta(f(x)f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q)) = A_\vartheta(x,q) \vee A_\vartheta(y,q)$, since $V_\vartheta(f(x,q)) = A_\vartheta(x,q)$ which implies that $A_\vartheta(xy,q) \leq A_\vartheta(x,q) \vee A_\vartheta(y,q)$.

(vii) $A_\vartheta(x \vee y,q) = V_\vartheta(f(x \vee y,q))$, since $V_\vartheta(f(x,q)) = A_\vartheta(x,q) = V_\vartheta(f(x) \vee f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q)) = A_\vartheta(x,q) \vee A_\vartheta(y,q)$, since $V_\vartheta(f(x,q)) = A_\vartheta(x,q)$ which implies that $A_\vartheta(x \vee y,q) \leq A_\vartheta(x,q) \vee A_\vartheta(y,q)$.

(viii) $A_\vartheta(x \wedge y,q) = V_\vartheta(f(x \wedge y,q))$, since $V_\vartheta(f(x,q)) = A_\vartheta(x,q) = V_\vartheta(f(x) \wedge f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q)) = A_\vartheta(x,q) \vee A_\vartheta(y,q)$, since $V_\vartheta(f(x,q)) = A_\vartheta(x,q)$ which implies that $A_\vartheta(x \wedge y,q) \leq A_\vartheta(x,q) \vee A_\vartheta(y,q)$.

As a result, A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Theorem3.3. Let $(R,+,\square,\vee,\wedge)$ and $(R',+,\square,\vee,\wedge)$ to be any two ℓ-semirings Q to be a non-empty set. The anti-homomorphic image of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.

Proof: Let $(R,+,\square,\vee,\wedge)$ and $(R',+,\square,\vee,\wedge)$ to be any two ℓ-semirings Q to be a non-empty set. Let $f: R \to R'$ be an ℓ-semiring anti-homomorphism. Then

(i) $f(x+y) = f(y) + f(x)$,

(ii) $f(xy) = f(y)f(x)$

(iii) $f(x \vee y) = f(y) \vee f(x)$ and

(iv) $f(x \wedge y) = f(y) \wedge f(x)$, for all $x, y \in R$. Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

We have to prove that V is an Q-intuitionistic L-fuzzy ℓ-subsemiring of $f(R) = R'$.

Now, for $f(x), f(y)$ in R' and q in Q,

(i) $V_\mu(f(x)+f(y),q) = V_\mu(f(y+x),q) \geq A_\mu(y+x,q) \geq A_\mu(y,q) \wedge A_\mu(x,q) = A_\mu(x,q) \wedge A_\mu(y,q)$which implies that $V_\mu(f(x)+f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q))$.

(ii) $V_\mu(f(x)f(y),q) = V_\mu(f(yx,q)) \geq A_\mu(yx,q) \geq A_\mu(y,q) \wedge A_\mu(x,q) = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $V_\mu(f(x)f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q))$.

(iii) $V_\mu(f(x) \vee f(y),q) = V_\mu(f(y \vee x,q)) \geq A_\mu(y \vee x,q) \geq A_\mu(y,q) \wedge A_\mu(x,q) = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $V_\mu(f(x) \vee f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q))$.

(iv) $V_\mu(f(x) \wedge f(y),q) = V_\mu(f(y \wedge x,q)) \geq A_\mu(y \wedge x,q) \geq A_\mu(y,q) \wedge A_\mu(x,q) = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $V_\mu(f(x) \wedge f(y),q) \geq V_\mu(f(x,q)) \wedge V_\mu(f(y,q))$. Now, for $f(x), f(y)$ in R' and q in Q.

(v) $V_\vartheta(f(x)+f(y),q) = V_\vartheta(f(y+x),q) \leq A_\vartheta(y+x,q) \leq A_\vartheta(y,q) \vee A_\vartheta(x,q) = A_\vartheta(x,q) \vee A_\vartheta(y,q)$which implies that $V_\vartheta(f(x)+f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q))$.

(vi) $V_\vartheta(f(x)f(y),q) = V_\vartheta(f(yx),q) \leq A_\vartheta(yx,q) \leq A_\vartheta(y,q) \vee A_\vartheta(x,q) = A_\vartheta(x,q) \vee A_\vartheta(y,q)$which implies that $V_\vartheta(f(x)f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q))$.

(vii) $V_\vartheta(f(x) \vee f(y),q) = V_\vartheta(f(y \vee x),q) \leq A_\vartheta(y \vee x,q) \leq A_\vartheta(y,q) \vee A_\vartheta(x,q) = A_\vartheta(x,q) \vee A_\vartheta(y,q)$which implies that $V_\vartheta(f(x) \vee f(y),q) \leq V_\vartheta(f(x,q)) \vee V_\vartheta(f(y,q))$.

(viii) $V_\vartheta(f(x) \wedge f(y), q) = V_\vartheta(f(y \wedge x), q) \leq A_\vartheta(y \wedge x, q) \leq A_\vartheta(y, q) \vee A_\vartheta(x, q) = A_\vartheta(x, q) \vee A_\vartheta(y, q)$which implies that $V_\vartheta(f(x) \wedge f(y), q) \leq V_\vartheta\big(f(x, q)\big) \vee V_\vartheta\big(f(y, q)\big)$.

As a result, V is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.

Theorem 3.4. Let $(R, +, \square, \vee, \wedge)$ and $(R', +, \square, \vee, \wedge)$ to be any two ℓ-semirings Q to be a non-empty set. The anti-homomorphic preimage of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R' is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let $(R, +, \square, \vee, \wedge)$ and $(R', +, \square, \vee, \wedge)$ to be any two ℓ-semirings Q to be a non-empty set. Let $f: R \to R'$ be an ℓ-semiring anti-homomorphism. Then

(i) $f(x + y) = f(y) + f(x)$,
(ii) $f(xy) = f(y)f(x)$
(iii) $f(x \vee y) = f(y) \vee f(x)$ and
(iv) $f(x \wedge y) = f(y) \wedge f(x)$, for all x and y in R.

Let V be a Q-intuitionistic L-fuzzy ℓ-subsemiring of $f(R) = R'$. We have to prove that A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R. Let x and y in R. Then

(i) $A_\mu(x + y, q) = V_\mu(f(x + y), q)$, since $V_\mu\big(f(x, q)\big) = A_\mu(x, q) = V_\mu(f(y) + f(x), q) \geq V_\mu\big(f(y, q)\big) \wedge V_\mu\big(f(x, q)\big) = V_\mu\big(f(x, q)\big) \wedge V_\mu\big(f(y, q)\big) = A_\mu(x, q) \wedge A_\mu(y, q)$, since $V_\mu\big(f(x, q)\big) = A_\mu(x, q)$ which implies that $A_\mu(x + y, q) \geq A_\mu(x, q) \wedge A_\mu(y, q)$.

(ii) $A_\mu(xy, q) = V_\mu(f(xy), q)$, since $V_\mu\big(f(x, q)\big) = A_\mu(x, q) = V_\mu(f(y)f(x), q) \geq V_\mu\big(f(y, q)\big) \wedge V_\mu\big(f(x, q)\big) = V_\mu\big(f(x, q)\big) \wedge V_\mu\big(f(y, q)\big) = A_\mu(x, q) \wedge A_\mu(y, q)$, since $V_\mu\big(f(x, q)\big) = A_\mu(x, q)$ which implies that $A_\mu(xy, q) \geq A_\mu(x, q) \wedge A_\mu(y, q)$.

(iii) $A_\mu(x \vee y, q) = V_\mu(f(x \vee y), q)$, since $V_\mu\big(f(x, q)\big) = A_\mu(x, q) = V_\mu(f(y) \vee f(x), q) \geq V_\mu\big(f(y, q)\big) \wedge V_\mu\big(f(x, q)\big) = V_\mu\big(f(x, q)\big) \wedge V_\mu\big(f(y, q)\big) = A_\mu(x, q) \wedge A_\mu(y, q)$, since $V_\mu\big(f(x, q)\big) = A_\mu(x, q)$ which implies that $A_\mu(x \vee y, q) \geq A_\mu(x, q) \wedge A_\mu(y, q)$.

(iv) $A_\mu(x \wedge y, q) = V_\mu(f(x \wedge y), q)$, since $V_\mu\big(f(x, q)\big) = A_\mu(x, q) = V_\mu(f(y) \wedge f(x), q) \geq V_\mu\big(f(y, q)\big) \wedge V_\mu\big(f(x, q)\big) = V_\mu\big(f(x, q)\big) \wedge V_\mu\big(f(y, q)\big) = A_\mu(x, q) \wedge A_\mu(y, q)$, since $V_\mu\big(f(x, q)\big) = A_\mu(x, q)$ which implies that $A_\mu(x \wedge y, q) \geq A_\mu(x, q) \wedge A_\mu(y, q)$.

(v) $A_\vartheta(x + y, q) = V_\vartheta(f(x + y), q)$, since $V_\vartheta\big(f(x, q)\big) = A_\vartheta(x, q) = V_\vartheta(f(y) + f(x), q) \geq V_\vartheta\big(f(y, q)\big) \wedge V_\vartheta\big(f(x, q)\big) = V_\vartheta\big(f(x, q)\big) \wedge V_\vartheta\big(f(y, q)\big) = A_\vartheta(x, q) \wedge A_\vartheta(y, q)$, since $V_\vartheta\big(f(x, q)\big) = A_\vartheta(x, q)$ which implies that $A_\vartheta(x + y, q) \geq A_\vartheta(x, q) \wedge A_\vartheta(y, q)$.

(vi) $A_\vartheta(xy, q) = V_\vartheta(f(xy), q)$, since $V_\vartheta\big(f(x, q)\big) = A_\vartheta(x, q) = V_\vartheta(f(y)f(x), q) \geq V_\vartheta\big(f(y, q)\big) \wedge V_\vartheta\big(f(x, q)\big) = V_\vartheta\big(f(x, q)\big) \wedge V_\vartheta\big(f(y, q)\big) = A_\vartheta(x, q) \wedge A_\vartheta(y, q)$, since $V_\vartheta\big(f(x, q)\big) = A_\vartheta(x, q)$ which implies that $\vartheta(xy, q) \geq A_\vartheta(x, q) \wedge A_\vartheta(y, q)$.

(vii) $A_\vartheta(x \vee y, q) = V_\vartheta(f(x \vee y), q)$, since $V_\vartheta\big(f(x, q)\big) = A_\vartheta(x, q) = V_\vartheta(f(y) \vee f(x), q) \geq V_\vartheta\big(f(y, q)\big) \wedge V_\vartheta\big(f(x, q)\big) = V_\vartheta\big(f(x, q)\big) \wedge V_\vartheta\big(f(y, q)\big) = A_\vartheta(x, q) \wedge A_\vartheta(y, q)$, since $V_\vartheta\big(f(x, q)\big) = A_\vartheta(x, q)$ which implies that $A_\vartheta(x \vee y, q) \geq A_\vartheta(x, q) \wedge A_\vartheta(y, q)$.

(viii) $A_\vartheta(x \wedge y, q) = V_\vartheta(f(x \wedge y), q)$, since $V_\vartheta\big(f(x, q)\big) = A_\vartheta(x, q) = V_\vartheta(f(y) \wedge f(x), q) \geq V_\vartheta\big(f(y, q)\big) \wedge V_\vartheta\big(f(x, q)\big) = V_\vartheta\big(f(x, q)\big) \wedge V_\vartheta\big(f(y, q)\big) = A_\vartheta(x, q) \wedge A_\vartheta(y, q)$, since $V_\vartheta\big(f(x, q)\big) = A_\vartheta(x, q)$ which implies that $A_\vartheta(x \wedge y, q) \geq A_\vartheta(x, q) \wedge A_\vartheta(y, q)$.

As a result, A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

In the following Theorem ∘ is the composition operation of functions

Theorem 3.5. Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring H and f is an isomorphism from a ℓ-semiring R onto H. Then $A \circ f$ is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let x and y in R and A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring H and Q to be a non-empty set. Then we have,

(i) $(A_\mu \circ f)(x + y, q) = A_\mu(f(x + y), q) = A_\mu(f(x) + f(y), q) \geq A_\mu\big(f(x, q)\big) \wedge A_\mu\big(f(y, q)\big) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$, which implies that $\big(A_\mu \circ f\big)(x + y, q) \geq \big(A_\mu \circ f\big)(x, q) \wedge \big(A_\mu \circ f\big)(y, q)$.

(ii) $\big(A_\mu \circ f\big)(xy, q) = A_\mu(f(xy), q) = A_\mu(f(x)f(y), q) \geq A_\mu\big(f(x, q)\big) \wedge A_\mu\big(f(y, q)\big) \geq \big(A_\mu \circ f\big)(x, q) \wedge \big(A_\mu \circ f\big)(y, q)$, which implies that $\big(A_\mu \circ f\big)(xy, q) \geq \big(A_\mu \circ f\big)(x, q) \wedge \big(A_\mu \circ f\big)(y, q)$.

(iii) $(A_\mu \circ f)(x \vee y, q) = A_\mu(f(x \vee y), q) = A_\mu(f(x) \vee f(y), q) \geq A_\mu(f(x, q)) \wedge A_\mu(f(y, q)) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$,which implies that $(A_\mu \circ f)(x \vee y, q) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$.

(iv) $(A_\mu \circ f)(x \wedge y, q) = A_\mu(f(x \wedge y), q) = A_\mu(f(x) \wedge f(y), q) \geq A_\mu(f(x, q)) \wedge A_\mu(f(y, q)) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$, which implies that $(A_\mu \circ f)(x \wedge y, q) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$. Then we have

(v) $(A_\vartheta \circ f)(x + y, q) = A_\vartheta(f(x + y), q) = A_\vartheta(f(x) + f(y), q) \geq A_\vartheta(f(x, q)) \wedge A_\vartheta(f(y, q)) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$, which implies that $(A_\vartheta \circ f)(x + y, q) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$.

(vi) $(A_\vartheta \circ f)(xy, q) = A_\vartheta(f(xy), q) = A_\vartheta(f(x)f(y), q) \geq A_\vartheta(f(x, q)) \wedge A_\vartheta(f(y, q)) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$, which implies that $(A_\vartheta \circ f)(xy, q) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$.

(vii) $(A_\vartheta \circ f)(x \vee y, q) = A_\vartheta(f(x \vee y), q) = A_\vartheta(f(x) \vee f(y), q) \geq A_\vartheta(f(x, q)) \wedge A_\vartheta(f(y, q)) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$, which implies that $(A_\vartheta \circ f)(x \vee y, q) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$.

(viii) $(A_\vartheta \circ f)(x \wedge y, q) = A_\vartheta(f(x \wedge y), q) = A_\vartheta(f(x) \wedge f(y), q) \geq A_\vartheta(f(x, q)) \wedge A_\vartheta(f(y, q)) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$, which implies that $(A_\vartheta \circ f)(x \wedge y, q) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$, for all x and y in R and q in Q.

As a result, $(A \circ f)$ is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R.

Theorem 3.6. Let A be an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring H and f is an anti-isomorphism from a ℓ-semiring R onto H. Then $A \circ f$ is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let x and y in R and A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring H and Q be a non-empty set. Then we have,

(i) $(A_\mu \circ f)(x + y, q) = A_\mu(f(x + y), q) = A_\mu(f(y) + f(x), q) \geq A_\mu(f(x, q)) \wedge A_\mu(f(y, q)) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$, which implies that $(A_\mu \circ f)(x + y, q) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$.

(ii) $(A_\mu \circ f)(xy, q) = A_\mu(f(xy), q) = A_\mu(f(y)f(x), q) \geq A_\mu(f(x, q)) \wedge A_\mu(f(y, q)) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$, which implies that $(A_\mu \circ f)(xy, q) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$.

(iii) $(A_\mu \circ f)(x \vee y, q) = A_\mu(f(x \vee y), q) = A_\mu(f(y) \vee f(x), q) \geq A_\mu(f(x, q)) \wedge A_\mu(f(y, q)) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$, which implies that $(A_\mu \circ f)(x \vee y, q) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$.

(iv) $(A_\mu \circ f)(x \wedge y, q) = A_\mu(f(x \wedge y), q) = A_\mu(f(y) \wedge f(x), q) \geq A_\mu(f(x, q)) \wedge A_\mu(f(y, q)) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$, which implies that $(A_\mu \circ f)(x \wedge y, q) \geq (A_\mu \circ f)(x, q) \wedge (A_\mu \circ f)(y, q)$. Then we have

(v) $(A_\vartheta \circ f)(x + y, q) = A_\vartheta(f(x + y), q) = A_\vartheta(f(y) + f(x), q) \geq A_\vartheta(f(x, q)) \wedge A_\vartheta(f(y, q)) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$, which implies that $(A_\vartheta \circ f)(x + y, q) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$.

(vi) $(A_\vartheta \circ f)(xy, q) = A_\vartheta(f(xy), q) = A_\vartheta(f(y)f(x), q) \geq A_\vartheta(f(x, q)) \wedge A_\vartheta(f(y, q)) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$, which implies that $(A_\vartheta \circ f)(xy, q) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$.

(vii) $(A_\vartheta \circ f)(x \vee y, q) = A_\vartheta(f(x \vee y), q) = A_\vartheta(f(y) \vee f(x), q) \geq A_\vartheta(f(x, q)) \wedge A_\vartheta(f(y, q)) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$, which implies that $(A_\vartheta \circ f)(x \vee y, q) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$.

(viii) $(A_\vartheta \circ f)(x \wedge y, q) = A_\vartheta(f(x \wedge y), q) = A_\vartheta(f(y) \wedge f(x), q) \geq A_\vartheta(f(x, q)) \wedge A_\vartheta(f(y, q)) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$, which implies that $(A_\vartheta \circ f)(x \wedge y, q) \geq (A_\vartheta \circ f)(x, q) \wedge (A_\vartheta \circ f)(y, q)$.

As a result, $A \circ f$ is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R.

Theorem3.7. Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring $(R, +, ⍰, \vee, \wedge)$, then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R, for every a in R and p in P.

Proof: Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R. For every x and y in R and q in Q, we have,

(i) $\left((aA_\mu)^p\right)(x + y, q) = p(a)A_\mu(x + y, q) \geq p(a)\{(A_\mu(x, q) \wedge A_\mu(y, q)\} = \{p(a)A_\mu(x, q) \wedge p(a)A_\mu(y, q)\} = \left\{\left((aA_\mu)^p\right)(x, q) \wedge \left((aA_\mu)^p\right)(y, q)\right\}$.Therefore, $\left((aA_\mu)^p\right)(x + y, q) \geq \left\{\left((aA_\mu)^p\right)(x, q) \wedge \left((aA_\mu)^p\right)(y, q)\right\}$.

(ii) $\left((aA_\mu)^p\right)(xy,q) = p(a)A_\mu(xy,q) \geq p(a)\{(A_\mu(x,q) \wedge A_\mu(y,q)\} = \{p(a)A_\mu(x,q) \wedge p(a)A_\mu(y,q)\} = \left\{\left((aA_\mu)^p\right)(x,q) \wedge \left((aA_\mu)^p\right)(y,q)\right\}$. Therefore, $\left((aA_\mu)^p\right)(xy,q) \geq \left\{\left((aA_\mu)^p\right)(x,q) \wedge \left((aA_\mu)^p\right)(y,q)\right\}$.

(iii) $\left((aA_\mu)^p\right)(x \vee y,q) = p(a)A_\mu(x \vee y,q) \geq p(a)\{(A_\mu(x,q) \wedge A_\mu(y,q)\} = \{p(a)A_\mu(x,q) \wedge p(a)A_\mu(y,q)\} = \left\{\left((aA_\mu)^p\right)(x,q) \wedge \left((aA_\mu)^p\right)(y,q)\right\}$. Therefore, $\left((aA_\mu)^p\right)(x \vee y,q) \geq \left\{\left((aA_\mu)^p\right)(x,q) \wedge \left((aA_\mu)^p\right)(y,q)\right\}$.

(iv) $\left((aA_\mu)^p\right)(x \wedge y,q) = p(a)A_\mu(x \wedge y,q) \geq p(a)\{(A_\mu(x,q) \wedge A_\mu(y,q)\} = \{p(a)A_\mu(x,q) \wedge p(a)A_\mu(y,q)\} = \left\{\left((aA_\mu)^p\right)(x,q) \wedge \left((aA_\mu)^p\right)(y,q)\right\}$. Therefore, $\left((aA_\mu)^p\right)(x \wedge y,q) \geq \left\{\left((aA_\mu)^p\right)(x,q) \wedge \left((aA_\mu)^p\right)(y,q)\right\}$, for every x and y in R and q in Q.

(v) $((aA_\vartheta)^p)(x+y,q) = p(a)A_\vartheta(x+y,q) \geq p(a)\{(A_\vartheta(x,q) \wedge A_\vartheta(y,q)\} = \{p(a)A_\vartheta(x,q) \wedge p(a)A_\vartheta(y,q)\} = \{((aA_\vartheta)^p)(x,q) \wedge ((aA_\vartheta)^p)(y,q)\}$. Therefore, $((aA_\vartheta)^p)(x+y,q) \geq \{((aA_\vartheta)^p)(x,q) \wedge ((aA_\vartheta)^p)(y,q)\}$.

(vi) $((aA_\vartheta)^p)(xy,q) = p(a)A_\vartheta(xy,q) \geq p(a)\{(A_\vartheta(x,q) \wedge A_\vartheta(y,q)\} = \{p(a)A_\vartheta(x,q) \wedge p(a)A_\vartheta(y,q)\} = \{((aA_\vartheta)^p)(x,q) \wedge ((aA_\vartheta)^p)(y,q)\}$. Therefore, $((aA_\vartheta)^p)(xy,q) \geq \{((aA_\vartheta)^p)(x,q) \wedge ((aA_\vartheta)^p)(y,q)\}$.

(vii) $((aA_\vartheta)^p)(x \vee y,q) = p(a)A_\vartheta(x \vee y,q) \geq p(a)\{(A_\vartheta(x,q) \wedge A_\vartheta(y,q)\} = \{p(a)A_\vartheta(x,q) \wedge p(a)A_\vartheta(y,q)\} = \{((aA_\vartheta)^p)(x,q) \wedge ((aA_\vartheta)^p)(y,q)\}$. Therefore, $((aA_\vartheta)^p)(x \vee y,q) \geq \{((aA_\vartheta)^p)(x,q) \wedge ((aA_\vartheta)^p)(y,q)\}$.

(viii) $((aA_\vartheta)^p)(x \wedge y,q) = p(a)A_\vartheta(x \wedge y,q) \geq p(a)\{(A_\vartheta(x,q) \wedge A_\vartheta(y,q)\} = \{p(a)A_\vartheta(x,q) \wedge p(a)A_\vartheta(y,q)\} = \{((aA_\vartheta)^p)(x,q) \wedge ((aA_\vartheta)^p)(y,q)\}$. Therefore, $((aA_\vartheta)^p)(x \wedge y,q) \geq \{((aA_\vartheta)^p)(x,q) \wedge ((aA_\vartheta)^p)(y,q)\}$. for every x and y in R and q in Q.

As a result, $(aA_\vartheta)^p$ is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R.

4 The Properties of Q-Intuitionistic L-Fuzzy Lower Q-Level ℓ-Subsemiring of a ℓ-Semiring in Homomorphism and Anti-Homomorphism

Theorem 4.1. Let $(R,+,\cdot,\vee,\wedge)$ and $(R',+,\cdot,\vee,\wedge)$ be any two ℓ-semirings and Q be a non-empty set. The homomorphic image of a lower Q-level ℓ-subsemiring of an Q-intuitionistic L-fuzzy ℓ-subsemiring of R is a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.

Proof: Let $(R,+,\cdot,\vee,\wedge)$ and $(R',+,\cdot,\vee,\wedge)$ be any two ℓ-semirings and Q be a non-empty set. Let $f: R \to R'$ be a ℓ-semiring homomorphism. Then,

(i) $f(x+y) = f(x) + f(y)$,
(ii) $f(xy) = f(x)f(y)$,
(iii) $f(x \vee y) = f(x) \vee f(y)$ and
(iv) $f(x \wedge y) = f(x) \wedge f(y)$, for all x and y in R.

Let $V = f(A)$, where A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Clearly V is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.

If x and y in R, then $f(x)$ and $f(y)$ in R'.

Let $A_{(\alpha,\beta)}$ be a lower Q-level ℓ-subsemiring of A.

Suppose x and y in $A_{(\alpha,\beta)}$, then $x+y, xy, x \vee y$ and $x \wedge y$ in $A_{(\alpha,\beta)}$. That is, $A_\mu(x,q) \geq \alpha$ and $A_\vartheta(x,q) \leq \beta, A_\mu(y,q) \geq \alpha$ and $A_\vartheta(y,q) \leq \beta, A_\mu(x+y,q) \geq \alpha, A_\mu(xy,q) \geq \alpha, A_\mu(x \vee y,q) \geq \alpha, A_\mu(x \wedge y,q) \geq \alpha$ and $A_\vartheta(x+y,q) \leq \beta, A_\vartheta(xy,q) \leq \beta, A_\vartheta(x \wedge y,q) \leq \beta, A_\vartheta(x \wedge y,q) \leq \beta$.

We have to prove that $f(A_{(\alpha,\beta)})$ is a lower Q-level ℓ-subsemiring of V.

Now, $V_\mu(f(x),q) \geq A_\mu(x,q) \geq \alpha$, implies that $V_\mu(f(x),q) \geq \alpha; V_\mu(f(y),q) \geq A_\mu(y,q) \geq \alpha$, implies that $V_\mu(f(y),q) \geq \alpha$,

(i) $V_\mu(f(x)+f(y),q) = V_\mu(f(x+y),q) \geq A_\mu(x+y,q) \geq \alpha$, which implies that $V_\mu(f(x)+f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R'.
(ii) $V_\mu(f(x)f(y),q) = V_\mu(f(xy),q) \geq A_\mu(xy,q) \geq \alpha$, which implies that $V_\mu(f(x)f(y),q) \geq \alpha$.
(iii) $V_\mu(f(x) \vee f(y),q) = V_\mu(f(x \vee y),q) \geq A_\mu(x \vee y,q) \geq \alpha$, which implies that $V_\mu(f(x) \vee f(y),q) \geq \alpha$.
(iv) $V_\mu(f(x) \wedge f(y),q) = V_\mu(f(x \wedge y),q) \geq A_\mu(x \wedge y,q) \geq \alpha$, which implies that $V_\mu(f(x) \wedge f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R'.

Now, $V_\vartheta(f(x),q) \leq A_\vartheta(x,q) \leq \beta$, implies that $V_\vartheta(f(x),q) \leq \beta; V_\vartheta(f(y),q) \leq A_\vartheta(y,q) \leq \beta$, implies that $V_\vartheta(f(y),q) \leq \beta$.

(v) $V_\vartheta(f(x)+f(y),q) = V_\vartheta(f(x+y),q) \geq A_\vartheta(x+y,q) \geq \alpha$, which implies that $V_\vartheta(f(x)+f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R'.

(vi) $V_\vartheta(f(x)f(y),q) = V_\vartheta(f(xy),q) \geq A_\vartheta(xy,q) \geq \alpha$, which implies that $V_\vartheta(f(x)f(y),q) \geq \alpha$.

(vii) $V_\vartheta(f(x) \vee f(y),q) = V_\vartheta(f(x \vee y),q) \geq A_\vartheta(x \vee y,q) \geq \alpha$, which implies that $V_\vartheta(f(x) \vee f(y),q) \geq \alpha$.

(viii) $V_\vartheta(f(x) \wedge f(y),q) = V_\vartheta(f(x \wedge y),q) \geq A_\vartheta(x \wedge y,q) \geq \alpha$, which implies that $V_\vartheta(f(x) \wedge f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R' and q in Q.

Therefore, $V_\mu(f(x)+f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R' and $V_\mu(f(x)f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R' and $V_\vartheta(f(x)+f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R' and $V_\vartheta(f(x)f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'. As a result, $f\left(A_{(\alpha,\beta)}\right)$ is a lower Q-level subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring V of a ℓ-semiring R'.

Theorem 4.2. Let $(R,+,\square,\vee,\wedge)$ and $(R',+,\square,\vee,\wedge)$ to be any two ℓ-semirings and Q to be a non-empty set. The homomorphic pre-image of a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R' is a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let $(R,+,\square,\vee,\wedge)$ and $(R',+,\square,\vee,\wedge)$ to be any two ℓ-semirings and Q to be a non-empty set. Let $f: R \to R'$ be a ℓ-semiring homomorphism. Then,

(i) $f(x+y) = f(x)+f(y)$,
(ii) $f(xy) = f(x)f(y)$,
(iii) $f(x \vee y) = f(x) \vee f(y)$ and
(iv) $f(x \wedge y) = f(x) \wedge f(y)$, for all x and y in R and $f(xy) = f(x)f(y)$, for all x and y in R.

Let $V = f(A)$, where V is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.

Clearly A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Let x and y in R and q in Q.

Let $f\left(A_{(\alpha,\beta)}\right)$ be a lower Q-level ℓ-subsemiring of V.

Suppose $f(x)$ and $f(y)$ in $f\left(A_{(\alpha,\beta)}\right)$, then $f(x)+f(y), f(x)f(y), f(x) \vee f(y)$ and $f(x) \wedge f(y)$ in $f\left(A_{(\alpha,\beta)}\right)$.

That is, $V_\mu(f(x),q) \geq \alpha$ and $V_\vartheta(f(x),q) \leq \beta; V_\mu(f(y),q) \geq \alpha$ and $V_\vartheta(f(y),q) \leq \beta; V_\mu(f(x)+f(y),q) \geq \alpha, V_\mu(f(x)f(y),q) \geq \alpha, V_\mu(f(x) \vee f(y),q) \geq \alpha, V_\mu(f(x) \wedge f(y),q) \geq \alpha$ and $V_\vartheta(f(x)+f(y),q) \leq \beta, V_\vartheta(f(x)f(y),q) \leq \beta, V_\vartheta(f(x) \vee f(y),q) \leq \beta, V_\vartheta(f(x) \wedge f(y),q) \leq \beta$.

We have to prove that $A_{(\alpha,\beta)}$ is a lower Q-level ℓ-subsemiring of A.

Now, $A_\mu(x,q) = V_\mu(f(x),q) \geq \alpha$, implies that $A_\mu(x,q) \geq \alpha; A_\mu(y,q) = V_\mu(f(y),q) \geq \alpha$, implies that $A_\mu(y,q) \geq \alpha$,we have

(i) $A_\mu(x+y,q) = V_\mu(f(x+y),q) = V_\mu(f(x)+f(y),q) \geq \alpha$, which implies that $A_\mu(x+y,q) \geq \alpha$, for all x and y in R and q in Q.

(ii) $A_\mu(xy,q) = V_\mu(f(xy),q) = V_\mu(f(x)f(y),q) \geq \alpha$, which implies that $A_\mu(xy,q) \geq \alpha$.

(iii) $A_\mu(x \vee y,q) = V_\mu(f(x \vee y),q) = V_\mu(f(x) \vee f(y),q) \geq \alpha$, which implies that $A_\mu(x \vee y,q) \geq \alpha$, for all x and y in R and q in Q.

(iv) $A_\mu(x \wedge y,q) = V_\mu(f(x \wedge y),q) = V_\mu(f(x) \wedge f(y),q) \geq \alpha$, which implies that $A_\mu(x \wedge y,q) \geq \alpha$.

$V_\vartheta(x,q) = V_\vartheta(f(x),q) \leq \beta$, implies that $A_\vartheta(x,q) \leq \beta; A_\vartheta(y,q) = V_\vartheta(f(y),q) \leq \beta$, implies that $A_\vartheta(y,q) \leq \beta$,we have

(v) $A_\vartheta(x+y,q) = V_\vartheta(f(x+y),q) = V_\vartheta(f(x)+f(y),q) \geq \beta$, which implies that $A_\vartheta(x+y,q) \geq \alpha$, for all x and y in R and q in Q.

(vi) $A_\vartheta(xy,q) = V_\vartheta(f(xy),q) = V_\vartheta(f(x)f(y),q) \geq \beta$, which implies that $A_\vartheta(xy,q) \geq \beta$.

(vii) $A_\vartheta(x \vee y,q) = V_\vartheta(f(x \vee y),q) = V_\vartheta(f(x) \vee f(y),q) \geq \beta$, which implies that $A_\vartheta(x \vee y,q) \geq \beta$.

(viii) $A_\vartheta(x \wedge y,q) = V_\vartheta(f(x \wedge y),q) = V_\vartheta(f(x) \wedge f(y),q) \leq \beta$ which implies that $A_\vartheta(x \wedge y,q) \leq \beta$, for all x and y in R and q in Q.

Therefore, $A_\mu(x+y,q) \geq \alpha, A_\mu(xy,q) \geq \alpha, A_\mu(x \vee y,q) \geq \alpha$, and $A_\mu(x \wedge y,q) \geq \alpha$, for all x and y in R and q in Q and $A_\vartheta(x+y,q) \leq \beta, A_\vartheta(xy,q) \leq \beta, A_\vartheta(x \vee y,q) \leq \beta$ and $A_\vartheta(x \wedge y,q) \leq \beta$, for all x and y in R and q in Q.

As a result, $A_{(\alpha,\beta)}$ is a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring A of R.

Theorem4.3. Let $(R,+,\square,\vee,\wedge)$ and $(R',+,\square,\vee,\wedge)$ to be any two ℓ-semirings and Q to be a non-empty set. The anti-homomorphic image of a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R is a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.

Proof: Let $(R,+,\cdot,\vee,\wedge)$ and $(R',+,\cdot,\vee,\wedge)$ to be any two ℓ-semirings and Q to be a non-empty set. Let $f: R \to R'$ be an anti-homomorphism. Then
(i) $f(x+y) = f(y)+f(x)$,
(ii) $f(xy) = f(y)f(x)$
(iii) $f(x \vee y) = f(y) \vee f(x)$ and
(iv) $f(x \wedge y) = f(y) \wedge f(x)$ for all x and y in R.
Let $V = f(A)$, where A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R.
Clearly V is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.
If x and y in R and q in Q, then $f(x)$ and $f(y)$ in R'.
Let $A_{(\alpha,\beta)}$be a lower Q-level ℓ-subsemiring of A.
Suppose x and y in $A_{(\alpha,\beta)}$, then $y+x, yx, y \vee x$ and $y \wedge x$in $A_{(\alpha,\beta)}$.
That is, $A_\mu(x,q) \geq \alpha$ and $A_\vartheta(x,q) \leq \beta, A_\mu(y,q) \geq \alpha$and $A_\vartheta(y,q) \leq \beta, A_\mu(y+x,q) \geq \alpha, A_\mu(yx,q) \geq \alpha, A_\mu(y \vee x,q) \geq \alpha, A_\mu(y \wedge x,q) \geq \alpha$ and $A_\vartheta(y+x,q) \leq \beta, A_\vartheta(yx,q) \leq \beta, A_\vartheta(y \vee x,q) \leq \beta, A_\vartheta(y \wedge x,q) \leq \beta$.
We have to prove that $f(A_{(\alpha,\beta)})$ is a lower Q-level ℓ-subsemiring of V.
Now, $V_\mu(f(x),q) \geq A_\mu(x,q) \geq \alpha$, implies that $V_\mu(f(x),q) \geq \alpha$; $V_\mu(f(y),q) \geq A_\mu(y,q) \geq \alpha$, implies that $V_\mu(f(y),q) \geq \alpha$,
(i) $V_\mu(f(x)+f(y),q) = V_\mu(f(y+x),q) \geq A_\mu(y+x,q) \geq \alpha$, which implies that $V_\mu(f(x)+f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R'.
(ii) $V_\mu(f(x)f(y),q) = V_\mu(f(yx),q) \geq A_\mu(yx,q) \geq \alpha$, which implies that $V_\mu(f(x)f(y),q) \geq \alpha$.
(iii) $V_\mu(f(x) \vee f(y),q) = V_\mu(f(y \vee x),q) \geq A_\mu(y \vee x,q) \geq \alpha$, which implies that $V_\mu(f(x) \vee f(y),q) \geq \alpha$.
(iv) $V_\mu(f(x) \wedge f(y),q) = V_\mu(f(y \wedge x),q) \geq A_\mu(y \wedge x,q) \geq \alpha$, which implies that $V_\mu(f(x) \wedge f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R'. And
$V_\vartheta(f(x),q) \leq A_\vartheta(x,q) \leq \beta$, implies that $V_\vartheta(f(x),q) \leq \beta$; $V_\vartheta(f(y),q) \leq A_\vartheta(y,q) \leq \beta$, implies that $V_\vartheta(f(y),q) \leq \beta$.
(v) $V_\vartheta(f(x)+f(y),q) = V_\vartheta(f(y+x),q) \leq A_\vartheta(y+x,q) \leq \beta$, which implies that $V_\vartheta(f(x)+f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'.
(vi) $V_\vartheta(f(x)f(y),q) = V_\vartheta(f(yx),q) \leq A_\vartheta(yx,q) \leq \beta$, which implies that $V_\vartheta(f(x)f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'.
(vii) $V_\vartheta(f(x) \vee f(y),q) = V_\vartheta(f(y \vee x),q) \leq A_\vartheta(y \vee x,q) \leq \beta$, which implies that $V_\vartheta(f(x) \vee f(y),q) \leq \beta$.
(viii) $V_\vartheta(f(x) \wedge f(y),q) = V_\vartheta(f(y \wedge x),q) \leq A_\vartheta(y \wedge x,q) \leq \beta$, which implies that $V_\vartheta(f(x) \wedge f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'.
Therefore, $V_\mu(f(x)+f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in $R', V_\mu(f(x)f(y),q) \geq \alpha, V_\mu(f(x) \vee f(y),q) \geq \alpha$, and $V_\mu(f(x) \wedge f(y),q) \geq \alpha$, for all $f(x)$ and $f(y)$ in R' and $V_\vartheta(f(x)+f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in $R', V_\vartheta(f(x) \vee f(y),q) \leq \beta, V_\vartheta(f(x) \wedge f(y),q) \leq \beta$ and $V_\vartheta(f(x)f(y),q) \leq \beta$, for all $f(x)$ and $f(y)$ in R'.
As a result, $f(A_{(\alpha,\beta)})$ is a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring V of a ℓ-semiring R'.

Theorem4.4. Let $(R,+,\cdot,\vee,\wedge)$and $(R',+,\cdot,\vee,\wedge)$ to be any two ℓ-semirings and Q to be a non-empty set. The anti-homomorphic pre-image of a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R' is a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let $(R,+,\cdot,\vee,\wedge)$and $(R',+,\cdot,\vee,\wedge)$ to be any two ℓ-semirings and Q to be a non-empty set. Let $f: R \to R'$ be an anti-homomorphism. Then,
(i) $f(x+y) = f(y)+f(x)$,
(ii) $f(xy) = f(y)f(x)$
(iii) $f(x \vee y) = f(y) \vee f(x)$ and
(iv) $f(x \wedge y) = f(y) \wedge f(x)$, for all x and y in R.
Let $V = f(A)$, where V is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R'.
Clearly A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.
Let x and y in R and q in Q.
Let $f(A_{(\alpha,\beta)})$ be a lower Q-level ℓ-subsemiring of V.
Suppose $f(x)$ and $f(y)$ in $f(A_{(\alpha,\beta)})$, then $f(y)+f(x), f(y)f(x), f(y) \vee f(x)$ and $f(y) \wedge f(x)$ in $f(A_{(\alpha,\beta)})$. That is, $V_\mu(f(x),q) \geq \alpha$ and $V_\vartheta(f(x),q) \leq \beta$; $V_\mu(f(y),q) \geq \alpha$ and $V_\vartheta(f(y),q) \leq \beta$; $V_\mu(f(y)+f(x),q) \geq \alpha, V_\mu(f(y)f(x),q) \geq \alpha, V_\mu(f(y) \vee f(x),q) \geq \alpha, V_\mu(f(y) \wedge f(x),q) \geq \alpha$and $V_\vartheta(f(y)+f(x),q) \leq \beta, V_\vartheta(f(y)f(x),q) \leq \beta, V_\vartheta(f(y) \vee f(x),q) \leq \beta, V_\vartheta(f(y) \wedge f(x),q) \leq \beta$.
We have to prove that $A_{(\alpha,\beta)}$ is a lower Q-level ℓ-subsemiring of A.
Now, $A_\mu(x,q) = V_\mu(f(x),q) \geq \alpha$, implies that $A_\mu(x,q) \geq \alpha$; $A_\mu(y,q) = V_\mu(f(y),q) \geq \alpha$, implies that $A_\mu(y,q) \geq \alpha$,we have

(i) $A_\mu(x+y,q)=V_\mu(f(x+y),q)=V_\mu(f(y)+f(x),q)\geq\alpha$, which implies that $A_\mu(x+y,q)\geq\alpha$, for all x and y in R.
(ii) $A_\mu(xy,q)=V_\mu(f(xy),q)=V_\mu(f(y)f(x),q)\geq\alpha$, which implies that $A_\mu(xy,q)\geq\alpha$, for all x and y in R.
(iii) $A_\mu(x\vee y,q)=V_\mu(f(x\vee y),q)=V_\mu(f(y)\vee f(x),q)\geq\alpha$, which implies that $A_\mu(x\vee y,q)\geq\alpha$, for all x and y in R.
(iv) $A_\mu(x\wedge y,q)=V_\mu(f(x\wedge y),q)=V_\mu(f(y)\wedge f(x),q)\geq\alpha$, which implies that $A_\mu(x\wedge y,q)\geq\alpha$, for all x and y in R. And, $A_\vartheta(x,q)=V_\vartheta(f(x),q)\leq\beta$, implies that $A_\vartheta(x,q)\leq\beta$; $A_\vartheta(y,q)=V_\vartheta(f(y),q)\leq\beta$, implies that $A_\vartheta(y,q)\leq\beta$,we have
(v) $A_\vartheta(x+y,q)=V_\vartheta(f(x+y),q)=V_\vartheta(f(y)+f(x),q)\leq\beta$ which implies that $A_\vartheta(x+y,q)\leq\beta$, for all x and y in R.
(vi) $A_\vartheta(xy,q)=V_\vartheta(f(xy),q)=V_\vartheta(f(y)f(x),q)\leq\beta$ which implies that $A_\vartheta(xy,q)\leq\beta$, for all x and y in R.
(vii) $A_\vartheta(x\vee y,q)=V_\vartheta(f(x\vee y),q)=V_\vartheta(f(y)\vee f(x),q)\leq\beta$ which implies that $A_\vartheta(x\vee y,q)\leq\beta$, for all x and y in R.
(viii) $A_\vartheta(x\wedge y,q)=V_\vartheta(f(x\wedge y),q)=V_\vartheta(f(y)\wedge f(x),q)\leq\beta$ which implies that $A_\vartheta(x\wedge y,q)\leq\beta$, for all x and y in R.

Therefore, $A_\mu(x+y,q)\geq\alpha$, for all x and y in R and q in $Q, A_\mu(xy,q)\geq\alpha, A_\mu(x\vee y,q)\geq\alpha$, and $A_\mu(x\wedge y,q)\geq\alpha$, for all x and y in R and $A_\vartheta(x+y,q)\leq\beta, A_\vartheta(xy,q)\leq\beta, A_\vartheta(x\vee y,q)\leq\beta$, for all x and y in R and $A_\vartheta(x\wedge y,q)\leq\beta$, for all x and y in R and q in Q.

As a result, $A_{(\alpha,\beta)}$ is a lower Q-level ℓ-subsemiring of a Q-intuitionistic L-fuzzy ℓ-subsemiring A of R.

Theorem4.5. Let A be an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring $(R,+,\square,\vee,\wedge)$. Then for α and β in L such that $\alpha\leq A_\mu(x,q)$ and $\beta\geq A_\mu(x,q)$, $A_{(\alpha,\beta)}$is a lower Q-level ℓ-subsemiring of R.

Proof: For all x and y in $A_{(\alpha,\beta)}$ and q in Q,we have, $A_\mu(x,q)\geq\alpha$ and $A_\vartheta(x,q)\leq\beta$ and $A_\mu(y,q)\geq\alpha$ and $A_\vartheta(y,q)\leq\beta$.Now,
(i) $A_\mu(x+y,q)\geq A_\mu(x,q)\wedge A_\mu(y,q)\geq\alpha\wedge\alpha=\alpha$,which implies that, $A_\mu(x+y,q)\geq\alpha$.
(ii) $A_\mu(xy,q)\geq A_\mu(x,q)\wedge A_\mu(y,q)\geq\alpha\wedge\alpha=\alpha$,which implies that, $A_\mu(xy,q)\geq\alpha$.
(iii) $A_\mu(x\vee y,q)\geq A_\mu(x,q)\wedge A_\mu(y,q)\geq\alpha\wedge\alpha=\alpha$,which implies that, $A_\mu(x\vee y,q)\geq\alpha$.
(iv) $A_\mu(x\wedge y,q)\geq A_\mu(x,q)\wedge A_\mu(y,q)\geq\alpha\wedge\alpha=\alpha$,which implies that, $A_\mu(x\wedge y,q)\geq\alpha$. And also,
(v) $A_\vartheta(x+y,q)\leq A_\vartheta(x,q)\vee A_\vartheta(y,q)\leq\beta\vee\beta=\beta$, which implies that, $A_\vartheta(x+y,q)\leq\beta$.
(vi) $A_\vartheta(xy,q)\leq A_\vartheta(x,q)\vee A_\vartheta(y,q)\leq\beta\vee\beta=\beta$, which implies that, $A_\vartheta(xy,q)\leq\beta$.
(vii) $A_\vartheta(x\vee y,q)\leq A_\vartheta(x,q)\vee A_\vartheta(y,q)\leq\beta\vee\beta=\beta$, which implies that, $A_\vartheta(x\vee y,q)\leq\beta$.
(viii) $A_\vartheta(x\wedge y,q)\leq A_\vartheta(x,q)\vee A_\vartheta(y,q)\leq\beta\vee\beta=\beta$, which implies that, $A_\vartheta(x\wedge y,q)\leq\beta$.

Therefore, $A_\mu(x+y,q)\geq\alpha, A_\mu(xy,q)\geq\alpha, A_\mu(x\vee y,q)\geq\alpha, A_\mu(x\wedge y,q)\geq\alpha$ and $A_\vartheta(x+y,q)\leq\beta, A_\vartheta(xy,q)\leq\beta, A_\vartheta(x\vee y,q)\leq\beta, A_\vartheta(x\wedge y,q)\leq\beta$, we get $x+y, xy, x\vee y$ and $x\wedge y$in $A_{(\alpha,\beta)}$.

As a result, $A_{(\alpha,\beta)}$ is a lower Q-level ℓ-subsemiring of R.

Theorem4.6. Let A be an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring $(R,+,\square,\vee,\wedge)$. Then two lower Q-level ℓ-subsemiring $A_{(\alpha_1,\beta_1)}, A_{(\alpha_2,\beta_2)}$ and α_1,α_2in L and β_1,β_2 in L with $\alpha_2<\alpha_1$ and $\beta_1<\beta_2$ of A are equal if and only if there is no x in R such that $\alpha_1>A_\mu(x,q)>\alpha_2$ and $\beta_1<A_\vartheta(x,q)<\beta_2$.

Proof: Assume that $A_{(\alpha_1,\beta_1)}=A_{(\alpha_2,\beta_2)}$.
Suppose there exists $x\in R$ and q in Q such that $\alpha_1>A_\mu(x,q)>\alpha_2$ and $\beta_1<A_\vartheta(x,q)<\beta_2$.
Then $A_{(\alpha_1,\beta_1)}\subseteq A_{(\alpha_2,\beta_2)}$, which implies that x belongs to $A_{(\alpha_2,\beta_2)}$, but not in $A_{(\alpha_1,\beta_1)}$.
This is contradiction to $A_{(\alpha_1,\beta_1)}=A_{(\alpha_2,\beta_2)}$.
Therefore there is no $x\in R$ and q in Q such that $\alpha_1>A_\mu(x,q)>\alpha_2$ and $\beta_1<A_\vartheta(x,q)<\beta_2$.
Conversely, if there is no $x\in R$ and q in Q such that $\alpha_1>A_\mu(x,q)>\alpha_2$ and $\beta_1<A_\vartheta(x,q)<\beta_2$.
Then $A_{(\alpha_1,\beta_1)}=A_{(\alpha_2,\beta_2)}$.

Theorem4.7.: Let $(R,+,\square,\vee,\wedge)$ be a ℓ-semiring and A be an Q-intuitionistic L-fuzzy subset of R such that $A_{(\alpha,\beta)}$ be a lower Q-level ℓ-subsemiring of R. If α and β in L satisfying $\alpha\leq A_\mu(x,q)$ and $\beta\geq A_\mu(x,q)$, then A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let $(R,+,\square,\vee,\wedge)$ to be a ℓ-semiring for x and y in R and q in Q.
Let $A_\mu(x,q)=\alpha_1$ and $A_\mu(y,q)=\alpha_2, A_\vartheta(x,q)=\beta_1$ and $A_\vartheta(y,q)=\beta_2$.
Case (i): If $\alpha_1<\alpha_2$ and $\beta_1>\beta_2$,then x and y in $A_{(\alpha_1,\beta_1)}$.
As $A_{(\alpha_1,\beta_1)}$ is a lower Q-level ℓ-subsemiring of $R, x+y, xy, x\vee y, x\wedge y$ in $A_{(\alpha_1,\beta_1)}$. Now,

(i) $A_\mu(x+y,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x+y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(ii) $A_\mu(xy,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(xy,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(iii) $A_\mu(x \vee y,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x \vee y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(iv) $A_\mu(x \wedge y,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x \wedge y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q. And also,

(v) $A_\vartheta(x+y,q) \leq \beta_1 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x+y,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(vi) $A_\vartheta(xy,q) \leq \beta_1 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(xy,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(vii) $A_\vartheta(x \vee y,q) \leq \beta_1 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x \vee y,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(viii) $A_\vartheta(x \wedge y,q) \leq \beta_1 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x \wedge y,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

Case (ii): If $\alpha_1 < \alpha_2$ and $\beta_1 < \beta_2$, then x and y in $A_{(\alpha_1,\beta_2)}$.

As $A_{(\alpha_1,\beta_2)}$ is a lower Q-level ℓ-subsemiring of $R, x+y, xy, x \vee y, x \wedge y$ in $A_{(\alpha_1,\beta_2)}$.Now,

(i) $A_\mu(x+y,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x+y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(ii) $A_\mu(xy,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(xy,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(iii) $A_\mu(x \vee y,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x \vee y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(iv) $A_\mu(x \wedge y,q) \geq \alpha_1 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x \wedge y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q. And also,

(v) $A_\vartheta(x+y,q) \leq \beta_2 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x+y,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(vi) $A_\vartheta(xy,q) \leq \beta_2 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(xy,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(vii) $A_\vartheta(x \vee y,q) \leq \beta_2 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x \vee y,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(viii) $A_\vartheta(x \wedge y,q) \leq \beta_2 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x \wedge y,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

Case (iii): If $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$,then x and y in $A_{(\alpha_2,\beta_1)}$.

As $A_{(\alpha_2,\beta_1)}$ is a lower Q-level ℓ-subsemiring of $R, x+y, xy, x \vee y, x \wedge y$ in $A_{(\alpha_2,\beta_1)}$.Now,

(i) $A_\mu(x+y,q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x+y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(ii) $A_\mu(xy,q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(xy,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(iii) $A_\mu(x \vee y,q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x \vee y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(iv) $A_\mu(x \wedge y,q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x \wedge y,q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q. And also,

(v) $A_\vartheta(x+y,q) \leq \beta_1 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x+y,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(vi) $A_\vartheta(xy,q) \leq \beta_1 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(xy,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(vii) $A_\vartheta(x \vee y,q) \leq \beta_1 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x \vee y,q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(viii) $A_\vartheta(x \wedge y, q) \leq \beta_1 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x \wedge y, q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

Case (iv): If $\alpha_1 > \alpha_2$ and $\beta_1 < \beta_2$,then x and y in $A_{(\alpha_2,\beta_2)}$.

As $A_{(\alpha_2,\beta_2)}$ is a lower Q-level ℓ-subsemiring of $R, x + y$ *and* $xy, x \vee y, x \wedge y$ in $A_{(\alpha_2,\beta_1)}$.Now,

(i) $A_\mu(x + y, q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x + y, q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(ii) $A_\mu(xy, q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(xy, q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(iii) $A_\mu(x \vee y, q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x \vee y, q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q.

(iv) $A_\mu(x \wedge y, q) \geq \alpha_2 = \alpha_1 \wedge \alpha_2 = A_\mu(x,q) \wedge A_\mu(y,q)$,which implies that $A_\mu(x \wedge y, q) \geq A_\mu(x,q) \wedge A_\mu(y,q)$, for all x and y in R and q in Q. And also,

(v) $A_\vartheta(x + y, q) \leq \beta_2 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x + y, q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(vi) $A_\vartheta(xy, q) \leq \beta_2 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(xy, q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(vii) $A_\vartheta(x \vee y, q) \leq \beta_2 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x \vee y, q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

(viii) $A_\vartheta(x \wedge y, q) \leq \beta_2 = \beta_1 \vee \beta_2 = A_\vartheta(x,q) \vee A_\vartheta(y,q)$,which implies that $A_\vartheta(x \wedge y, q) \leq A_\vartheta(x,q) \vee A_\mu(y,q)$, for all x and y in R and q in Q.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$.

It is trivial.

In all the cases, as a result, A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R.

Theorem4.8. Let A be an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring $(R, +, \square, \vee, \wedge)$. If any two lower Q-level ℓ-subsemiring of A belongs to R, then their intersection is also lower Q-level subsemiring of A in R.

Proof: For $\alpha_1, \alpha_2, \beta_1, \beta_2$in L and $\alpha_1 \leq A_\mu(x,q), \alpha_2 \leq A_\mu(x,q)$ and $\beta_1 \geq A_\mu(x,q), \beta_2 \geq A_\mu(x,q)$.

Case (i): If $\alpha_1 < A_\mu(x,q) < \alpha_2$ and $\beta_1 > A_\vartheta(x,q) > \beta_2$, then $A_{(\alpha_2,\beta_1)} \subseteq A_{(\alpha_1,\beta_1)}$.

Therefore, $A_{(\alpha_1,\beta_1)} \cap A_{(\alpha_2,\beta_2)} = A_{(\alpha_2,\beta_2)}$, but $A_{(\alpha_2,\beta_2)}$ is a lower Q-level ℓ-subsemiring of A.

Case(ii): If $\alpha_1 > A_\mu(x,q) < \alpha_2$ and $\beta_1 < A_\vartheta(x,q) < \beta_2$, then $A_{(\alpha_1,\beta_1)} \subseteq A_{(\alpha_2,\beta_1)}$.

Therefore, $A_{(\alpha_1,\beta_1)} \cap A_{(\alpha_2,\beta_2)} = A_{(\alpha_1,\beta_1)}$, but $A_{(\alpha_1,\beta_1)}$ is a lower Q-level ℓ-subsemiring of A.

Case (iii): If $\alpha_1 < A_\mu(x,q) < \alpha_2$ and $\beta_1 < A_\vartheta(x,q) < \beta_2$, then $A_{(\alpha_2,\beta_1)} \subseteq A_{(\alpha_1,\beta_2)}$.

Therefore, $A_{(\alpha_2,\beta_1)} \cap A_{(\alpha_1,\beta_2)} = A_{(\alpha_2,\beta_1)}$, but $A_{(\alpha_2,\beta_1)}$ is a lower Q-level ℓ-subsemiring of A.

Case (iv): If $\alpha_1 > A_\mu(x,q) > \alpha_2$ and $\beta_1 > A_\vartheta(x,q) > \beta_2$, then $A_{(\alpha_1,\beta_2)} \subseteq A_{(\alpha_2,\beta_1)}$.

Therefore, $A_{(\alpha_1,\beta_1)} \cap A_{(\alpha_2,\beta_2)} = A_{(\alpha_1,\beta_2)}$, but $A_{(\alpha_1,\beta_2)}$ is a lower Q-level ℓ-subsemiring of A.

Case (v): If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$.

In all cases, intersection of any two lower Q-level ℓ-subsemiring is a lower Q-level ℓ-subsemiring of A.

Theorem 4.9. Let A be an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring $(R, +, \square, \vee, \wedge)$. If α_j and $\beta_j$in $L, \alpha_i \leq A_\mu(x,q)$ and $\beta_j \geq (x,q)$ and $A_{(\alpha_i,\beta_j)}, i$ and j in I, is a collection of lower Q-level ℓ-subsemiring of A, then their intersection is also a lower Q-level ℓ-subsemiring of A.

Proof:

It is trivial.

5 Conclusion

This study work's major goal is to succinctly describe and prove the findings, theorems, and facts related to the homomorphism and anti-homomorphism lower Q-level subsets of a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring. Future work can expand on this work to include inter-valued Q-level subsets of a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring and ideals of (Q, L) fuzzy soft ℓ-subsemiring. We believe that our effort will have a significant impact on future research in this area and other soft algebraic investigations, opening up new avenues for advancement and premium.

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