Time Tensors

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Abstract: - Time Tensors functions have been used to describe the flows of time. The magnitude of the value of time tensor function means the temporal coordinates in a flow of time. We also use a function to describe the motion of particles in quantum mechanics but it has different meanings. The function is called time tensor function. Time tensor imposes space and time measurements and space and time probing. Although using optimised space and time probe fields will allow to deep probing in a position and time measurement beyond the space and time measurements of the probe field stil result in a time tensor. Fluctuation and dissipation relations in time tensor characterise the mechanical effects of time fluctuations, which lead to an ultimate temporal effects on space and time. For time tensors,the temporal effects on space and time are dominated by fluctuations and dissipations fluctuations and take temporal form deduced from fluctuations of space and time curvatures in temporal space. They can be considered as ultimate space and time fluctuations, fixing ultimate space and time measurements in time tensors.

Key-Words: - Time Tensor,Time Tensor Fields

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1 Introduction

Time Tensors concern the whole of physical reality, considered in usefully physical fields. The physical world appears to have temporal aspects, so the existence and nature of time are general fields. We analyze time tensors and space and time curvatures, using the framework of fluctuation and dissipation mechanisms arising when time tensors and spacetime metric are combined. Fluctuations and dissipations of time tensor and spacetime curvatures are shown flows of time at space and time orders. Time tensors correspond to space and time curvatures,the regions of space-like and light-like.

We deduce spatial and temporal effects for geodesic deviations registered by probe fields which determine ultimate space and time measurements from these fluctuations and dissipations of time tensor.In particular, a relation between spatial and temporal characterizing spaceand time fluctuations and dissipations of time tensor.Fluctuations of time tensors lead to observable mechanical effects. Time tensor carry time and exert temporal fields effect on space and time curvatures. These time tensors are themselves fluctuating quantities,like stress tensors which describe temporal fluctations. Such temporal fluctuations are associated with temporal effects.

Those time tensors persist in a state in space and time. Such time tensors fluctuate, like stress tensors Relations between fluctuation and dissipation still hold in space and time.Dissipative temporal effects in space and time may be identified . When attention is focussed time tensors, they appear that fluctuation and dissipation mechanisms of space and time for determining ultimate space and time measurements.

From an analysis of interferometric space and time measurements or from a general analysis of spat,al and temporal effects and dissipations in space and time measurements. Since time tensor can be accepted that sensitivity in space and time probing goes beyond space and time.Time tensor appears that temporal effect has to be taken into account when analysing ultimate temporal fields.When time tensor developed on spatial and tenmporal fields theories , time tensor exhibits metric and curvature fluctuations.Like spatial and temporal curvature perturbations associated with time tensors , spatial and temporal curvature fluctuations are felt to probe spacetime. Estimating time tensor for fluctuating geodesic deviations stemming from these spatial and temporal fields leads to spatial and temporal for measurements.Einstein equation for spatial and temporal curvatures can be regarded as a lot of response equations which describe the metric response to a stress tensor perturbation with time tensor perturbation , and used to derive spatial and temporal fluctuations of metric. Time tensor also leads to spatial and temporal curvature fluctuations arising from spatial and temporal fluctuations of stress tensors .Time tensor feels mean values of stress tensors has sometimes been expressed, but it is known to develop the consistency of spatial and temporal predictions . Even if the existence of time tensor fluctuations associated with Einstein equation are combined, unavoidable coupling to stress tensors combine metric fluctuations into space and time.Fluctuations of stress tensors are associated with time tensors on a metric perturbation that can be identified with spatial and temporal curvatures.

Here, we already proved fruitful for studying spatial fields coupled to temporal fields to analyse spatail and temporal fluctuations coupled to stress tensors.We derive the curvature fluctuations associated with time tensors for fluctuations of stress tensors. Spatial and temporal fluctuations based upon Lorentz invariance and conservation laws in Minkowski spacetime for time tensors. We then show how to build fluctuations of metric and stress tensors, which is similar to that used for spatial and temporal fluctuations for time tensors.

2 Temporal Space

Temporal space is a set of temporal elements or points satisfying specified time dimensions. Von Neumann says that "First of all we must admit that this objection points at an essential weakness which is, in fact, the chief weakness of quantum mechanics: its non-relativistic character, which distinguishes the time t from the three space coordinates x, y, z and presupposes an objective simultaneity concept. In fact, while all other quantities especially those x, y, z closely connected with t by the Lorentz transformation are represented by operators, there corresponds to the time an ordinary number-parameter t, just as in classical mechanics."

Temporal space consists from time.

Reference Frames

A frame of reference or reference frame is a coordinate system or set of axes used by an observer to measure the position, orientation, everything of objects in space. The position of the observer itself is assumed fixed relative to its own frame. The term of frame of reference is a relative to which motion and rest will be measured and any set of points that are at rest relative to one another enables us for in principle, to describe the relative motions of points.

A frame of reference is therefore a kinematical device, for the geometrical and temporal description of motion without regard to the forces involved. We define the coordinates of events in one reference frame to the coordinates of the same event in other reference frame. An event will have different coordinates in different reference frames.If we had chosen a particular set of axes, we would have and so on where the values of the components of reference frame depend on the set of axes chosen. The Galilean transformation equations define Newtonian Mechanics. The Lorentz transformation equations define Special and General Relativistic Mechanics.

Inertial Reference Frame

A dynamical term of motion leads to the term of inertial frame or a reference frame relative to which motions have distinguished dynamical properties. The term of inertial frame has to be understood as a spatial and temporal reference frame together with means of measuring time, so that uniform motions can be distinguished from accelerated motions. As to the laws of Newtonian dynamics inertial frame is a reference frame with geometric and temporal spaces, relative to which the motion of a point not subject to forces is rectilinear and uniform, accelerations are always proportional to and in the direction of applied forces, and applied forces are always met with equal and opposite reactions. We follow that, in an inertial frame, the center of system of points is always at rest or in uniform motion. We follow that any other references frame moving uniformly relative to an inertial frame is also an inertial frame.

Isolated system of particles

An isolated system of particles is a system of particles subject only to their mutual interactions subject to no external interactions. Any system of particles subject to external interactions that somehow cancel one another in order to make the system’s motion identical to that of an isolated system will also be considered an “isolated” system. An isolated system consisting of a single particle is called a free particle.

Reference Frames in Geometric Space

Geometric processes involve the dynamics of particles and fields moving or propagating through geometric space.All of the fundamental laws of physics involve position in geometric space.

Newton’s second law of motion

$$F = ma$$

when applied to a particle responding to the action of a force will yield the geometric space of the particle.Maxwell’s equations will yield the wave equation

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

for the propagation of a light wave through geometric space.

Reference Frames in Temporal Space

Temporal processes involve the dynamics of particles and fields moving or propagating through temporal space. A temporal frame of reference can be constructed in essentially a lot of ways, provided it meets the requirements in temporal spaces for the time of any event.

Newton's second law can be written as

$$F = m\frac{d^2r}{dt^2}$$

expressed in a way that makes no mention of a reference frame though note the appearance of a singled out time variable t.

when applied to a particle responding to the action of a force will yield the temporal space of the particle.

Maxwell's equations will yield the wave equation

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

for the propagation of a light wave through temporal space.

Events

The motion of a field through space could be thought of as a continous series of events, while the collision of two fields would be an event.

Events in Geometric Space

An event is realizing at a region in geometric space,the coordinate system will present an event will present at a point in geometric space.

Events in Temporal Space

An event is realizing at a region in temporal space,the coordinate system will present an event at a time in temporal space.

The direction of time

The concept of time has an objective direction that there is an objective distinction between past and future with future. Concerning this ingredient, we can get a good sense of what is at issue by comparing some remarks from writers on opposite sides.

Orientability of time

We are interested with the direction of time is guided by many attempts to make sense of the notion of the flow of time. If time has a direction in a sense relevant to any coordinate system. This implies that the direction of time is that it is a temporal direction at every place and time.

It means that the direction of time is that the spacetime within which we live be orientable temporally.

Group Theory and symmetries

Group theory is studying physical system with symmetry. In particular, the representation theory of the group simpli.es the physical solutions to the systems which have symmetries.

We suppose that an one-dimensional Hamiltonian has the symmetry $x \to -x$.

$$H(x) = H(-x)$$

Then from the time-independent Schrödinger's equation,

$$H(x)\psi(x) = E\psi(x)$$

$$H(-x)\psi(-x) = H(x)\psi(-x) = E\psi(-x)$$

which means that $\psi(-x)$ is also an eigenstate with same eigenvalue E.

Thus we can form the linear combinations of these two states,

$$\psi_{\pm} = \frac{1}{\sqrt{2}}(\psi(x) \pm \psi(-x))$$

which are parity eigenstates and are either symmetric or antisymmetric under $x \to -x$. These are the consequences of symmetry. This means only that the eigenstates can be chosen to be either symmetric or antisymmetric and does not imply that the system has degenerate eigenstates. This is because the even or odd state can be identically zero.

A Hamiltonian system is called time-reversal invariant if from any given solution $x(t)$, $p(t)$ of Hamilton's equations an independent solution $x'(t')$, $p'(t')$ is obtained with $t \to -t$ and some operation relating x' and p' to the original coordinates x and momenta p. The simplest such invariance, to be referred to as conventional, holds when the Hamiltonian is an even function of all momenta

$$t \to -t$$
$$x \to x$$
$$p \to -p$$
$$H(x, p) = H(x, -p)$$

Symmetries in classical physics

In classical mechanics, one usually considers the Lagrange formulation defined in terms of the Lagrangian L, which is a function of the generalized coordinates, q_i, and the generalized velocities, q_i' of the system.

If, for instance, the Lagrangian L remains unchanged under displacements,

$$q_i \to q_i + \delta q_i$$

$$\frac{\partial L}{\partial q_i} = 0$$

which implies that:

$$\frac{\partial p_i}{\partial t} = 0$$

since

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i'}\right) - \frac{\partial L}{\partial q_i'} = 0 \text{ and}$$

$$p_i = \frac{\partial L}{\partial v_i} = 0$$

$\frac{\partial p_i}{\partial t} = 0$ means that p_i is a conserved quantity.

In the Hamiltonian formulation

$$H(p_i, q_i)$$

the Hamilton equations are given by

$$p_i' = -\frac{\partial H}{\partial q_i}$$

$$q_i = -\frac{\partial H}{\partial p_i}$$

and

$$\frac{d}{dt} p_i = 0$$

Whenever

$$\frac{dH}{dq_i} = 0$$

If the Hamiltonian desn't explicity depend on q_i which is equivalent to saying that H is unchanged under $q_i \to q_i + \delta q_i$.We have a conserved quantity in this case the momentum p_i

Symmetries in quantum mechanics

In quantum mechanics:

We associate a unitary operator $\hat{U}$ to a transformation that conserves probability. For instance, a rotation is described by a unitary operator. This operator is often called a symmetry operator We classify symmetries as continuous (rotation, translation) and discrete (parity, lattice translations, time reversal).

Symmetry operations that differ infinitesimally from the identity transformation (continuous symmetries) are written as:

$$\hat{U} = 1 - \frac{i\varepsilon}{\hbar}\hat{G}$$

where $\hat{G}$ is the hermitian generator of the symmetry operator we are describing.

Time Tensor

A temporal object under the action of external temporal effects undergoes distortion and the effect due to this system of temporal effects is transmitted throughout the temporal object developing internal temporal effects in it.

To examine these internal temporal effects at a point O in Figure 2.1, inside the temporal object, consider a plane MN passing through the point O.

If the plane is divided into a number of small areas, as in Figure 2.2, and the effects acting on each of these measured, it will be observed that these effects vary from one small area to the next.

On the small area ΔA at point O, there will be acting a effect of ΔG as shown in Figure 2.2.

From this, it can be understood that the concept of temporal stress is the internal effect per unit area. Assuming the temporal material is continuous, the term "temporal stress" at any point across a small area ΔA can be defined by the limiting equation (2.1).

$$Temporalstresstensor = \lim_{\Delta A \to 0} \frac{\Delta G}{\Delta A}$$

$$Timetensor = \lim_{\Delta A \to 0} \frac{\Delta G}{\Delta A}$$

where ΔG is the internal temporal effect on the area ΔA surrounding the given point.

Temporal stress tensor or time tensor is sometimes referred to as effect intensity.

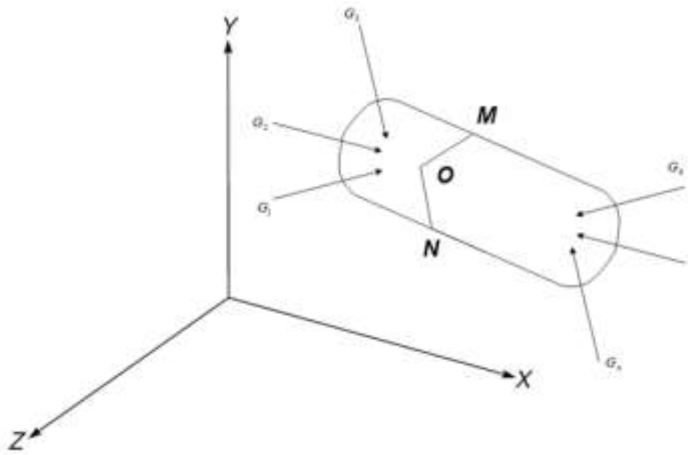


Figure 2.1 Effects acting on a temporal object

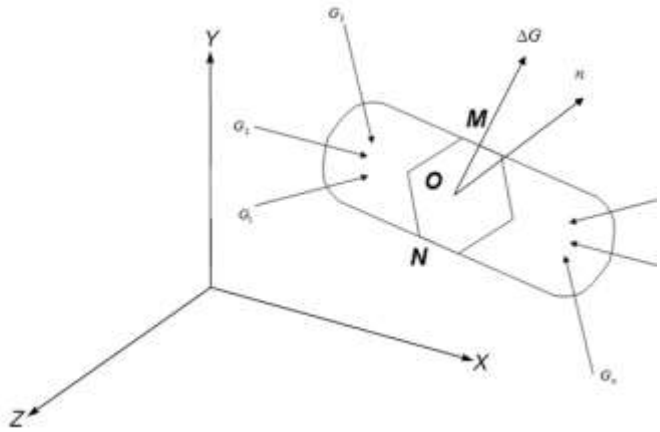


Figure 2.2 Effects acting on a temporal object

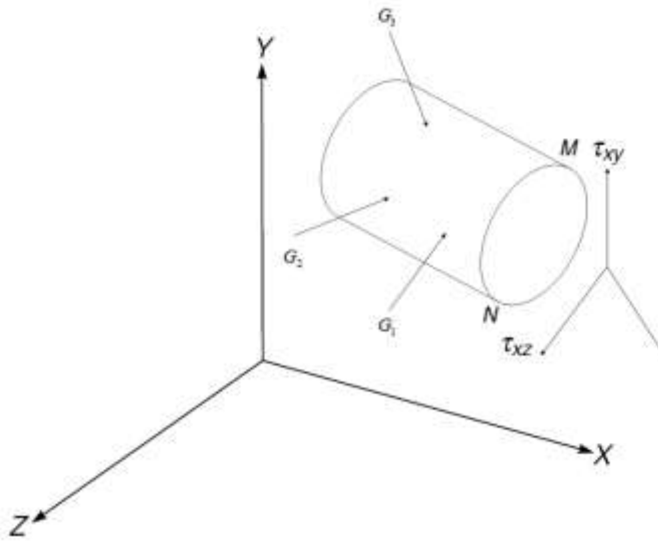


Figure 2.3 Temporal stress tensor or Time tensor components at point O

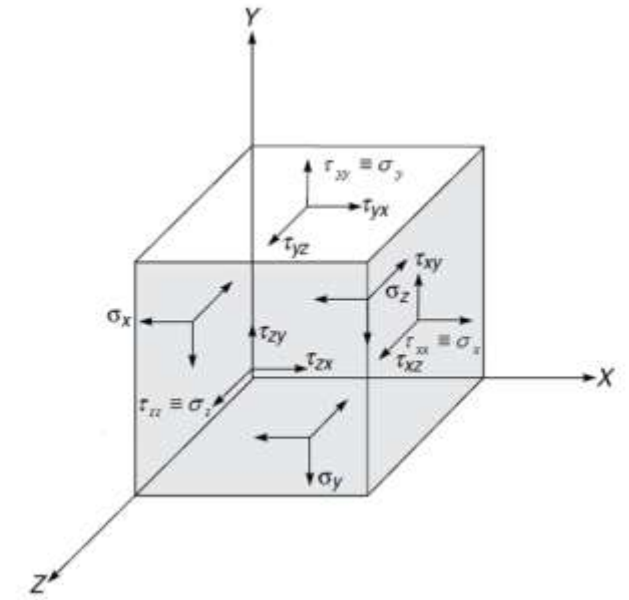


Figure 2.4 Temporal Stress components acting on parallelepiped

Indicial notation

In indicial notation, the coordinate axes x, y, z are replaced by numbered axes x_1, x_2, x_3 respectively.

The components of the temporal effect ΔG of Figure 2.1 are written as ΔG_1, $\Delta G_2, \Delta G_3$ where the numerical subscript indicates the component with respect to the numbered coordinate axes.

The definitions of the components of temporal stress acting on the x_1 face can be written in indicial form as follows:

$$\sigma_{11} = \lim_{\Delta A_1 \to 0} \frac{\Delta G_1}{\Delta A_1}$$

$$\sigma_{12} = \lim_{\Delta A_1 \to 0} \frac{\Delta G_2}{\Delta A_1}$$

$$\sigma_{13} = \lim_{\Delta A_1 \to 0} \frac{\Delta G_3}{\Delta A_1}$$

Here, the symbol σ is used for both temporal normal and shear stresses.

In general, all components of temporal stress can now be defined by a single equation:

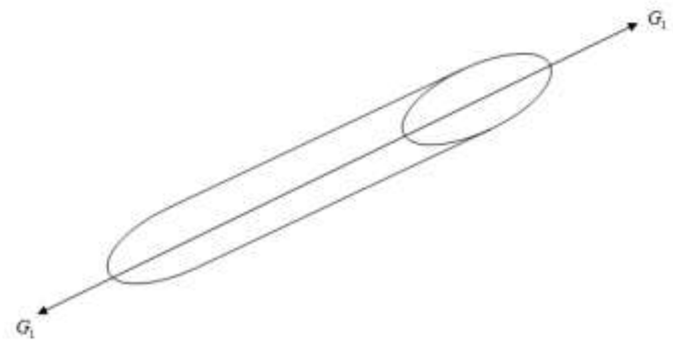
$$\sigma_{ij} = \lim_{\Delta A_i \to 0} \frac{\Delta G_j}{\Delta A_i}$$

Here $i = \{1,2,3\}$ and $j = \{1,2,3\}$.

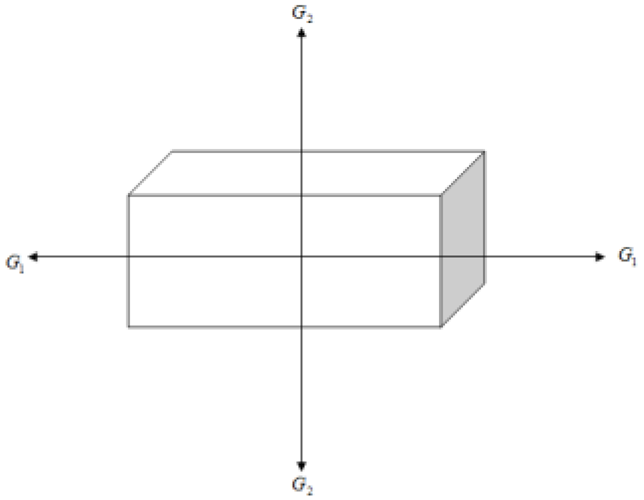
Forms of temporal stress

Temporal stress may be classified in a lot of ways, according to the form of temporal object on which they act, or the nature of the temporal stress itself.

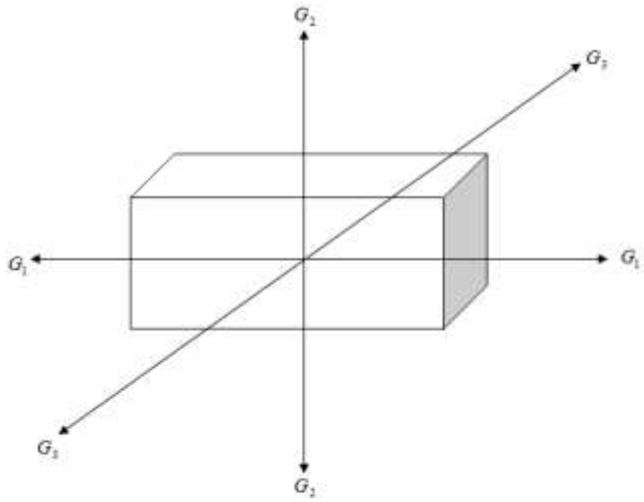
Thus, temporal stresses could be one-dimensional, two-dimensional or three dimensional as shown in Figure 2.5.



(a) One-dimensional temporal stress



(b) Two-dimensional temporal stress



(c) Three-dimensional temporal stress

Figure 2.5 Forms of temporal stress

Temporal stress tensor

Let O be the point in a temporal object shown in Figure 2.1. Passing through that points infinitely, many planes may be drawn. As the resultant forces acting on these planes are the same, the temporal stresses on these planes are different because the areas and the inclinations of these planes are different. Therefore, for a complete description of temporal stress or time tensor. We have to specify not only its magnitude, direction and sense but also the surface on which it acts.

For this reason, the temporal stress is called a “time tensor”.

Figure 2.4 depicts three orthogonal coordinate planes representing a parallelopiped on which are nine components of temporal stress.

Of these three are direct temporal stresses and six temporal shear stresses.

In tensor notation, these can be expressed by the time tensor τ_{ij}, where $i = x, y, z$ and $j = x, y, z$

In matrix notation, it is often written as

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

otherwise it is written as

$$S = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

If we use ordinary expression in matrix form.
Consider a space of infinitesimal dimensions shown.
All stresses acting on this space are identified.
The subscripts τ are the shear stress, associate the stress with a plane perpendicular to a given axis, the second designate the direction of the stress.

$$\tau_{xy} = \tau_{FaceDirection}$$

x presents the face of time tensor τ_{xy}

y presents the direction of time tensor τ_{xy}

Unit time tensor

Any second rank unit time tensor I can be uniquely expressed I. This is the identity matrix. In the composition above, the second matrix indicates the unit tensor is composed of the column of the three unit vectors. In remaining discussion we will not place the accent over the unit vectors or the double bar over the tensors for convenience only. Also the unit vectors are written as column vectors or row vectors as appropriate.

Zero time tensor

A zero tensor is a tensor of any rank and with any pattern of covariant and contravariant indices all of whose components are equal to 0.

Equal time tensors

For two tensors to be equal, they must have
The same dimensions.
Corresponding elements must be equal In other words, say that

$A_{n\times m} = (a_{ij})$ and that $B_{p\times q} = (b_{ij})$

Then $A = B$ if and only $n = p$, $m = q$ and $a_{ij} = b_{ij}$

for all i and j in range

Symmetry and antisymmetry on time tensor

In practice it often happens that tensors display a certain amount of symmetry, like what we know from matrices. Such symmetries have a strong effect on the properties of these tensors. Often many of these properties or even tensor equations can be derived solely on the basis of these symmetries.

A tensor t is called symmetric in the indices μ and ν if the components are equal upon exchange of the index-values. So, for a 2nd rank contravariant tensor,

$t^{\mu\nu} = t^{\nu\mu}$ symmetric (2,0)-time tensor

A tensor t is called anti-symmetric in the indices μ and ν if the components are equal but- opposite upon exchange of the index-values. So, for a 2nd rank contravariant tensor,

$t^{\mu\nu} = -t^{\nu\mu}$ anti-symmetric (2,0)-time tensor .

It is not useful to speak of symmetry or anti-symmetry in a pair of indices that are not of the same type covariant or contravariant.

The properties of symmetry only remain invariant upon basis transformation if the indices are of the same type.

Decomposition into Symmetric and Anti-Symmetric Parts

Any second rank time tensor t_{ij} can be uniquely expressed as the sum of a symmetric and an anti-symmetric time tensor; for

$$t_{ij} = S_{ij} + A_{ij}$$

Where

$S_{ij} = \frac{1}{2}\left(t_{ij} + t_{ji}\right)$ is symmetric time tensor

$A_{ij} = \frac{1}{2}\left(t_{ij} - t_{ji}\right)$ is anti-symmetric time tensor

Temporal asymmetries

Spatial asymmetry will not require that space itself be anisotropic or that the direction of space be distinguished by the orientation of time.

Temporal asymmetry will be same with spatial asymmetry. The concept of time might be temporally asymmetric, without time itself having any asymmetry. Accordingly, we need to be cautious in making inferences from observed temporal asymmetries to the anisotropy of time itself.

Second-order real time tensors

Definition1

Let V be a vector space of dimension n .

A second-order tensor is defined as a bilinear function $T : V \times V \to R$.

The dual of a vector field:

V^* is the set of linear forms $V \to R$

There is a correspondance between second-order tensors and linear maps between V and is dual V^* .

Given $A : V \to V^*$

We define $T_A(v, w)$ as $A(v)(w)$

It is bilinear.

Given T second-order tensor

We define $A_T(v)$ a linear from by $A_T(v)(w) = T(v, w)$

Matrix notation for the second-order tensor $T : Let\{e_1, e_2, ..., e_n\}$ be an orthonormal basis of Euclidean space V

$$T(v, w) = (v_1 ... v_n).M.\begin{pmatrix} w_1 \\ . \\ . \\ w_n \end{pmatrix}$$

Where

$$v = \sum_{i=1}^{n} v_i e_i$$

$$w = \sum_{i=1}^{n} w_i e_i$$

M is $n \times n$ matrix representing T

Time tensors of any order

Let V be a vector space of dimension n .

Definition2

A general (k, l) tensor is a function

$T : V^* \times ... \times V^* \times V \times ... \times V \to R$

$V^* \times ... \times V^*$ k copies

$V \times ... \times V$ l copies

Linear in every variable.

$A(0,0)$ time tensor is a scalar

$A(1,0)$ tensor is a vector

$\xi : V^* \to R$ corresponds a vector v since when V is finite-dimensional

V is isomophic to V^{**}

Consider the isomorophism ψ

$V \to V^{**}$ defined by $\psi(v)(\xi) = \xi(v)$ and $\xi \in V^*$

$A(2,0)$ tensor is what we called a second order tensor.

Through the choice a basis on V

We can see it as a $n \times n$ matrix.

Time tensor fields

Definition3

A (k,l)time tensor field over $U \subset R^n$ is the giving of a (k,l) time tensor in every point of U , varying smoothly with the point.

Definition4

Let S be a regular surface. A time tensro field T on S is the assignment to each point $p \in S$ of a tensor $T(p)$on $T_p S$ such that these time tensors vary in a smooth manner.

Considering second-order time tensor field in $U \in R^2$

We can see it as a field of 2×2 matrices $T: U \rightarrow M_2(R)$.

Considering second-order time tensor field in $U \in R^3$

We can see it as a field of 3×3 matrices $T: U \rightarrow M_3(R)$.

Let S be a regular surface patch given by a parameterization $f: U \rightarrow R^3$.

In every point p , the second-order time tensor field T gives a second-order time tensor T(p) on the tangent plane $T_p S$.

Since the vectors $\frac{\partial f}{\partial x}(u,v)and\frac{\partial f}{\partial y}(u,v)$form a basis $T_{f(u,v)}S$.

We can write the secosnd order time tensor as 2×2 matrix.

We will have a map $T: U \rightarrow M_2(R)$.

Change of basis for time tensors

Suppose that we have two basis $\{e_1, e_2, ..., e_n\}$and $\{f_1, f_2, ..., f_n\}$of V .

For a linear map

$L: V \rightarrow V$:

$B = P^{-1}AP$

A matrix of L in the first basis of V ,B in the second, P is the matrix with column vectors f_i expressed in the old basis. A and B are similar.

For a second-order time tensor or equivalently a bilinear form

$$T: V \times V \rightarrow R: (X')^T BY' = T(x,y)$$

Where A is the matrix of the time tensor in the first basis, B is the matrix of the time tensor in second basis.

X, Y and X', Y' are the coordinates of x, y in the first and second basis.

A, B are congruent

$B = P^T AP$

It means that to a second-order time tensor corresponds a congruence class of matrices.

Tensor diagonalization

The matrix representation of a time tensor becomes especially simple in a basis made of eigenvectors.

Remember that a 3×3 symmetric matrix always has 3 real eigenvalues, and that the associated eigenvectors u_1, u_2, u_3 are orthogonal. The complete transformation of T from an arbitrary basis into the eigenvector basis is then given by

$$U^T TU = \begin{pmatrix} \xi_1 & 0 & 0 \\ 0 & \xi_2 & 0 \\ 0 & 0 & \xi_3 \end{pmatrix}$$

where $\xi_1, \xi_2, \xi_3$are the eigenvalues and U is the orthogonal matrix that is composed of the unit eigenvectors u_1, u_2, u_3 .$U = (u_1 \quad u_2 \quad u_3)$.

Time Tensor properties

Definition5

A second-order time tensor is said to be symmetric if $S(v,w) = S(w,v)$for all $v, w \in V$.

In matrix notation: $s_{ij} = s_{ji}$ for all $i, j \in \{1,...,n\}$.

Number of degrees of freedom:

$\frac{1}{2}n(n+1)$

A second-order time tensor is said to be antisymmetric if $A(v,w) = -A(w,v)$for all $v, w \in V$.

In matrix notation: $s_{ij} = -s_{ji}$ for all $i, j \in \{1,...,n\}$.

Number of degrees of freedom:

$\frac{1}{2}n(n-1)$

A second-order time tensor is said to be traceless time tensor $tr(T) = 0$for T a matrix representing the tensor.Since the trace is invariant with respect to congruence,it is well defined the trace of a second-order time tensor.

Let T be a symmetric second-order time tensor.

Definition6

T is said positive definite if $T(v,v) > 0$, for every non-zero vector v.

It means that all eigenvalues are positive.

T is said positive semi-definite if $T(v,v) \geq 0$,for every non-zero vector v.

It means that all eigenvalues are non-negative.

T is said negative definite if $T(v,v) < 0$, for every non-zero vector v.
It means that all eigenvalues are negative.
T is said negative semi-definite if $T(v,v) \leq 0$, for every non-zero vector v.
It means that all eigenvalues are non-positive.
T is indefinite if it is neither positive definite nor negative definite.
The eigenvalues have different signs.

Decomposition in symmetric and anti-symmetric parts in time tensor

The decomposition of time tensors in distinctive parts can help in analyzing them.
Each part can reveal information that might not be easily obtained from the original tensor.
Let T be a second-order time tensor.
If it is not symmetric, it is common to decompose it in a symmetric part S and an antisymmetric part A

$$S = \frac{1}{2}\left(T + T^T\right)$$

$$A = \frac{1}{2}\left(T - T^T\right)$$

$$T = \frac{1}{2}\left(T + T^T\right) + \frac{1}{2}\left(T - T^T\right) = S + A$$

Decomposition in shape and orientation

Let T be a symmetric second-order time tensor on R^3
The eigenvalues give information about the shape.
The eigenvectors give information about the direction.
For a time tensor field, the orientation field defined by the eigenvectors is not a vector field, due to the bidirectionality of eigenvectors.
It is sometimes of interest to consider shape and orientation separately, for the interpolation, or in order to define features on them.

The flows of time

We would the world have to be like, for the flow of time to be an objective feature of reality. We can distinguish four distinct views the flow of time.
1. The view that the past,present and future moment is objectively distinguished.
2. The view that time has a direction; that it is an objective matter which of two nonsimultaneous events is the earlier and which the later.
3. The view that there is something objectively dynamic about time.
4. The view that there is something objectively fluxlike or flowlike about time.
Four views have been sufficiently distinguished, either by defenders or critics of the notion of objective fields.

We present time tensor fluctuations and their effects in driving fluctuations of the spatial and temporal field. The correlations and anticorrelations of time tensor is emphasized. We begin with the properties of the time tensor correlation function.
We consider times tensors fluctuations and the fluctuations of spacetime geometry.
Time tensor, $T_{\mu\nu}$ is the source of spatial and temporal field in the quantity which describes stresses on spatial and temporal fields.
Time tensor doesn't become an operator in quantum field.
Time tensor or temporal stres tensor response to deformation or strain on space and time.
the spatial and temporal tensor is a continuum of the response of space and time.
Time tensor has been applied on a continuum on quantum mechanical phenomena in their response to stres tensor.
Time tensor has been used to assess the spatial structures on time. It can have temporal effects on spatial effects.

The past ,present and future moments

The component of the intuitive flow of time is that it involves a distinguished but continually variable present moment in a frame whose contents are continually changing.
The flow of time is at the concept of presentism, a view which holds that the present moment. We combine the past,present and the future moments.
If the concept of time is coherent or incoherent then why past, present and future moments have changed?
There are many distinguished properties about the past,present and future moments .

Lorentz Tensor

Lorentz tensor is, by definition, an object whose indices transform like a tensor under Lorentz transformations; what we mean by this precisely will be explained below.
4-vector is a tensor with a first rank tensor.
We write a 4-vector in components as

$$G^\mu = \begin{pmatrix} G^0 \\ G^1 \\ G^2 \\ G^3 \end{pmatrix}$$

where we use Greek indices to run over all the spacetime indices, $\mu \in [0,3]$.

The Lorentz transformation

We write the components of the Lorentz transformation matrix in index notation.

We transform the components of a 4-vector from an unprimed frame to a frame which is moving at speed v in the x direction relative to F.

We use the Lorentz transformation

$$\begin{pmatrix}\Delta x'^0\\ \Delta x'^1\\ \Delta x'^2\\ \Delta x'^3\end{pmatrix}=\begin{pmatrix}\gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta x^0\\ \Delta x^1\\ \Delta x^2\\ \Delta x^3\end{pmatrix}$$

where $\gamma=\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$ and $\beta=\dfrac{v}{c}$.

Now we write the components of the Lorentz transformation matrix as S_ν^μ where μ is a row index and ν is a column index, such that

$$S=\begin{pmatrix}S_0^0 & S_1^0 & S_2^0 & S_3^0\\ S_0^1 & S_1^1 & S_2^1 & S_3^1\\ S_0^2 & S_1^2 & S_2^2 & S_3^2\\ S_0^3 & S_1^3 & S_2^3 & S_3^3\end{pmatrix}$$

Then, the Lorentz transformation for Δx^μ can be written in the compact notation

$$(\Delta x')^\mu=\sum_{\nu=0}^{3}S_\nu^\mu\Delta x^\nu=S_\nu^\mu\Delta x^\nu$$

$$(\Delta x')^\mu=S_0^\mu\Delta x^0+S_1^\mu\Delta x^1+S_2^\mu\Delta x^2+S_3^\mu\Delta x^3$$

$$(\Delta x')^\mu=S_0^\mu c\Delta t+S_1^\mu\Delta x+S_2^\mu\Delta y+S_3^\mu\Delta z$$

$$(\Delta x')^0=(c\Delta t')=\gamma(c\Delta t)-\gamma\beta\Delta x$$

$$(\Delta x')^1=(\Delta x')=-\gamma\beta(c\Delta t)+\gamma\Delta x$$

$$(\Delta x')^2=(\Delta y')=\Delta y$$

$$(\Delta x')^3=(\Delta z')=\Delta z$$

is the usual Lorentz transformation to a frame moving in the x direction.

$$\begin{pmatrix}\Delta x'^0\\ \Delta x'^1\\ \Delta x'^2\\ \Delta x'^3\end{pmatrix}=\begin{pmatrix}(c\Delta t')\\ (\Delta x')\\ (\Delta y')\\ (\Delta z')\end{pmatrix}=\begin{pmatrix}\gamma(c\Delta t)-\gamma\beta\Delta x\\ -\gamma\beta(c\Delta t)+\gamma\Delta x\\ \Delta y\\ \Delta z\end{pmatrix}$$

The inverse Lorentz transformation should satisfy

$$(S^{-1})_\nu^\xi S_\nu^\xi=\delta_\nu^\xi$$

where $\delta_\nu^\beta\equiv diag(1,1,1,1)$ is the Kronecker delta.

$$(S^{-1})_\nu^\xi(\Delta x')^\mu=\delta_\nu^\xi\Delta x^\nu=\Delta x^\xi$$

The inverse $(S^{-1})_\nu^\xi$ is also written as S_ν^ξ.

The left index denotes a row while the right index denotes a column, while the top index denotes the frame we're transforming to and the bottom index denotes the frame we're transforming from.

We present the components of S and S^{-1} in their transformations.

$$S=\begin{pmatrix}S_0^0 & S_1^0 & S_2^0 & S_3^0\\ S_0^1 & S_1^1 & S_2^1 & S_3^1\\ S_0^2 & S_1^2 & S_2^2 & S_3^2\\ S_0^3 & S_1^3 & S_2^3 & S_3^3\end{pmatrix}$$

$$S^{-1}=\begin{pmatrix}S_0^0 & -S_0^1 & -S_0^2 & -S_0^3\\ -S_1^0 & S_1^1 & S_1^2 & S_1^3\\ -S_2^0 & S_2^1 & S_2^2 & S_2^3\\ -S_3^0 & S_3^1 & S_3^2 & S_3^3\end{pmatrix}$$

The inverse to the transformation

$$\begin{pmatrix}\Delta x'^0\\ \Delta x'^1\\ \Delta x'^2\\ \Delta x'^3\end{pmatrix}=\begin{pmatrix}\gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta x^0\\ \Delta x^1\\ \Delta x^2\\ \Delta x^3\end{pmatrix}$$

$$\begin{pmatrix}\gamma & \gamma\beta & 0 & 0\\ \gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta x'^0\\ \Delta x'^1\\ \Delta x'^2\\ \Delta x'^3\end{pmatrix}=\begin{pmatrix}\Delta x^0\\ \Delta x^1\\ \Delta x^2\\ \Delta x^3\end{pmatrix}$$

The metric

The metric $L_{\mu\nu}$ is a special Lorentz tensor, which for Minkowski spacetime in our convention is given by

$$L=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}=diag(1,-1,-1,-1)$$

The other convention is to use $L_{\mu\nu}=diag(1,-1,-1,-1)$, which will change around minus signs in various places.

We use the metric to raise and lower Lorentz indices.

By de_nition $G_\mu=L_{\mu\nu}G^\nu$ given a 4-vector G^ν with an upstairs index.

We think that G^ν as a column vector, and G_μ as a row vector.

The inverse metric $L^{\mu\psi}$ with upstairs indices satisfies $L^{\mu\psi}L_{\psi\nu}=\delta_\nu^\mu$ then, we can show that

$L^{\mu\psi} = diag(1,-1,-1,-1)$.

In other words, the Minkowski metric is its own inverse. We can then use the inverse metric to raise indices, as in $G^{\mu} = L^{\mu\psi} G_{\psi}$ given a 4-vector with a lower index.

The Lorentz group

We can write down the condition for an object S to be a Lorentz transformation.

$L_{\mu\nu} = S^{\psi}_{\mu} S^{\xi}_{\nu} L_{\psi\xi}$

It translates to $S^T L S = L$ for S^T the matrix transpose of S.

$$S^T L S = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = L$$

This condition is both necessary and sufficient for a 4×4 matrix S to leave the inner product of any two 4-vectors invariant.

Any group is a set of elements with an operation that combines any two elements to form a third, which satisfies certain properties are closure, associativity, identity, and inverse.

Here, the elements are the S and the group operation is matrix multiplication.

Closure

The product of any 2 Lorentz transformations is another Lorentz transformation.

Associativity

Associativity of Lorentz transformations which follows from the properties of matrix multiplication.

Identity

The identity is $S^{\mu}_{\nu} = \delta^{\mu}_{\nu}$

Inverse

The inverse of S^{μ}_{ν} is $\left(S^{-1}\right)^{\mu}_{\nu} = S^{\mu}_{\nu}$.

Time Tensor analysis in $O(3)$

Rotation in R^3 and temporal space

Coordinate axes are .xed and the physical system is undergoing a rotation in temporal space.

Let be the components of new and old vectors.

$x'_a = \sum_b R_{ab} x_b$

where R_{ab} are elements of matrix which represents rotation.

the relation between x'_a, x_b is linear and homogeneous.

The properties of transformation in temporal space

1. R is an orthogonal matrix,

$RR^T = R^T R = 1$

$R_{ab} R_{ac} = \delta_{bc}$

$R_{ab} R_{cb} = \delta_{ac}$

We will write the orthogonality relations as

$R_{ab} R_{cd} \delta_{ac} = \delta_{bd}$

$R_{ab} R_{cd} \delta_{bd} = \delta_{ac}$

in the product of 2 rotation matrix elements, making row or column indices the same and summed over will give Kronecker δ

2. The combination $\vec{x}^2 = x_a x_a$ invariant under rotations,

$x_a x_a = R_{ac} R_{ab} x_c x_b = x_b x_b$

It can be generalized to the case of 2 arbitrary vectors, $\vec{A}, \vec{B}$ with transformation property

$A'_a = R_{ab} A_b$

$B'_c = R_{cd} B_d$

Then $\vec{A}.\vec{B} = A_a B_a = A_a B_b \delta_{ab}$

Which is called the contraction indices.

3. Transformation of the gradient operators,

$$\frac{\partial}{\partial x'_a} = \frac{\partial}{\partial x_c} \frac{\partial x_c}{\partial x'_a}$$

From $x_b = \left(R^{-1}\right)_{ab} x'_a$

We get then

$$\frac{\partial}{\partial x'_a} = \left(R^{-1}\right)_{ca} \frac{\partial}{\partial x_c}$$

Thus gradient operator transforms by $\left(R^{-1}\right)^T$

However for rotations R is orthogonal $\left(R^{-1}\right)^T = R$

$$\frac{\partial}{\partial x'_a} = R_{ac} \frac{\partial}{\partial x_c}$$

$\partial_k = \frac{\partial}{\partial x_a}$ transform the same x_a.

Time Tensors

We have two vectors and they have the transformation properties,

$A_a \to A'_a = R_{ab} A_b$

$B_c \to B'_c = R_{cd} B_d$

$A'_a B'_c = R_{ab} R_{cd} A_b B_d$

The second rank time tensors are those objects which have the same transformation properties as the product of 2 vectors,

$T_{ac} \to T'_{ac} = (R_{ab}R_{cd})T_{bd}$

Definition of n-th rank time tensors

Cartesian time tensors

$T_{i_1 i_2 \ldots} \to T'_{i_1 i_2 \ldots i_n} = (R_{i_1 j_1})(R_{i_2 j_2})\ldots(R_{i_n j_2})T_{j_1 i_j \ldots j_n}$

These transformations are linear and homogeneous which implies that

If

$T_{j_1 i_j \ldots j_n} = 0$

for all

j_m

Then they zero in other coordinate system.

Time Tensor operations

1. Multiplication by constants

$(cT)_{i_1 i_2 \ldots i_n} = cT_{i_1 i_2 \ldots i_n}$

2. Add tensors of same rank

$(T_1 + T_2)_{i_1 i_2 \ldots i_n} = (T_1)_{i_1 i_2 \ldots i_n} + (T_2)_{i_1 i_2 \ldots i_n}$

3. Multiplication of 2 tensors

$(ST)_{i_1 i_2 \ldots i_n j_1 j_2 \ldots j_m} = S_{1 i_2 \ldots i_n} T_{j_1 j_2 \ldots j_m}$

This will give a tensor of rank which is the sum of the ranks of 2 constituent tensors.

4. Contraction

$S_{abc}T_{ae} = S_{abc}T_{de}\delta_{ad}$

3rd rank time tensor

5. Symmetrization

T_{ab} 2nd rank time tensor

$T_{ab} \pm T_{ba}$ are also 2nd rank tensors.

6. Special numerical time tensors

$RR^T = 1$

$R_{ij}R_{kj} = \delta_{ik}$

$R_{ij}R_{kl}\delta_{jl} = \delta_{ik}$

This means that δ_{ij} _ij can be treated as 2nk rank tensor.

$(\det R)\varepsilon_{abc} = \varepsilon_{ijk}R_{ai}R_{bj}R_{ck}$

ε_{abc} a 3rd rank tensor.

Useful identities for ε_{abc}

$\varepsilon_{ijk}\varepsilon_{ijl} = 2\delta_{kl}$

$\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$

General notation for time tensor transformation

$x'_a = R_{ab}x_b$

$R_{ab} = \frac{x'_a}{x_b}$

Transformation Law of Time Tensors in SU(N)

The $SU(n)$ group consists of $n \times n$ unitary matrices with unit determinant. We can regard these matrices as linear transformations on an n dimensional complex vector space C_n.

Thus any vector

$\psi_i = (\psi_1, \psi_2 \ldots, \psi_n)$

in C_n is mapped by an $SU(n)$ transformation U_{ij}; as

$\psi_i \to \psi'_i = U_{ij}\psi_j$

Thus ψ'_i also belong to C_n with $UU^t = U^tU = 1$ and $\det U = 1$.

We can define a scalar product for two vectors

$(\psi, \phi) \equiv \psi_i^* \phi_i$

which is invariant under $SU(n)$ transformation.

The transformation law for the conjugate vector is given by,

$\psi_i^* \to \psi_i'^* = U_{ij}^* \psi_j^* = \psi_j^* U_{ji}^t$

It is convenient to introduce upper and lower indices to write

$\psi^i \equiv \psi_i^*$

$U_i^j \equiv U_{ij}$

$U_j^i \equiv U_{ij}^*$

Thus complex conjugation just changes the lower indices to upper ones, and vice versa.

In these notation,

$\psi_i \to \psi'_i = U_i^j \psi_j$

$\psi^i \to \psi'^i = U_j^i \psi^j$

The $SU(n)$ invariant scalar product is then

$(\psi, \phi) = \psi^i \phi_i$

and the unitary condition becomes

$U_k^i U_j^k = \delta_j^i$

where the Kronecker delta is defined as

$$\delta_j^i = \delta_{ij} = \begin{cases} 1 & if \quad i = j \\ 0 & otherwise \end{cases}$$

We call this a contraction of indices. The ψ_i are the basis for the $SU(n)$ defining representation also called the fundamental or vector representation and denoted as n, while the ψ^i are the basis for the conjugate representation, n^*:

Higher rank time tensors are de.ned as those quantities which have the same transformations properties as the direct products of vectors.

Thus tensors generally have both upper and lower indices with the transformation law,

$$\psi'^{i_1 i_2 \ldots i_p}_{j_1 j_2 \ldots j_q} = \left(U^{i_1}_{k_1} \quad U^{i_2}_{k_2} \quad \ldots \quad U^{i_p}_{k_p}\right)\left(U^{l_1}_{j_1} \quad U^{l_2}_{j_2} \quad \ldots \quad U^{l_q}_{j_q}\right)\psi^{k_1 k_2 \ldots k_p}_{l_1 l_2 \ldots l_q}$$

They correspond to the basis for higher-dimensional representations.

Invariant time tensors

The Kronecker delta and Levi-Civita symbol are invariant tensors under $SU(n)$ transformations.

1. From the unitarity condition of $U^i_k U^k_j = \delta^i_j$

We immediately have

$$\delta^i_j = U^i_k U^l_j \delta^k_l$$

Hence δ^i_j,even though do not change under the $SU(n)$ transformations, behaves as if they are second rank tensors.

They can be used to contract indices of other tensor to produce a tensor of lower rank.

If ψ^k_{ij} is a 3rd rank tensor,

$$\psi^k_{ij} \to \psi'^k_{ij} = U^k_a U^b_i U^c_j \psi^a_{bc}$$

the contracting with δ^k_i gives

$$\psi'^k_{ij}\delta^k_i = \delta^k_i U^k_a U^b_i U^c_j \psi^a_{bc} = U^c_j \delta^a_b \psi^a_{bc}$$

where we have used

$$\delta^i_j = U^i_k U^l_j \delta^k_l$$

This gives a tensor of rank 1or vector.

2. The Levi-Civita symbol is de.ned as the totally antisymmetric quantity,

$$\varepsilon^{i_1 i_2 \ldots i_n} = \varepsilon_{i_1 i_2 \ldots i_n}$$

This is also an invariant tensor, because from the property of the determinant we have

$$(\det U)_{\varepsilon_{i_1 i_2 \ldots i_n}} = U^{j_1}_{i_1} U^{j_2}_{i_2} \ldots U^{j_n}_{i_n} \varepsilon_{i_1 i_2 \ldots i_n}$$

Since $\det U$ in $SU(n)$; $\varepsilon_{i_1 i_2 \ldots i_n}$ in can be treated as $n-th$ rank tensor.

We can use this to change the rank of a time tensor.

Permutation symmetry and time tensors

Generally time tensors we have just define are basis for reducible representation of $SU(n)$: To decompose them into irreducible representations we use the following property of these tensors.

The permutation of upper or lower indices commutes with the $SU(n)$ transformations, as the latter consists of product of identical U_{ij} or U^*_{ij} .

Consider the second rank time tensor ψ_{ij} whose transformation is given by

$$\psi'_{ij} = U^a_i U^b_j \psi_{ab}$$

Since U is the same, we can write the indices to get

$$\psi'_{ji} = U^b_j U^a_i \psi_{ba}$$

Thus the permutation of indices in the tensor does not change the transformation law.

If P_{12} is the permutation operator which interchanges the first two indices,

$$P_{12}\psi_{ij} = \psi_{ji}$$

then P_{12} commutes with the group transformation

$$P_{12}\psi'_{ij} = U^a_i U^b_j (P_{12}\psi_{ab})$$

This property can be used to decompose ψ_{ij} as follows.

First we form eigenstates of the permutation operator P_{12} by symmetrization or antisymmetrization,

$$S_{ij} = \frac{1}{2}(\psi_{ij} + \psi_{ji})$$

$$A_{ij} = \frac{1}{2}(\psi_{ij} - \psi_{ji})$$

$$P_{12}S_{ij} = S_{ji}$$

$$P_{12}A_{ij} = -A_{ji}$$

In group theory, S_{ij} form basis of an one-dimensional representation of the permutation group S_2 and A_{ij} the basis for another representation.

It is S_{ij} and A_{ij} will not mix under the $SU(n)$ transformations,

$$S'_{ij} = U^a_i U^b_j S_{ab}$$

$$A'_{ij} = U^a_i U^b_j A_{ab}$$

This shows that the second rank tensor ψ_{ij} decomposes into S_{ij} ; and A_{ij} in such a way that group transformations never mix parts with different symmetries.

It turns out that S_{ij} ; and A_{ij} can not be decomposed any further and they thus form the basis of irreducible representations of $SU(n)$.

This can be generalized to tensors of higher rank hence the possibility of mixed symmetries with the result that the basis for irreducible representations of $SU(n)$ correspond to tensors with definite permutation symmetry among the positions of its indices.

Time in classical mechanics

Quantum mechanics was based on classical Hamiltonian mechanics. In Hamiltonian mechanics a physical system is described by N pairs of canonical conjugate dynamical variables, ξ_k and Φ_k, which satisfy the Poisson-bracket relations:

$$\{\xi_k, \Phi_l\} = \delta_{kl}$$

$$\{\xi_k, \xi_l\} = \{\Phi_k, \Phi_l\} = 0$$

These variables define a point of the $2N$ dimensional so-called 'phase space' of the system. The time evolution of the system is generated by the Hamiltonian, a function of the canonical variables,

$$H = H\{\xi_k, \Phi_k\}$$

$$\frac{d\xi_k}{dy} = \{\xi_k, H\}$$

$$\frac{d\Phi_k}{dy} = \{\Phi_k, H\}$$

H does not explicitly depend on time.

The ξ_k and Φ_k are generalized variables; they need not be positions and momenta, but may be angles, angular momenta, et cetera. However, if the system is a system of point particles the canonical variables are usually taken to be the positions q_n and momenta p_n of the particles Tthree-vectors are in bold type and the subscript denotes the n-th particle. Let us consider the relation of this scheme with space and time.

In all of physics, with the exception of General Relativity, physical systems are supposed to be situated in a three-dimensional Euclidean space. The points of this space will be given by Cartesian coordinates $s = (x, y, z)$. Together with the time parameter t they form the coordinates of a continuous, independently given, space-time background. How the existence of this space and time is to be justified is an important and difficult problem into which we will not enter; we just take this assumption as belonging to the standard formulation of classical and quantum mechanics and of special relativity.

The (3+1) dimensional space-time must be sharply distinguished from the $2N$ dimensional phase space of the system, and the space-time coordinates (s, t) must be sharply distinguished from the dynamical variables (ξ_k, Φ_k) characterizing material systems in space-time. In particular, the position variable q of a point particle must be distinguished from the coordinate s of the space-point the particle occupies, although we have the numerical relation:

$$q_x = x$$

$$q_y = y$$

$$q_z = z$$

A point particle is a material system having a mass, a position, a velocity, an acceleration, while s is the coordinate of a fixed point of empty space.

The symmetries space and time are supposed to possess in physics. It is assumed that three-dimensional space is isotropic or rotation symmetric and homogeneous or translation symmetric and that there is translation symmetry in time. In special relativity the space-time symmetry is enlarged by Lorentz transformations which mix s and t, transforming them as the components of a four-vector.

Individual physical systems in space-time need not show these symmetries; only the physical laws, that is the totality of physically allowed situations and processes, must show them. A physical system need not be rotation invariant, and a position variable of a physical system need not be part of a four-vector.

The generators of translations in space and time are the total momentum P and the total energy H, respectively. The generator of space rotations is the total angular momentum J. We shall in particular be interested in the behavior of dynamical variables under translations in time and space. For an infinitesimal translation δT in time we have:

$$\delta\xi_k = \{\xi_k, H\}\delta T$$

$$\delta\Phi_k = \{\Phi_k, H\}\delta T$$

and for an infinitesimal translation δm in space:

$$\delta\xi_k = \{\xi_k, P\}.\delta m$$

$$\delta\Phi_k = \{\Phi_k, P\}.\delta m$$

The Hamiltonian and the generator of time translations of the time evolution of the system, is so much more prominent in classical mechanics than is the total momentum, the generator of translations in space. The reason for this is that the dynamical variables of the systems which are traditionally studied in classical mechanics, namely particles and rigid bodies, transform trivially under space translations.

We conclude that in classical physics a sharp distinction must be made between the universal space-time coordinates and the dynamical variables of specific physical systems situated in space-time. Particles and clocks are physical systems having dynamical variables which behave in much the same way as the space and time coordinates, respectively, and may thus serve to indicate the 'position' of the system in space and in time. Point particles and clocks are non-covariant concepts. If one is to look for physical systems which transform covariantly under relativistic space-time transformations one must consider fields.

3 Time in Quantum Mechanics

In quantum mechanics the situation is essentially not different. The theory supposes a fixed, unquantized space-time background, the points of which are given by classical number coordinates s,t. The space-time symmetry transformations are expressed in terms of these coordinates.

Dynamical variables of physical systems, on the other hand, are quantized: they are replaced by self-adjoint operators on a Hilbert space. All formulas of the preceding section remain valid if the Poisson-brackets are replaced by commutators according to $\{,\} \to (i\hbar)^{-1}[,]$.

In particular, the canonical variables are replaced by operators satisfying the commutation relations:

$$[\xi_k, \Phi_l] = i\hbar\delta_{kl}$$

$$[\xi_k, \xi_l] = [\Phi_k, \Phi_l] = 0$$

Symbols representing dynamical variables are supposed to be operators.

Thus, for a point particle,

$$[q_i, p_j] = i\hbar\delta_{ij}$$

$$[q_i, q_j] = [p_i, p_j] = 0$$

where $ij = x, y, z$ denote the Cartesian components of the position q and momentum p of the particle. These relations have the well-known representation where q is the multiplication operator and p the corresponding differentation operator. Both these operators are unbounded and have the full real axis as their spectrum. However, if the position wavefunctions are required to obey periodic boundary conditions the eigenvalues of p become discrete, and if the position wavefunctions are required to vanish at the endpoints of a finite interval particle in a box a self-adjoint momentum operator does not even exist. Corresponding statements hold for q. Similarly, since the wavefunctions of an angle variable must obey a periodic boundary condition, the eigenvalues of the corresponding angular momentum operator are discrete. Discrete energy eigenvalues are of course the hallmark of quantum mechanics. Nobody would conclude from these facts that something is totally wrong with the notions of position, momentum, angular momentum or energy in quantum mechanics.

The three picture of Quantum Mechanics

The Schrödinger Picture

Quantum systems are regarded as wave functions which solve the Schrödinger equation.

Observables are represented by Hermitian operators which act on the wave function. In the Schrödinger picture, the operators stay fixed while the Schrödinger equation changes the basis with time.

The Schrödinger Picture is

$$|\Psi\rangle = |\Psi(t)\rangle$$

$$\hat{O} \neq \hat{O}(t)$$

In the Schrödinger picture, the operators are constant while the basis changes is time via the Schrödinger equation.

$$\frac{d}{dt}|\Psi_s(t)\rangle = -\frac{i}{\hbar}\hat{H}_s|\Psi_s(t)\rangle$$

$$\hat{O} \neq \hat{O}(t)$$

The differential equation leads to an expression for the wave function.

$$|\Psi_s(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}_s t}|\Psi_s(0)\rangle$$

A quantum operator as the argument of the exponential function is defined in terms of its power series expansion.

$$e^{-\frac{i}{\hbar}\hat{H}_s t} = \sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{i}{\hbar}\hat{H}_s t\right)^n$$

$$e^{-\frac{i}{\hbar}\hat{H}_s t} = 1 - \frac{i}{\hbar}\hat{H}_s t - \frac{1}{2\hbar}\left(\hat{H}_s t\right)^2 + \ldots$$

We presents power series expansion of time in a quantum operator as the argument of the exponential function.

The Heisenberg Picture

In the Heisenberg picture, it is the operators which change in time while the basis of the space remains fixed.

Heisenberg's matrix mechanics actually came before Schrödinger's wave mechanics but were too mathematically different to catch on.

A fixed basis is, in some ways, more mathematically pleasing. This formulation also generalizes more easily to relativity.
It is the nearest analog to classical physics.
The Heisenberg Picture is

$$\left|\Psi\right\rangle = \left|\Psi(t)\right\rangle$$

$$\hat{O} \neq \hat{O}(t)$$

In the Heisenberg picture, the basis does not change with time. This is accomplished by adding a term to the Schrödinger states to eliminate the time-dependence.

$$\left|\Psi_H\right\rangle = e^{-\frac{i}{\hbar}\hat{H}_s t}\left|\Psi_s(t)\right\rangle = \left|\Psi_s(0)\right\rangle$$

The quantum operators, however, do change with time.

$$\hat{O} = \hat{O}(t)$$

We may define operators in the Heisenberg picture via expectation values.

$$\left\langle O\right\rangle = \left\langle \Psi_s(t)\right|\hat{O}\left|\Psi_s(t)\right\rangle$$

$$\left\langle O\right\rangle = \left\langle \Psi_s(0)\right|e^{\frac{i}{\hbar}\hat{H}_s t}\hat{O}e^{-\frac{i}{\hbar}\hat{H}_s t}\left|\Psi_s(0)\right\rangle$$

$$\left\langle O\right\rangle = \left\langle \Psi_H\right|e^{\frac{i}{\hbar}\hat{H}_s t}\hat{O}e^{-\frac{i}{\hbar}\hat{H}_s t}\left|\Psi_H\right\rangle$$

Operators in the Heisenberg picture, therefore, pick up time dependence through unitary transformations.

$$\hat{O}_H = e^{\frac{i}{\hbar}\hat{H}_s t}\hat{O}e^{-\frac{i}{\hbar}\hat{H}_s t}$$

We may ascertain the operators' time-dependence through differentiation.

$$\frac{d\hat{O}}{dt} = \frac{i}{\hbar}\hat{H}_s e^{\frac{i}{\hbar}\hat{H}_s t}\hat{O}e^{-\frac{i}{\hbar}\hat{H}_s t} - \frac{i}{\hbar}e^{\frac{i}{\hbar}\hat{H}_s t}\hat{O}\hat{H}_s e^{-\frac{i}{\hbar}\hat{H}_s t} + \frac{\partial\hat{O}}{\partial t}$$

$$\frac{d\hat{O}}{dt} = \frac{i}{\hbar}\left[\hat{H}_s, \hat{O}\right] + \frac{\partial\hat{O}}{\partial t}$$

The Dirac Picture

In the Dirac or, interaction picture, both the basis and the operators carry time-dependence.
The interaction picture allows for operators to act on the state vector at different times and forms the basis for quantum field theory and many other newer methods.
The Dirac Picture is

$$\left|\Psi\right\rangle = \left|\Psi(t)\right\rangle$$

$$\hat{O} = \hat{O}(t)$$

The Dirac picture is a sort of intermediary between the Schrödinger picture and the Heisenberg picture as both the quantum states and the operators carry time dependence.
Consider some Hamiltonian in the Schrödinger Picture containing both a free term and an interaction term.
It is especially useful for problems including explicitly timedependent interaction terms in the Hamiltonian.

$$\hat{H}_s = \hat{H}_{0,s} + \hat{V}_S$$

In the interaction picture, state vectors are again defined as transformations of the Schrödinger states. These state vectors are transformed only by the free part of the Hamiltonian.

$$\left|\Psi_I(t)\right\rangle == e^{\frac{i}{\hbar}\hat{H}_{0,s}t}\left|\Psi_s(t)\right\rangle$$

The Dirac operators are transformed similarly to the Heisenberg operators.

$$\hat{O}_I(t) = e^{\frac{i}{\hbar}\hat{H}_{0,s}t}\hat{O}_s e^{-\frac{i}{\hbar}\hat{H}_{0,s}t}$$

Consider the interaction picture counterparts to the Schrödinger Hamiltonian operators.

$$\hat{H}_{0,I}(t) = e^{\frac{i}{\hbar}\hat{H}_{0,s}t}\hat{H}_{0,s}e^{-\frac{i}{\hbar}\hat{H}_{0,s}t}$$

$$\hat{H}_{0,I}(t) = \hat{H}_{0,s}$$

$\hat{H}_{0,s}$ commuting with itself in the series expansion of the exponential.

The interacting term of the Schrödinger Hamiltonian is defined similarly.

$$\hat{V}_I(t) = e^{\frac{i}{\hbar}\hat{H}_{0,s}t}\hat{V}_I e^{-\frac{i}{\hbar}\hat{H}_{0,s}t}$$

States in the interaction picture evolve in time similarly to Heisenberg states.

$$\frac{d}{dt}|\Psi_I(t)\rangle = \frac{i}{\hbar}\hat{H}_{0,s}|\Psi_I(t)\rangle + e^{\frac{i}{\hbar}\hat{H}_{0,s}t}\frac{d}{dt}|\Psi_s(t)\rangle$$

$$\frac{d}{dt}|\Psi_I(t)\rangle = e^{\frac{i}{\hbar}\hat{H}_{0,s}t}\hat{V}_s e^{-\frac{i}{\hbar}\hat{H}_{0,s}t}|\Psi_I(t)\rangle$$

$$\frac{d}{dt}|\Psi_I(t)\rangle = \hat{V}_I(t)|\Psi_I(t)\rangle$$

Therefore, the state vectors in the interaction picture evolve in time according to the interaction term only.

$$\frac{d}{dt}|\Psi_I(t)\rangle = \hat{V}_I(t)|\Psi_I(t)\rangle$$

It can be easily shown through differentiation that operators in the interaction picture evolve in time according only to the free Hamiltonian.

$$\frac{d\hat{O}_I}{dt} = \frac{i}{\hbar}\left[\hat{H}_{0,I}, \hat{O}\right] + \left(\frac{d\hat{O}_I}{dt}\right)$$

Tensor equations

Scalar valued function of second-order tensors

Let $f(t)$ be a scalar valued function of the second order time tensor t

$$f = f(t)$$

f is a scalar valued function

$t = t_{pq}e_p \otimes e_q$ is a tensor valued time function

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial t_{pq}} e_p \otimes e_q$$

Ψ is a scalar valued probability function

$H = H_{km}e_k \otimes e_m$ is a tensor valued energy function

$G = G_{km}e_k \otimes e_m$ is a tensor valued energy function

$R = R_{km}e_k \otimes e_m$ is a tensor valued energy function

Tensor valued function of second-order tensors

Let $F(t)$ be a second order tensor valued function of the second order time tensor t

$$F = F(t)$$

$F = F_{ij}e_i \otimes e_j$ is a tensor valued function

$t = t_{pq}e_p \otimes e_q$ is a tensor valued time function

$$\frac{\partial F}{\partial t} = \frac{\partial F_{ij}}{\partial t_{pq}} e_i \otimes e_j \otimes e_p \otimes e_q$$

$\Psi = \Psi_{ij}e_i \otimes e_j$ is a tensor valued probability function

$H = H_{km}e_k \otimes e_m$ is a tensor valued energy function

$G = G_{km}e_k \otimes e_m$ is a tensor valued energy function

$R = R_{km}e_k \otimes e_m$ is a tensor valued energy function

Time Tensors on Schrodinger's Equation

Elementary quantum mechanics start by considering a single point particle. The particle position is commonly denoted s instead of q and the time-dependent wave function is written $\Psi(s,t)$.

This notation is misleading in several ways. It gives the false impression that the wave function is just an ordinary wave in three-dimensional space, an impression which is reinforced by the usual discussions of double slit interference, quantum tunneling.

$$i\hbar\frac{\partial\Psi(s,t)}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi(s,t)}{\partial x^2}+\frac{\partial^2\Psi(s,t)}{\partial y^2}+\frac{\partial^2\Psi(s,t)}{\partial z^2}\right)+U(s)\Psi(s,t)$$

$$H=\frac{\hbar^2}{2m}\nabla^2+U(s,t)$$

$$i\hbar\frac{\partial\Psi(s,t)}{\partial t}=H\Psi(s,t)$$

is Schrodinger's equation.

Scalar valued function of second-order tensoral Schrodinger's Equation

$i\hbar\frac{\partial\Psi(s,t)}{\partial t}=H\Psi(s,t)$ transform to

$$i\hbar\frac{\partial\Psi(s,t_{pq}e_p\otimes e_q)}{\partial t_{pq}}e_p\otimes e_q=H_{km}\Psi(s,t_{pq}e_p\otimes e_q)e_k\otimes e_m$$

Tensor valued function of second-order tensoral Schrodinger's Equation

$i\hbar\frac{\partial\Psi(s,t)}{\partial t}=H\Psi(s,t)$ transform to

$$i\hbar\frac{\partial\Psi_{ij}\left(s,t_{pq}e_p\otimes e_q\right)}{\partial t_{pq}}e_i\otimes e_j\otimes e_p\otimes e_q=H_{km}\Psi_{ij}(s,t_{pq}e_p\otimes e_q)e_i\otimes e_j\otimes e_k\otimes e_m$$

Time Tensors on Relativistic Quantum Mechanics

The particles and rigid bodies effect in classical physics the notion of the position of a physical system seems that in non-relativistic quantum mechanics, although we have seen that position-operators may have discrete eigenvalues.

In relativistic quantum mechanics the concept of a position-operator encounters serious problems. T.D. Newton and E.P. Wigner showed that the required behavior of a position operator under space translations and rotations almost uniquely determines this operator.

The resulting operator q is non-covariant and, due to its energy being positive, has the ugly property that a state which is an eigenstate of it at a given time or "localized' state" will be spread out over all of space an infinitesimal time later.

$$-\hbar^2\frac{\partial^2\Psi(s,t)}{\partial t^2}=-\hbar^2c^2\left(\frac{\partial^2\Psi(s,t)}{\partial x^2}+\frac{\partial^2\Psi(s,t)}{\partial y^2}+\frac{\partial^2\Psi(s,t)}{\partial z^2}\right)+m^2c^4\Psi(s,t)$$

$$G=\hbar^2c^2\nabla^2+m^2c^4$$

$$-\hbar^2\frac{\partial^2\Psi(s,t)}{\partial t^2}=G\Psi(s,t)$$

is Klein-Gordon Equation.

Scalar valued function of second-order tensoral Klein-Gordon Equation

$-\hbar^2\frac{\partial^2\Psi(s,t)}{\partial t^2}=G\Psi(s,t)$ transform to

$$-\hbar^2\frac{\partial^2\Psi\left(s,t_{pq}e_p\otimes e_q\right)}{\partial t_{pq}^2}e_p\otimes e_q=G_{km}\Psi(s,t_{pq}e_p\otimes e_q)e_k\otimes e_m$$

Tensor valued function of second-order tensoral Klein-Gordon Equation

$-\hbar^2\frac{\partial^2\Psi(s,t)}{\partial t^2}=G\Psi(s,t)$ transform to

$$-\hbar^2\frac{\partial^2\Psi_{ij}(s,T_{pq}e_p\otimes e_q)}{\partial T_{pq}^2}e_i\otimes e_j\otimes e_p\otimes e_q=G_{km}\Psi_{ij}(s,T_{pq}e_p\otimes e_q)e_i\otimes e_j\otimes e_k\otimes e_m$$

Dirac equation

In the case of a Dirac $spin\frac{1}{2}$ particle the Newton-Wigner position operator turns out to be identical with the Foldy-Wouthuysen "mean position" operator. This case is particularly interesting for when the Dirac-equation was conceived in 1928 the space-part s of the four-vector appearing as the argument of Dirac's four-spinor "wavefunction $\Psi(s,ct)$, was identified with the position of the electron. This had the embarrassing consequence that the corresponding 'velocity' of the electron would always be found to be the velocity of light. It took twenty years before this problem was solved and the proper position-operator q was identified. The notation s for both the particle position and the space-coordinate certainly has obscured the issue.

The basic quantity is the operator field $\Psi(s,t)$ which is parametrized by the classical number coordinates of spacetime points.

$$i\hbar\frac{\partial\Psi(s,t)}{\partial t} = \beta mc^2\Psi(s,t) + c\left(\sum_{k=1}^{3}\alpha_k p_k\right)\Psi(s,t)$$

$$R = \beta mc^2 + c\left(\sum_{k=1}^{3}\alpha_k p_k\right)$$

$$i\hbar\frac{\partial\Psi(s,t)}{\partial t} = R\Psi(s,t)$$

is Dirac Equation.

Scalar valued function of second-order tensoral Dirac Equation

$i\hbar\frac{\partial\Psi(s,t)}{\partial t} = R\Psi(s,t)$ transform to

$$i\hbar\frac{\partial\Psi(s,t_{pq}e_p\otimes e_q)}{\partial t_{pq}}e_p\otimes e_q = R_{km}\Psi(s,t_{pq}e_p\otimes e_q)e_k\otimes e_m$$

Tensor valued function of second-order tensoral Dirac Equation

$i\hbar\frac{\partial\Psi(s,t)}{\partial t} = R\Psi(s,t)$ transform to

$$i\hbar\frac{\partial\Psi_{ij}(s,t_{pq}e_p\otimes e_q)}{\partial t_{pq}}e_i\otimes e_j\otimes e_p\otimes e_q = R_{km}\Psi_{ij}(s,t_{pq}e_p\otimes e_q)e_i\otimes e_j\otimes e_k\otimes e_m$$

4 Conclusion

Time tensor follows that stress tensors and spacetime metric have to be regarded as spatial and temporal states coupled through the determination of ultimate spatial and temporal states in spacetime probing requires their fluctuations. We want to define time tensor for consistent description of the fluctuations of stress tensors and space and time curvatures, of the associated spatial and temporal mechanisms. We established temporal fluctations for studying the interplay between space and time fluctuations and curvatures. We presented Three Pictures of Quantum Mechanics on time tensors.

References

[1] Einstein A., Ann. Physik 17 132 (1905); Phys. Z. 10 185 (1909); Mitt. Phys. Ger. Zurich 16 97 (1916)
[2] Einstein A., Ann. Physik 17 549 (1905); Phys. Z. 18 121 (1917)
[3] Casimir H.B.G., Proc. K. Ned. Akad. Wet. 51 793 (1948); Brown L.S. andMaclay G.J., Phys. Rev. 184 1272 (1969); Plunien G., M¨uller B. and Greiner W., Phys. Rep. 134
87 (1986); Jaekel M.T. and Reynaud S., J. Physique I-1 1395 (1991) and references therein
[4] Barton G., J. Physics A24 991, 5533 (1991); in Cavity Quantum Electrodynamics (Supplement: Advances in Atomic, Molecular and Optical Physics), P.Berman (ed) (Academic Press, 1994)
[5] Callen H.B. and Welton T.A., Phys. Rev. 83 34 (1951)
[6] Jaekel M.T. and Reynaud S., Quant. Opt. 4 39 (1992); Jaekel M.T. and Reynaud S., J. Physique I-2 149 (1992)
[7] Moore G.T., J. Math. Phys. 11 2679 (1970); De Witt B.S., Phys. Rep. 19 295 (1975); Fulling S.A. and Davies P.C.W., Proc. R. Soc. London A348 393 (1976); Ford L.H. and Vilenkin A., Phys. Rev. D25 2569 (1982)
[8] Jaekel M.T. and Reynaud S., EuroPhys. Lett. 13 301 (1990); Pace A.F., Collett M.J. and Walls D.F., Phys.Rev. A47 3173 (1993)
[9] Braginskii V.B. and Khalili F.Ya., Quantum measurement (Cambridge, 1992)
[10] Jaekel M.T. and Reynaud S., J. Physique I-3 1 (1993)
[11] Wheeler J.A., Ann. Physics 2 604 (1957); DeWitt B.S.,in Gravitation, An Introduction to Current Research,L.Witten (ed) (Wiley, 1962), p.266; v. Borzeszkowski H.H. and Treder H.J., Ann. Physik 40 287 (1983); Padmanabhan T., Class. Quant. Grav. 4 L107 (1987)
[12] Feynman R.P., Acta Phys. Polonica 24 711 (1963); Zeldovich Ya.B. and Grishchuk L.P., Usp. Fiz. Nauk 149 695 (1986) [Sov. Phys. Usp. 29 780 (1986)][13] Weinberg S., Phys. Rev. 138 B988 (1965)
[14] Grishchuk L.P., Usp. Fiz. Nauk 121 629 (1977) [Sov.Phys. Usp. 20 319 (1977)]; Grishchuk L.P. and Sidorov Y.V., Phys. Rev. D42 3413 (1990); Grishchuk L., Haus H.A. and Bergman K., Phys. Rev. D46 1440 (1992)
[15] Braginskii V.B. andMenskii M.B., Pisma Zh. Eksp. Teor.Fiz. 13 585 (1971) [Sov. Phys. JETP Letters 13 417 (1971)]; Misner C.W., Thorne K.S. and Wheeler J.A.,Gravitation (Freeman and Company, 1973)

[16] Jaekel M.T. and Reynaud S., Phys. Lett. A185 143 (1994)
[17] Einstein A., Ann. Physik 49 769 (1916)
[18] Blohincev D.I., Nuovo Cim. XVI 382 (1960)
[19] Rosenfeld L., Nucl. Phys. 40 353 (1963)
[20] De Witt B.S., J. Math. Phys. 3 619 (1962); Unruh W.G.,in Quantum Theory of Gravity, Christensen S.M. (ed) (Bristol, 1984) p.234
[21] Zeldovich Ya.B. and Pitaevskii L.P., Comm. Math. Phys.23 185 (1971); Zeldovich Ya.B. and Starobinskii A.A.,Zh. Eksp. Teor. Fiz. 61 216 (1971) [Sov. Phys. JETP 34 1159 (1972)]; Pisma Zh. Eksp. Teor. Fiz. 26 373 (1977) [Sov. Phys. JETP Letters 26 252 (1977)]
[22] Birrell N.D. and Davies P.C.W., Quantum Fields in Curved Space (Cambridge, 1982); Fulling S.A., Aspects of Quantum Field Theory in Curved Spacetime (Cambridge,1989)
[23] Itzykson C. and Zuber J.B., Quantum Field Theory (Mac Graw Hill, 1985)
[24] Landau L.D. and Lifshitz E.M., Cours de Physique Th´eorique: Th´eorie des Champs (Mir, 1970) §95.
[25] Utiyama R. and DeWitt B., J. Math. Phys. 3 608 (1962); Wald R.M., Phys. Rev. D17 1477 (1978)
[26] Brown M.R. and Ottewill A.C., Phys. Rev. D34 1776 (1986)
[27] Capper D.M., Duff M.J. and Halpern L., Phys. Rev. D10 461 (1974)
[28] Horowitz G.T., Phys. Rev. D21 1445 (1980)
[29] Horowitz G.T. and Wald R.M., Phys. Rev. D17 414 (1978); Hartle J.B. and Horowitz G.T., Phys. Rev. D24 257 (1981)
[30] 't Hooft G. and Veltman M., Ann. Inst. Henri Poincar´e 20 69 (1974); Duff M.J., in Quantum Gravity, C.J.Isham,R.Penrose and D.W.Sciama (eds) (Clarendon Press,1975), p.78; Deser S. ibidem, p.136
[31] Weinberg S., Gravitation and Cosmology (Wiley, 1972), ch.7.6
[32] Will C.M., Science 250 770 (1990); Damour T. in Gravitation and Quantizations, B.Julia and J.Zinn-Justin (eds) (North Holland, 1994)
[33] Capper D.M., Leibbrandt G. and Ramon Medrano M., Phys. Rev. D8 4320 (1973)

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