Effectiveness of k_T for Forecasting Solar Radiation Using a Five Parameter Mathematical Model

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Abstract: The effectiveness of clearness index for forecasting solar radiation using a five parameter mathematical model has been used to produce efficient model equations for Calabar, Enugu, Gusau, Jos Maiduguri, Yola and Zaria. High correlation coefficients have been obtained for both yearly and seasonal fits, where, for most of the stations $R_a^2 > 0.97$, Se < 0.02, SeH < 0.2, LPE < 5 % for all the stations. Calabar, Gusau, Maiduguri, Yola and Zaria showed high correlation with k_T as the dependent variable, Enugu and Jos showed high correlation with H and H' respectively as the dependent variables respectively.

Keywords: clearness index, forecasting, solar radiation, five parameter

1. Introducation

The usefulness of solar radiation and its various applications to which it can be put to useful forms by mankind has already being enumerated in a number of literatures [1] – [9]. Various equations correlating total solar radiation on a horizontal surface, H, unavailable solar radiation, $H' = H_0 - H$ (H_0 being the total extraterrestrial solar radiation on a horizontal surface) or clearness index, k_T (= H/H_0), in terms of some important climatological parameters such as relative sunshine duration, S/S_0 (S is the bright sunshine duration and S_0 is the day – length, both measured in hours), relative humidity, R and maximum air temperature, T_m have been used to estimate solar radiation [1] – [9]. Ododo and Adam [1] have used relative sunshine duration, maximum air temperature and relative humidity to model total solar radiation, the equations were used to analyze the data for Calabar and Yola, the result shows high correlation for yearly fits in a six parameter (k = 6) and eight parameter (k = 8) equations respectively. Aidan *et al*

[5] have used relative sunshine duration, relative humidity and cloud cover to model k_T or H' in a five parameter model equation, the equations where applied to seven (7) stations (Bauchi, Jos, Kano, Maiduguri, Nguru, Potiskum and Yola) the results show high correlation for majority of the stations when H' is used as the dependent variable. In this paper, we have used relative sunshine duration, maximum air temperature and relative humidity in a proposed five parameter model equation which utilizes k_T, H or H' as the dependent variable to model available data for seven (7) meteorological stations in Nigeria, the stations are Calabar, Enugu, Gusau, Jos, Maiduguri, Yola and Zaria, results obtained will be compared with existing results where available.

2 The Model Equations

In the present work, we have proposed equations of the following format to analyze available data:

$$k_T = \alpha_{000} + \alpha_{100}\left(S/S_0\right) + \alpha_{010}T_m + \alpha_{102}\left(S/S_0\right)R^2 + \alpha_{462}\left(S/S_0\right)^4 T_m^6 R^2 \tag{1}$$

$$H = \alpha_{000} + \alpha_{100}\left(S/S_0\right) + \alpha_{010}T_m + \alpha_{102}\left(S/S_0\right)R^2 + \alpha_{462}\left(S/S_0\right)^4 T_m^6 R^2 \tag{2}$$

and

$$H' = \alpha_{000} + \alpha_{100}\left(S/S_0\right) + \alpha_{010}T_m + \alpha_{102}\left(S/S_0\right)R^2 + \alpha_{462}\left(S/S_0\right)^4 T_m^6 R^2 \tag{3}$$

where the α_{ijk} (i,j,k = 0,1 2, 3, 4, 5, 6) are constant coefficients, for brevity, we have adopted some of the notations used by [1]. $k_T = H/H_0$ and $H' = H_0 - H$ Equations (1) – (3) are derivable from the general form proposed by [8]

$$X = \sum_{i,j,k=0}^{6} \alpha_{ijk}\left(S/S_0\right)^i T_m^j R^k \tag{4}$$

where X assumes any of the forms k_T, H or H'

3 Data and analysis

The data for the seven (7) stations (Calabar, Enugu, Gusau, Jos, Maiduguri, Yola and Zaria) have been tabulated elsewhere [1], [7] – [9]. Multiple linear regression was carried out on equations (1) – (3) for both yearly and seasonal variations, the seasonal variations considered were the dry season (November – April) and the wet season (May - October). The goodness-of-fit indices used are the adjusted coefficient of determination (R_a^2), standard error Se or SeH as appropriate, largest percentage error (LPE), Absolute Average Percentage Error (AAPE) and the residual sum of squares (Δ).

4 Results and Discussion

Tables 1 – 7 for parameters of regression analysis are shown in the appendix

4.1 Calabar (4.9757^0 N, 8.3417^0 E)

The parameters of regression analysis are shown in Table 1. From the Table, it is obvious that yearly fit with R_a^2 = 0.9952, Se = 0.0044, Δ = 0.0001, LPE = 1.9 % and AAPE = 0.7 % is quite satisfactory, thus, one does not need to consider seasonal fits.

4.2 Enugu (6.4584^0 N, 7.5464^0 E)

Regression parameters for both yearly and seasonal fits are shown in Table 2, equation (2) gives the best model equation, however, the values of SeH = 0.2643, Δ = 0.4889 are relatively large. If we consider seasonal fits, equation (1) gives best model equation for the two seasons and also, $\Delta \approx 0$, LPE $\le$ 0.3 % and AAPE $\le$ 0.2 %, thus, seasonal fits are quite satisfactory

4.3 Gusau (12.1628^0 N, 6.6745^0 E)

The results of regression analysis for both yearly and seasonal fits are shown in Table 3, for the yearly variation, model equation (1) gives a satisfactory result with R_a^2 = 0.977, Se = 0.012, LPE = 3.9 % and AAPE = 1.4 %. For the seasonal variation equations (1) and (3) gives best model equations for the dry and the wet seasons respectively, $R_a^2 \ge$ 0.8381 for the two seasons.

4.4 Jos (9.8965^0 N, 8.8583^0 E)

The data in Table 4 is the result of regression analysis, where, for yearly fit equation (2) with R_a^2 = 0.9991, SeH = 0.132, Δ = 0.122, LPE = 1.3 % and

AAPE = 0.5% gives a satisfactory result, this is an improvement over the values obtained [5] which had R_a^2 = 0.9965 (corresponding to R_d^2 = 0.9978), SeH = 0.275, LPE = 2.8 % and AAPE = 0.9 %. For the seasonal variation, equations (1) and (3) gives a near perfect fit for the data for which $R_a^2 \geq 0.9999$, $\Delta \approx 0$, LPE$\leq$ 0.1% and AAPE ≈ 0 % for the two seasons, therefore, seasonal fits are satisfactory.

4.5 Maiduguri (11.8311^0 N, 13.1510^0 E)

Table5 shows the parameters of regression analysis for the yearly and seasonal fits, model equation (1) with R_a^2 = 0.9089, LPE = 4.4 % and AAPE = 1.3 % gives the best fit for the yearly variation. The result of seasonal variation shows that equation (1) gives best fit for the two seasons with $R_a^2 \geq 0.9997$, LPE $\leq$ 0.1%, AAPE ≈ 0% for the two seasons, Se = 0.0001 for the dry season and 0.0011 for the wet season, clearly, seasonal variation gives a quite satisfactory fit for the data.

4.6 Yola (9.2035^0 N, 12.4954^0 E)

Table 6 shows the regression parameters and goodness of fit indices for this station, as can be seen, equation (1) with R_a^2 = 0.9834, Se = 0.0096, LPE = 3.2 % and AAPE = 1.2 % gives a quite satisfactory fit for the yearly variation. The corresponding result obtained by [5] had R_a^2 = 0.9530 (R_d^2 =0.9701 for the same dependent variable), SeH = 0.578 and LPE = 5.5 %, obviously, our proposed formula gives a better model. For the seasonal fits, R_a^2 = 0.9636 for dry season and 0.9213 for wet season, LPE $\leq$ 1.9 %, AAPE $\leq$ 1.1 %, where equations (1) and (2) gives the best model equations for the dry and the wet seasons.

4.7 Zaria (11.0855^0 N, 7.7199^0 E)

The entries in Table 7 are the regression parameters for both yearly and seasonal fits, equation (1) gives satisfactory fit for the yearly data, where R_a^2 = 0.9191, Se = 0.0187, LPE = 4.3 % and AAPE = 2.2 %, thus, there is no need for seasonal fits, but a consideration of the seasonal fit indicates that equation (1) gives best fit, this is a confirmation of the applicability of equation (1) to the data for Zaria'

4.8 Plot of Observed and Best Fit Model Equations

Fig.1 (a) – (g) (shown in the appendix) shows the plots of yearly observed clearness index, daily total solar radiation (as appropriate) and best fit equation of each station versus months of the year for the seven stations, each plot shows a near perfect fit between the observed and fitted solar radition.

5. Conclusion

In this communication, we have demonstrated the effectiveness of k_T using a five parameter model equation applied to seven meteorological stations in Nigeria, for the yearly variation; Calabar, Gusau, Maiduguri, Yola and Zaria showed high correlation with k_T as the dependent variable, where, R_a^2 > 0.9, Se < 0.02, LPE < 5 %. Enugu and Jos showed high correlation with H and H' as the dependent variable with R_a^2 = 0.9739, SeH = 0.2643 and LPE = 2.3 % for Enugu and R_a^2 = 0.9991, SeH = 0.2045 and LPE = 0.1 % for Jos respectively. For seasonal variation, all of the stations showed high correlation with k_T as the dependent variable in at least one of the two seasons.

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APPENDIX

Table 1. Regression parameters and goodness-of-fit indices for Calabar

	α_{000}	α_{100}	α_{010}	α_{102}	$\alpha_{462}/10^{-9}$	R_a^2	Se	SeH	Δ	$LPE\,(\%)$	$AAPE\,(\%)$
yearly variation											
k_T	-0.1114	0.4363	0.0113	0.2352	-19.1984	0.9952	0.0044	…	0.0001	1.9	0.7
H	-20.6094	-8.8525	1.0298	33.9869	-381.7668	0.9800	…	0.2986	0.6243	2.6	1.4
H'	17.9446	-48.8877	0.4231	25.0595	1294.6376	0.9822	…	0.3484	0.8495	2.0	1.1
dry season (November – April)											
k_T	0.9164	-0.3454	-0.0114	-0.3273	70.0542	0.5691	0.0072	…	0.0001	1.1	0.6
H	-26.8212	3.6996	1.1371	31.4963	-1586.1207	-0.4191	…	0.6065	0.3678	2.5	1.4
H'	-54.4734	18.5982	1.9687	56.5878	-6086.9466	0.9621	…	0.2332	0.0544	0.7	0.4
wet season (May – October)											
k_T	-0.4584	-1.0805	0.0236	2.2867	49.1187	0.9985	0.0023	…	0.0000	0.5	0.2
H	-39.4331	-32.6989	1.7573	52.5946	964.9774	0.9981	…	0.0904	0.0082	0.6	0.3
H'	5.3758	39.9694	1.0043	-124.0946	-2870.9468	0.9921	…	0.2011	0.0404	0.6	0.3

Table 2. Regression parameters and goodness-of-fit indices for Enugu

	α_{000}	α_{100}	α_{010}	α_{102}	$\alpha_{462}/10^{-9}$	R_a^2	Se	SeH	Δ	$LPE\,(\%)$	$AAPE\,(\%)$
yearly variation											
k_T	0.1137	0.4205	0.0042	0.0428	1.6755	0.9753	0.0085	…	0.0005	2.8	1.2
H	-11.7200	0.8855	0.7560	14.1168	94.8363	0.9739	…	0.2643	0.4889	2.3	1.1
H'	16.4094	-31.0623	0.4625	10.9311	68.7102	0.9681	…	0.4420	1.3673	3.0	1.6
dry season (November – April)											
k_T	0.8354	-0.1748	-0.0080	-0.5823	31.9537	0.9943	0.0021	…	0.0000	0.3	0.2
H	-15.0414	0.8691	0.8563	11.1315	233.2250	0.9829	…	0.1003	0.0101	0.4	0.2
H'	-47.2926	17.9174	1.5966	58.2400	-2232.8132	0.9955	…	0.1230	0.0151	0.4	0.3
wet season (May – October)											
k_T	2.1385	-7.3986	-0.0548	9.9834	269.8688	0.9999	0.0005	…	0.0000	0.1	0.0
H	57.0853	-271.4686	-1.1951	353.8920	9596.2975	0.9797	…	0.2116	0.0448	1.2	0.4
H'	-65.1664	243.4776	2.9521	-346.8665	-9366.5764	0.9612	…	0.3274	0.1072	1.1	0.4

Table 3. Regression parameters and goodness-of-fit indices for Gusau

	α_{000}	α_{100}	α_{010}	α_{102}	$\alpha_{462}/10^{-9}$	R_a^2	Se	SeH	Δ	$LPE\,(\%)$	$AAPE\,(\%)$
yearly variation											
k_T	0.2788	0.3621	0.0044	-0.4603	4.7426	0.9770	0.0120	…	0.0010	3.9	1.4
H	-8.3453	9.6253	0.6601	2.5153	-100.9353	0.8506	…	0.7601	4.0446	5.6	2.7
H'	12.1067	-12.5322	0.1882	30.5386	-444.6816	0.9521	…	0.8256	4.7711	9.7	4.1
dry season (November – April)											
k_T	-0.4428	0.8319	0.0165	3.7971	-84.3448	0.8381	0.0087	…	0.0001	1.0	0.4
H	83.5982	-50.1731	-0.8712	-584.5448	11907.0567	0.9407	…	0.4136	0.1710	1.5	0.7
H'	100.1486	-69.3106	-1.2968	-514.8949	10884.5924	0.8238	…	0.6678	0.4460	4.4	1.9
wet season (May – October)											
k_T	0.4783	-0.3692	0.0050	-0.2032	23.7984	0.8998	0.0209	…	0.0004	3.4	1.4
H	9.3223	2.1903	0.2451	-5.5842	251.6751	0.4792	…	1.2487	1.5592	5.6	2.4
H'	6.3058	40.1015	-0.0855	5.6987	-1739.7550	0.9953	…	0.2045	0.0418	0.7	0.4

Table 4. Regression parameters and goodness-of-fit indices for Jos

	α_{000}	α_{100}	α_{010}	α_{102}	$\alpha_{462}/10^{-9}$	R_a^2	Se	SeH	Δ	$LPE(\%)$	$AAPE(\%)$
yearly variation											
k_T	-0.0040	0.5112	0.0078	-0.0136	-0.3528	0.9969	0.0055	…	0.0002	2.0	0.5
H	-8.1293	9.9517	0.7209	3.5102	-31.9707	0.9775	…	0.3863	1.0447	3.4	1.4
H'	33.8162	-24.6900	-0.0642	0.9342	3.4118	0.9991	…	0.1320	0.1220	1.3	0.5
dry season (November – April)											
k_T	-0.1617	0.5657	0.0116	-0.0060	9.7058	0.9999	0.0004	…	0.0000	0.1	0.0
H	-28.4572	16.3725	1.2539	6.6019	-23.7238	0.7659	…	0.3888	0.1511	1.5	0.6
H'	27.2180	-22.0440	0.1000	9.6450	-611.4560	0.9987	…	0.1067	0.0114	0.8	0.3
wet season (May – October)											
k_T	0.1689	0.7940	-0.0072	0.3263	0.3219	0.9901	0.0062	…	0.0000	1.3	0.5
H	5.6105	28.6040	-0.3110	20.3526	14.2800	0.9585	…	0.4022	0.1618	2.3	0.8
H'	30.1568	-31.0751	0.2675	-6.9609	-11.4978	1.0000	…	0.0034	0.0000	0.0	0.0

Table 5. Regression parameters and goodness-of-fit indices for Maiduguri

	α_{000}	α_{100}	α_{010}	α_{102}	$\alpha_{462}/10^{-9}$	R_a^2	Se	SeH	Δ	$LPE\,(\%)$	$AAPE\,(\%)$
yearly variation											
k_T	-0.2601	0.8026	0.0105	0.4891	-3.9818	0.9089	0.0175	…	0.0021	4.4	1.3
H	-20.5910	17.7719	0.8753	27.2658	-205.3985	0.8296	…	0.6994	3.4242	3.8	1.7
H'	39.5470	-32.1319	-0.1639	-11.8409	102.5416	0.9297	…	0.7238	3.6677	16.4	4.1
dry season (November – April)											
k_T	-0.3056	0.7557	0.0127	1.1807	-11.6079	1.0000	0.0001	…	0.0000	0.0	0.0
H	9.9019	6.9386	0.2994	-130.2154	960.5213	0.9981	…	0.0792	0.0063	0.2	0.1
H'	46.8632	-30.4522	-0.3883	-89.7045	787.4209	0.9986	…	0.0591	0.0035	0.4	0.3
wet season (May – October)											
k_T	0.5143	1.0736	-0.0116	-0.3276	-8.9069	0.9997	0.0011	…	0.0000	0.1	0.0
H	-14.6093	33.8419	0.5500	15.8685	-581.0505	0.9978	…	0.0816	0.0067	0.3	0.1
H'	10.3716	-42.9918	0.6908	17.8717	267.0476	0.9980	…	0.1172	0.0137	0.6	0.2

Table 6. Regression parameters and goodness-of-fit indices for Yola

	α_{000}	α_{100}	α_{010}	α_{102}	$\alpha_{462}/10^{-9}$	R_a^2	Se	SeH	Δ	$LPE(\%)$	$AAPE(\%)$
yearly variation											
k_T	-1.2761	1.0601	0.0321	0.8333	-14.8298	0.9834	0.0096	…	0.0006	3.2	1.2
H	-61.2789	31.0095	1.6603	44.6715	-596.3478	0.9614	…	0.4205	1.2380	3.0	1.3
H'	76.1964	-44.4695	-0.9123	-23.3051	534.1704	0.9839	…	0.4211	1.2411	3.4	1.6
dry season (November – April)											
k_T	-2.6606	1.7832	0.0561	2.8251	-38.0599	0.9636	0.0041	…	0.0000	0.4	0.2
H	-99.3774	50.7084	2.3274	92.4634	-1188.1012	0.6719	…	0.8043	0.6469	2.2	1.3
H'	112.7391	-63.6260	-1.5401	-86.1645	1236.2269	0.9730	…	0.2485	0.0617	1.2	0.7
wet season (May – October)											
k_T	-1.3275	1.9280	0.0265	0.2802	-34.8002	0.9115	0.0173	…	0.0003	2.3	1.3
H	-39.7629	22.6030	1.1761	32.8817	-266.8197	0.9213	…	0.5305	0.2815	1.9	1.1
H'	102.5429	-125.2336	-0.9431	11.4525	2514.1609	0.8952	…	0.7639	0.5836	2.3	1.4

Table 7. Regression parameters and goodness-of-fit indices for Zaria

	α_{000}	α_{100}	α_{010}	α_{102}	$\alpha_{462}/10^{-9}$	R_a^2	Se	SeH	Δ	$LPE\,(\%)$	$AAPE\,(\%)$
yearly variation											
k_T	0.7713	0.4893	-0.0148	-0.4230	10.7250	0.9191	0.0187	…	0.0024	4.3	2.2
H	28.9629	5.4986	-0.3577	-9.5462	156.7066	0.0936	…	1.6283	18.5597	11.1	5.2
H'	9.5908	-26.0891	0.6437	17.9594	-460.5464	0.9301	…	0.8432	4.9774	10.3	4.1
dry season (November – April)											
k_T	0.5756	0.2464	-0.0037	1.4452	-43.4717	0.9669	0.0049	…	0.0000	0.4	0.3
H	27.5516	-18.1022	0.1354	177.6711	-5255.3382	0.5862	…	1.0001	1.0003	2.8	1.7
H'	18.2798	-24.9370	0.3259	38.0131	-1073.2025	0.7352	…	0.9262	0.8578	4.3	2.5
wet season (May – October)											
k_T	0.2986	0.6250	-0.0041	-0.0963	6.3466	0.7584	0.0260	…	0.0007	3.6	1.7
H	-0.4790	23.8670	0.1850	10.2064	-598.9235	0.6466	…	0.9407	0.8849	3.6	1.7
H'	24.9306	-23.7841	0.2076	6.3811	-580.6229	0.8233	…	0.9561	0.9142	3.9	1.9

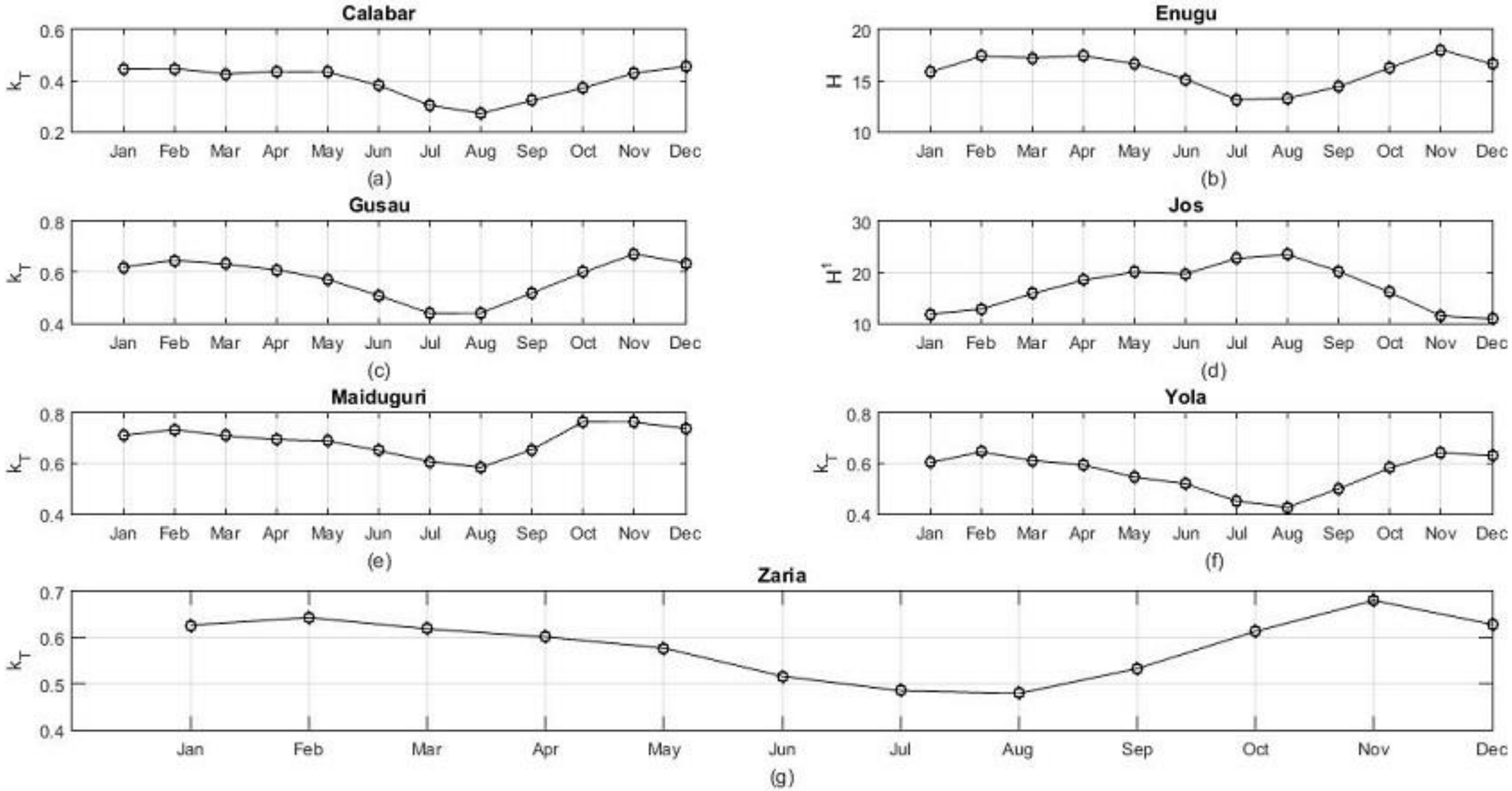

Fig. 1 plots of observed and best fit equation versus months of the year