Stochastic Repair and Replacement with a single repair channel

MOHAMMED A. HAJEEH
Techno-Economics Division
Kuwait Institute for Scientific Research
P.O. Box 24885; Safat-13109, KUWAIT
mhajeeh@kis.edu.kw http://www.kisr.edu.kw

Abstract: Systems consist of one or more component arranged in different configurations. In a series configuration, a system fails upon the failure of any component, in a parallel configuration, the system fails when all components fail, while in a standby configuration, a failed component is replaced by an available standby. Upon failure of a system/component is perfectly, minimally, or imperfectly repaired. Perfect repair is the most commonly used in literature, but in real life, systems undergo several imperfect repairs before complete replacement. This research work examines a one component system where upon failure it is either replaced or imperfectly repaired with different probabilities. In this regard, we provide an analytical expression for the steady-state availability. An example is provided for demonstration.

Key Words: repairable models, imperfect, availability, exponential distribution

1 Introduction

Operating systems such as mechanical systems consist of components; these components may fail in different modes such as fatigue, leak, rupture, creep, wear, corrosion, deformation. Failed components may either be replaced or repaired; repairable systems when failed usually receive repair or maintenance actions that restore their functions. These actions affect the system behavior and alter the makeup of the system. Repair either brings the system to as good as new "perfect repair" or a status before failure "minimal repair" or to an inferior state called 'imperfect repair". The aim of the different maintenance actions is to enhance systems' performance; reliability is one of the performance measures used to insure system effectiveness and produce quality characteristics products required by consumers. Another important performance criterion is availability that encompasses both reliability and maintainability; it is the probability that a system is operational and executes its required functions at a given point of time

Systems' performance has been addressed extensively in the literature; for example, Abdel-Hameed [1] examined an age dependent minimal repair model where the failed system is either is perfectly repaired with probability p(t) or minimally repaired with probability 1−p (t). Brown and Proschan [2] developed optimal replacement policies for a system that upon failure is either undergoes perfect repair with probability p or is minimally repaired with probability (1-p). Meanwhile, Beichelt [3] considered the total repair cost limit replacement policy, where the system is replaced as soon as its total repair cost reaches a specific level. Moustafa [4] studied Markov models for the transient analysis of the reliability of k-out-of-N: G systems with and without repair subject to M failure modes. Zhao [5] studied the failure pattern of repairable components when a failed component is either perfectly or imperfectly repaired. In this work, the lifetime of a component is assumed to behave according to a general distribution. Several asymptotic quantities were derived, such as the mean number of failures in a specific component position, the fraction of time the system is down due to failure in a particular component position and the availability of the system

Dimitrov et al. [6] examined an age-dependent repair model with imperfect repair, obtaining the warranty costs for the products under a nonhomogeneous Poisson process scenario. The behavior of multiple repairable systems was inspected by Pan and Rigdon [7] using Bayesian methods for models that are between as bad-as-old and the good-as-new. Monte Carlo methods were used to approximate properties of the posterior distributions. Pandey et al. [8] developed a mathematical model to assist decision makers in selecting proper maintenance scenarios under imperfect repair. Examples were used to validate the applicability of the proposed method. Results indicated that introducing of imperfect repair better facilitates the allocation of maintenance resources. Meanwhile, El-Damcese and Shama [9] studied the performance of a 2-state repairable system with two types of failure. Laplace transform techniques were utilized to develop expressions for several performance measures including availability, reliability and mean time to failure under exponentially distributed times between failures and repair times. Neguyen et al. [10] studied repairable systems under imperfect repair where the time between failures of a new system follows Weibull distribution. Furthermore, an analytical approach for the distribution of the inter-failure times was obtained in addition to producing under steady conditions an optimal preventive maintenance policy under a static, a dynamic or a failure limit policy.

2 System Descriptions

The purpose of this paper is to study the availability of a system that upon failure either is replaced with certain probability or is or imperfectly. The failure and repair rate times are assumed to follow exponential distribution several. The system is analyzed using Kolmogorov's forward equations method. Other assumptions are: the travel time to repair station is negligible, availability of only one repair person...

At each failure, the system is imperfectly repaired with probability q or is perfectly repaired p. After perfect repair the system becomes as new, while following imperfect repair the system's performance decreases as time progresses. Thus the failure rate increases after each repair ($\alpha_{i+1} \geq \alpha_i$ for i = 1, 2, n); similarly, the rate of repair declines after each repair; $\varphi_{i+1} \leq \varphi_i$ for i = 1, 2,.., n. The system is replaced after undergoing a specified number n of repairs.

A pictorial presentation of the state transitions is shown in Figure 1. The rectangular shapes in the figure represent the operational state, while the oval shapes are the failed states. The process starts at state 1, where the system is new, after failure (α_1) at state 1, the systems moves to state 2 for repair. At state 2 the system is either perfectly repaired with probability $p\varphi_{n+1}$ to state 1, or imperfectly repaired with probability $q\varphi_1$ (q+p =1) to state 3. Similarly, after failure with failure rate α_1, the system transitions to state 4 At state 4, is system is either perfectly repaired to state 1, or imperfectly to state 2 with probability $q\varphi_2$. Finally, after the n+1 failure, the system is replaced and moves to state 1 and the process is regenerated.

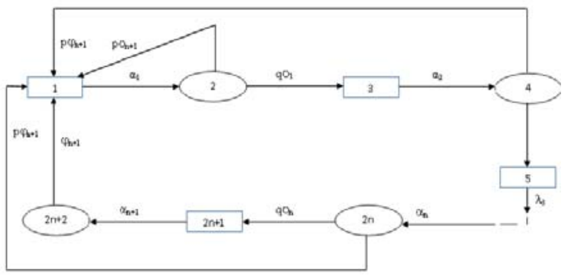

Fig. 1. A pictorial presentation of the transition process of the system

2.1 Availability Analysis

The transition probability P (t) for the systems is as follows:

$$\begin{aligned}
\tilde{p}_1(t) &= -\alpha_1 p_1(t) + p\varphi_{n+1} p_2(t) + p\varphi_{n+1} p_4(t) + \ldots + p\varphi_{n+1} p_{2N}(t) \\
\tilde{p}_2(t) &= \alpha_1 p_1(t) - (p\varphi_{n+1} + q\varphi_1) p_2(t) \\
&\vdots \\
\tilde{p}_{2n}(t) &= \alpha_n p_{2n-1}(t) - (p\varphi_{n+1} + q\varphi_n) p_{2n}(t) \\
\tilde{p}_{2n+1}(t) &= \varphi_n p_{2n}(t) - \alpha_{n+1} p_{2n+1}(t) \\
\tilde{p}_{2n+2}(t) &= \alpha_{n+1} p_{2n+1}(t) - \varphi_{n+1} p_{2n+2}(t)
\end{aligned}$$

The steady state system with exponential Markov with continuous time is expressed as:

$Q\pi = 0$, where π is the state transition probability matrix, and Q is the state transition rate matrix, Q is:

$$Q = \begin{bmatrix}
-\alpha_1 & p\mu_{n+1} & 0 & p\mu_{n+1} & 0 & \cdots & p\mu_{n+1} & 0 & \mu_{n+1} \\
\alpha_1 & -(p\mu_{n+1}+q\mu_1) & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & q\mu_1 & -\alpha_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \alpha_2 & -(p\mu_{n+1}+q\mu_2) & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & q\mu_2 & -\alpha_3 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & -(p\mu_{n+1}+q\mu_n) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & q\mu_n & -\alpha_{n+1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{n+1} & -\mu_{n+1}
\end{bmatrix}$$

Solving the system of equations $Q\pi = 0$, the steady state transition probability is:

$$\pi_{2k} = \frac{\alpha_j}{(q\varphi_j + p\varphi_{n+1})} \pi_{2k-1}; \text{for } 1 \le k \le n \tag{1}$$

$$\pi_{2k+1} = \frac{q\varphi_j}{\alpha_{J+1}} \pi_{2k} \qquad \text{for } 1 \le k \le n \tag{2}$$

$$\pi_{2n+2} = \frac{\alpha_{n+1}}{\varphi_{n+1}} \pi_{2n+1} \tag{3}$$

$$\sum_{i=1}^{n} \pi_k = 1 \tag{4}$$

Equations can be written in terms of π_1 as follows:

$$\pi_{2k} = q^{k-1} \frac{\alpha_1}{(p\varphi_{n+1} + q\varphi_1)} \prod_{i=1}^{k-1} \frac{\varphi_i}{(p\varphi_{n+1} + q\varphi_{i+1})} \pi_1 \quad \text{for } 1 \le k \le n+1 \tag{5}$$

$$\pi_{2k+1} = q^k \frac{\varphi_k}{\alpha_{k+1}} \frac{\alpha_1}{(p\varphi_{n+1} + q\varphi_1)} \prod_{i=1}^{k-1} \frac{\varphi_i}{(p\varphi_{n+1} + q\varphi_{i+1})} \pi_1 \quad \text{for } 1 \le k \le n \tag{6}$$

Where $\prod_{i=1}^{0} \frac{\varphi_i}{(p\varphi_{n+1} + q\varphi_{i+1})} = 1$

Utilizing equations 4, 5, 6, the expression for the probability of being in the initial state 1 (π_1) is obtained. Define

$$a = \frac{\alpha_1}{q\varphi_1 + p\varphi_{n+1}}$$

$$b(k) = \prod_{i=1}^{k-1} \frac{\varphi_i}{q\varphi_{i+1} + p\varphi_{n+1}}$$

$$\pi_1 = \frac{1}{1 + a\left[\sum_{k=1}^{n+1} q^k b(k) + \sum_{k=1}^{n+1} q^k \frac{\varphi_k}{\alpha_{k+1}} b(k)\right]} \tag{7}$$

$$A = \frac{\sum_{k=1}^{n+1} q^k \frac{\varphi_k}{\alpha_{k+1}} b(k)}{1 + a\left[\sum_{k=1}^{n+1} q^k b(k) + \sum_{k=1}^{n+1} q^k \frac{\varphi_k}{\alpha_{k+1}} b(k)\right]} \tag{8}$$

$$A = \frac{\sum_{k=1}^{n+1} q^k \frac{\varphi_k}{\alpha_{k+1}} b(k)}{1 + a\left[\sum_{k=1}^{n+1} q^k b(k) + \sum_{k=1}^{n+1} q^k \frac{\varphi_k}{\alpha_{k+1}} b(k)\right]} \tag{9}$$

3. Examples

To illustrate the process an example is presented for a system that undergoes two imperfect repairs and is replaced after the third failure. The transition states for this system are given in Figure 2.

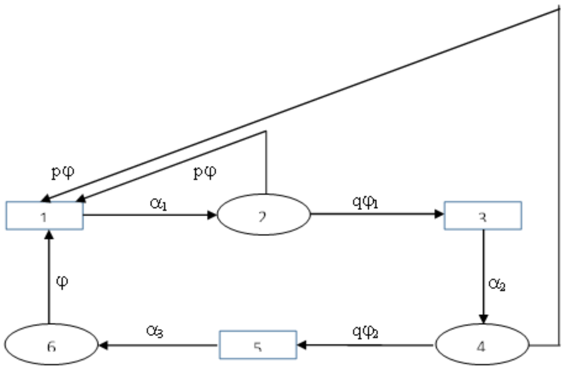

Fig. 2. A pictorial presentation of a system with 2 imperfect repairs before replacement

The state transition matrix for this problem is as presented below:

$$Q=\begin{bmatrix} -\alpha_1 & p\varphi_1 & 0 & p\varphi_2 & 0 & \varphi \\ \alpha_1 & -(p\varphi+q\varphi_1) & 0 & 0 & 0 & 0 \\ 0 & q\varphi_1 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 & -(p\varphi+q\varphi_2) & 0 & 0 \\ 0 & 0 & 0 & q\varphi_2 & -\alpha_3 & 0 \\ 0 & 0 & 0 & 0 & \alpha_3 & -\varphi \end{bmatrix}\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\\ \pi_4\\ \pi_5\\ \pi_6\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

From the above matrix, the systems transition probabilities are presented and the expression for the steady state probability of being at state 1 is derived as follows:

$$\pi_1=\frac{\alpha_2\alpha_3\varphi(p\varphi+q\varphi_1)(p\varphi+q\varphi_2)}{\begin{bmatrix}\alpha_2\alpha_3\varphi(p\varphi+q\varphi_1)(p\varphi+q\varphi_2)+\alpha_1\alpha_2\alpha_3\varphi(p\varphi+q\varphi_2)+\alpha_1\alpha_3 q\varphi\varphi_1(p\varphi+q\varphi_2)\\ +\alpha_1\alpha_2\alpha_3\varphi q\varphi_1+\alpha_1\alpha_2\varphi q\varphi_1 q\varphi_2+\alpha_1\alpha_2\alpha_3 q\varphi_1 q\varphi_2\end{bmatrix}}$$

Summing the probabilities of the operational states, we obtain the steady state availability of the systems:

Dividing the numerator and the denominator by $\alpha_1\alpha_2\alpha_3\varphi(p\varphi+q\varphi_1)(p\varphi+q\varphi_2)$ and rearranging and simplifying, the following expression is derived:

$$A=\frac{\left[\frac{1}{\alpha_1}+\frac{q\varphi_1}{(p\varphi+q\varphi_1)}\left[\frac{1}{\alpha_2}+\frac{q\varphi_2}{\alpha_3(p\varphi+q\varphi_2)}\right]\right]}{\left[\begin{matrix}\frac{1}{\alpha_1}+\frac{q\varphi_1}{(p\varphi+q\varphi_1)}\left\{\frac{1}{\alpha_2}+\frac{q\varphi_2}{\alpha_3(p\varphi+q\varphi_2)}\right\}\\ +\frac{1}{(p\varphi+q\varphi_1)}\left\{1+\frac{q\varphi_1}{(p\varphi+q\varphi_2)}\left[1+\frac{q\varphi_2}{\varphi}\right]\right\}\end{matrix}\right]}$$

Figures 3 and 4 exhibit the system availability for various values of repair and failure rates; in figure 4, the failure rates are increase for the same repair rates increased

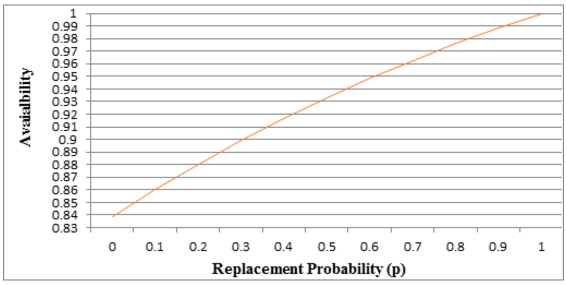

Fig. 3. Availability vs. replacement probability for the system
(α_1 = 0.05, α_2= 0.30, α_3= 0.70; φ_1= 2.0, φ_2= 1.75, φ_3= 1.5; φ = 3.0).

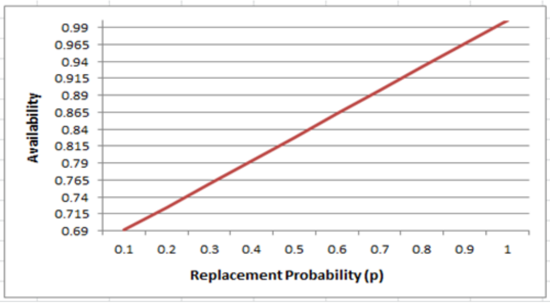

Fig. 4. Availability vs. replacement probability for the system (α_1= 0.6 α_2= 0.90, α_3 = 1.5; φ_1= 2.0, φ_2= 1.75, φ_3 = 1.5; φ = 3.0)

4 Conclusions and Future Research

Majority of maintenance problems in literature address the topic of perfect. This type of repair is useful for inexpensive items and is a must for critical components. However, in generally it is costly and unpractical. This study examines the behavior of repairable systems that upon each failure could be either perfectly or imperfectly repaired. An analytical expression for steady state availability is derived. This expression is applicable to both perfect and imperfect repair. The expression become more complicated as the number of states increases. Furthermore, for fixed values of repair rates, availability declines as the failure rates increases. Conversely, for fixed failure rates, availability raises as the repair rates increases.

Future research should address cases were the failure rates or the repair rates or both are not exponentially distributed and with more than one repair stations complements

Simulation may be utilized to address large scale dynamic maintenance models and problems with multiple compounds...

References

[1] Abdel-Hameed, M. (1978). Inspection and maintenance policies of devices subjected to deterioration. *Advances in Applied Probability* 10, 509-512.

[2] Brown M. & Proschan F. (1983). Imperfect repair, *Journal of Applied Probability*, 20: 85l–859.

[3] Beichelt F. (1997). Maintenance policies under stochastic repair cost development. *Economic Quality Control* 12, 173-181.

[4] Moustafa, M.S. (1998). Transient analysis of reliability with and without repair for K-out-of-N: G systems with M failure modes. *Reliability Engineering and System Safety* 59, 317-320.

[5] Zhao, M. (1994). Availability for repairable components and series systems. *IEEE Transactions on Reliability* 43(2), 329-334.

[6] Dimitrov B, Chukova S, Khalil Z. (2004). Warrantee costs: An age-dependent failure/repair model, *Naval Research Logistics*, 51, 959–976.

[7] Pan, R., Rigdon, S.E. (2009). Bayes inference for general repairable systems, Journal of Quality Technology, 41(1), 82-94

[8] Pandey, M., Zuo, M. J., Moghaddass R. & Tiwari, M.K.(2013). Selective maintenance for binary systems under imperfect repair, *Reliability Engineering and System Safety*,. 113, 42–51.

[9] El-Damcese, M. A. & Shama M. S. (2015) Reliability and availability analysis of a 2-state repairable system with two-types of failures, *Eng. Math. Letters*, 2, 1-9.

[10] Nguyen, D. T., Dijoux, Y. & Mitra F. (2017). Analytical properties of an imperfect repair model and application in preventive maintenance scheduling, European Journal of Operational Research, Vol. 256, No. 2, 2017, pp. 439-453