About informational utility of investment project expertise

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Abstract—In the context of this article the effectiveness of an investment project is evaluated by net present value, this index is considered a random variable which can be estimated by an investor as a segment. Main difficulties in decision-making process arise when this segment includes zero value. In order to reduce the rate of uncertainty an investor can decide to carry out an expertise. And so the main features of utility evaluation of an investment expertise are explained in the article.

Keywords—solution utility; risk aversion; loss of profit perception; utility function; efficiency evaluation of investment.

1. Introduction

The principle of investment efficiency is the fundamental basis for decision-making process in the conditions of modern economy [1, 2]. The aforementioned principle is based on calculation of efficiency indexes of an investment project, such as: profitability index (PI), net present value (NPV), internal rate of return (IRR) and some others. Net present value is one of the most common indexes which can be used to evaluate efficiency of an investment. Therefore further in the article we will review only methods which are based on calculation of NPV of a project.

Let's make the following assumptions: an investor evaluates NPV_1 (net present value calculated according to the pessimistic scenario) and NPV_2 (net present value calculated according to the optimistic scenario). In modern economy, indexes NPV_1 and NPV_2 are estimated in the majority of business plans. We presume that $NPV_1 < 0$ and $NPV_2 > 0$, because otherwise an investment project is rejected or accepted for realization. We consider NPV to be a random variable on the interval $[NPV_1; NPV_2]$ with known probability density function $P(NPV)$. In this situation the motive for realization of a project is a possibility of obtaining income which can be estimated by the following equation:

$$P = \int_{0}^{NPV_2} NPV \cdot P(NPV) d(NPV) \qquad (1)$$

The project risks consist in a possibility of financial losses, the size of this losses can be estimated by the following equation:

$$L = \int_{NPV_1}^{0} NPV \cdot P(NPV) d(NPV) \qquad (2)$$

Before making an investment an investor can additionally carry out an expertise with a price of $C \geq 0$. An expertise allows reducing of the initial uncertainty interval $[NPV_1; NPV_2]$. As a result of an expertise an investor receives more accurate interval $[a_1; b_1]$ for NPV of a project. Intervals $[a_1; b_1]$ and $[NPV_1; NPV_2]$ overlap with the following valid equations: $b_1 \geq \mathrm{NPV}_1$ (like depicted in fig.1) and $a_1 \leq \mathrm{NPV}_2$ (in a case $NPV_2 \in [a_1; b_1]$).

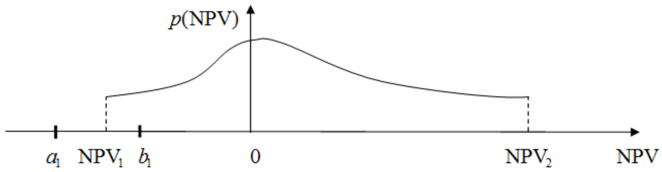

Fig. 1. Some possible positions of interval $[a_1; b_1]$ defined by expertise

We presume that before carrying out an expertise an investor knows parameter Δ, this parameter characterizes accuracy of an expertise. The value of Δ can be measured like this: $b_1 - a_1 = 2\Delta$.

Thereby an investor knows initial assessments of NPV_1 and NPV_2, and after he decides on type of function $P(NPV)$, he will be able to evaluate utility of the decision to carry out an investment expertise with parameter Δ and a price of $C \geq 0$ thus improving the effectiveness of his decision.

Type of function $P(NPV)$ may be determined by an investor after considering statistical information concerning realization of similar investment projects. Also function of continuous uniform distribution can be chosen as probability density function $P(NPV)$ in case if there isn't any information about realization of similar investment projects.

In this article we aim to quantitatively evaluate utility of investment expertise. In order to successfully solve this task we formulate a mathematical model of reviewed situation and analyze our model to find its similarities with existing economic-mathematical methods.

2. Mathematical Modelling of Investment Expertise

The decision tree of reviewed situation is depicted in fig.2.

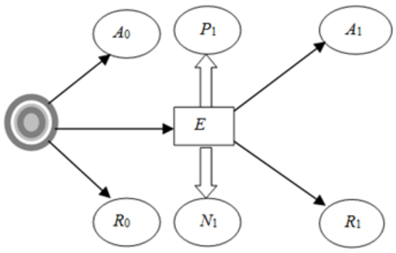

Fig. 2. The block diagram of existing decisions with opportunity to carry out an investment expertise

Decisions A_0 and R_0 are an acceptance and a rejection of an investment project respectively at the initial stage. Node E is an event of carrying out an expertise. Decision P_1 symbolizes acceptance of an investment project in case of $a_1 \geq 0$ and $b_1 > 0$ (meaning that interval $[a_1; b_1]$ is situated in the positive area of the numerical axis). Decision N_1 represents rejection of an investment project in case of $a_1 < 0$ and $b_1 \leq 0$.

Decisions A_1 and R_1 are an acceptance and a rejection of an investment project respectively after an expertise in case of $a_1 < 0$ and $b_1 > 0$ (meaning that zero value includes in the interval $[a_1; b_1]$).

Thus the task in this case reduces to utility evaluation of all existing decisions and calculation of utility of an investment expertise in accordance with the following equation:

$$\begin{aligned} U(E) &= U(P_1) + U(N_1) + \\ &+ \max\left(U(A_1); U(R_1)\right) - \\ &- \max\left(U(A_0); U(R_0)\right) - C, \end{aligned} \quad (3)$$

$U(\square)$ – utility of specified decision.

The subjective utility function which can be used in this situation is introduced in [3]. In accordance with (1) and (2), utilities of decisions A_0 and R_0 may be calculated as:

$$U(A_0) = (1+\beta)L + P = 0, \quad (4)$$

$$U(R_0) = -\beta L - \gamma P = 0, \quad (5)$$

β – coefficient measuring fear of risk experienced by an investor; γ – coefficient measuring sorrow experienced by an investor in case of lost profit. These coefficients are described in detail in [3, 4].

We presume that a_1^{Un} and b_1^{Un} are some boundary values that divide the initial interval $[NPV_1; NPV_2]$ into areas belonging to decisions P_1, N_1 and A_1, R_1.

In this case interval $[\mathrm{NPV}_1; a_1^{Un}]$ pertains to decision N_1, interval $[b_1^{Un}; \mathrm{NPV}_2]$ is used to calculate utility of decision P_1 and interval $[a_1^{Un}; b_1^{Un}]$ is a remaining uncertainty range after carrying out an expertise (interval $[a_1^{Un}; b_1^{Un}]$ can be used to calculate utility of decision A_1 or R_1).

The following equation allows to calculate values of a_1^{Un} and b_1^{Un}:

$$\frac{\mathrm{IN}[p(\mathrm{NPV}), a_1^{Un}, 0]}{\mathrm{IN}[p(\mathrm{NPV}), 0, b_1^{Un}]} = \frac{\mathrm{IN}[p(\mathrm{NPV}), \mathrm{NPV}_1, 0]}{\mathrm{IN}[p(\mathrm{NPV}), 0, \mathrm{NPV}_2]}. \quad (6)$$

We introduce the next identification mark in (4) and following equations:

$$\mathrm{IN}[f(\mathrm{NPV}), a_1, b_1] = \int_{a_1}^{b_1} f(\mathrm{NPV}) d(\mathrm{NPV}).$$

The next step is to formulate equations to calculate utilities of decisions P_1, N_1, A_1, R_1.

According to the definition of decision P_1, lower and upper estimation of NPV in this case are situated in non-negative area of the numerical axis. Thus the utility of decision P_1 can be calculated according to the following equation:

$$U(P_1) = \mathrm{IN}[\mathrm{NPV} \cdot p(\mathrm{NPV}), b_1^{Un}, \mathrm{NPV}_2] \quad (7)$$

The utility of decision N_1 is found with the following equation:

$$U(N_1) = -\beta \cdot \mathrm{IN}[\mathrm{NPV} \cdot p(\mathrm{NPV}), \mathrm{NPV}_1, a_1^{Un}] \quad (8)$$

The utility of decision A_1 is determined with the following equation:

$$\begin{aligned} U(A_1) &= (1+\beta) \cdot \mathrm{IN}[\mathrm{NPV} \cdot p(\mathrm{NPV}), a_1^{Un}, 0] + \\ &+ \mathrm{IN}[\mathrm{NPV} \cdot p(\mathrm{NPV}), 0, b_1^{Un}] \end{aligned} \quad (9)$$

We can calculate the utility of decision R_1 with similar equation:

$$\begin{aligned} U(R_1) &= -\beta \cdot \mathrm{IN}[\mathrm{NPV} \cdot p(\mathrm{NPV}), a_1^{Un}, 0] - \\ &- \gamma \cdot \mathrm{IN}[\mathrm{NPV} \cdot p(\mathrm{NPV}), 0, b_1^{Un}] \end{aligned} \quad (10)$$

Then we can calculate utility of an investment expertise $U(E)$ using (3) because we have determined values of all necessary variables with the price of an expertise C belonging to primary data.

If we find that $U(E) > 0$, then we can conclude that the decision of carrying out an expertise is characterized by positive utility in comparison with deciding on project realization without carrying out an expertise. So we can say that an expertise is advised in this case. Otherwise ($U(E) < 0$), we believe that carrying out an expertise doesn't deliver additional benefits.

3. Application of Described Mathematical Model and Conclusions

Our next step is to explore created mathematical model. First, we test our model with a numerical example. Input data for this example is provided by table I.

TABLE I. INPUT DATA FOR UTILITY EVALUATION OF AN INVESTMENT EXPERTISE

Identification mark of a parameter	Value of a parameter
NPV_1	- 4 000
NPV_2	14 000
m_1	0
m_2	0
α	0,7
γ	0,817
C	0
Δ	0; 1 000; 2 000; 3 000; 4 000; 5 000; 6 000; 7 000; 8 000; 9 000

The values of coefficients $m_1 = m_2 = 0$ define function $P(NPV)$ as a function of continuous uniform distribution. The values for parameter Δ are taken from an interval $[0;\ 9000]$, beginning with zero and increased by 1000.

Table 2 indicates the link between utility of an expertise $U(E)$ and parameter Δ. Also the term of marginal utility in relation with discussed situation is introduced in table II.

TABLE II. UTILITY VARIATIONS OF AN INVESTMENT EXPERTISE AND ITS MARGINAL UTILITY IN CONDITION OF CHANGING EXPERTISE ACCURACY

Value of parameter Δ	Value of an expertise utility	Value of marginal utility
0	4395,06	-
1000	4340,80	54,26
2000	4178,02	162,78
3000	3906,72	271,30
4000	3526,90	379,82
5000	3038,56	488,34
6000	2441,70	596,86
7000	1736,32	705,38
8000	922,42	813,90
9000	0,00	922,42

From table 2 we can see that value of expertise utility decreases monotonically with an increase of parameter Δ, which characterizes expertise accuracy. The maximum value of expertise utility occurs when carried out expertise is perfect with parameter $\Delta = 0$ (this condition means that length of an interval of uncertainty equals zero). In other case we can notice that the minimum value of expertise utility equals zero (in the case if the length of an interval of uncertainty after expertise is equal to the length of an initial interval of uncertainty). Now we can introduce the term of marginal utility in relation with discussed situation.

In economic theory, marginal utility is defined as a utility which an individual can gain from using an additional unit of some economic good [5]. The law of diminishing marginal utility is commonly known among economists all around the world. This law says that the first unit of consumption of a good or service yields more utility than the second and subsequent units, with a continuing reduction for greater amounts. Thus marginal utility decreases with an increase of consumption. The zero value of marginal utility occurs when total utility is at its maximum value. After total utility reaches its maximum value, it begins to decrease.

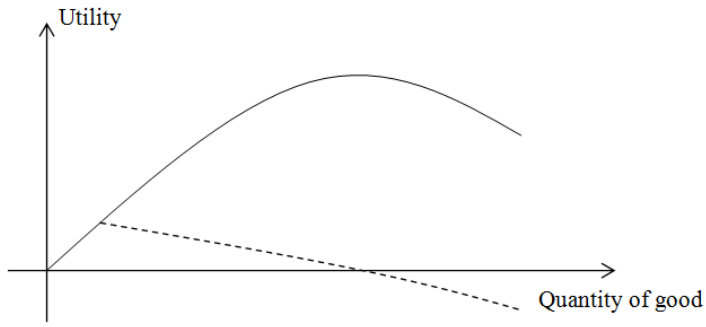

Fig. 3. Comparison between total and marginal utilities

In fig. 3 the unbroken line depicts variation of total utility with an increase of consumption and the dashed line characterizes variation of marginal utility.

As we can see from table II marginal utility decreases with an increase of expertise accuracy in reviewed situation. If an increase of consumption of a good or service is substituted with an increase of expertise accuracy (meaning decrease in values of parameter Δ) then we can notice the effect of diminishing marginal utility. Despite this in reviewed situation we can point out some peculiar properties. For example, marginal utility never reaches negative values because expertise accuracy varies to a limited extent (interval $[0;\ 9000]$) in contrast with quantity of a good or service which in theory is not limited.

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