A Pattern Search Algorithm Approach for Optimal Allocation of PMUs considering Sensitive Bus Constraints to obtain Complete Observability

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Abstract: - Allocation of Phasor Measurement Units (PMU) at all the buses in a network leads to high economic cost which is not feasible. So in order to increase redundancy and obtain complete observability, PMUs must be allocated at optimal places in the network with minimum cost. In this paper, a Pattern Search Algorithm (PSA) is proposed to minimize and optimally allocate PMUs considering nonlinear sensitive constraints of buses. Sensitive buses are generated by formulating Voltage Stability Index (VSI) considering mean of voltages obtained from load flows with increasing loads. Modeling of nonlinear constraints integrated with Zero Injection (ZI) buses is considered to further minimize the number of PMUs with and without presence of zero injection buses. The redundancy at each bus is measured with Bus Observability Index (BOI) to show the importance of PMUs allocation at sensitive buses. The proposed method is programmed in MATLAB and applied on IEEE 14, 24, 30 and 57 bus systems. The results obtained are analyzed and compared with other methods available in literature to show effectiveness of proposed method.

Key-Words: - Observability, Phasor Measurement Units(PMUs), Sensitive Bus, Pattern Search Algorithm (PSA), State stimation, Zero Injection (ZI).

1 Introduction

Due to inaccurate measurement of voltage and its phase angles of a network, processed data is inaccurate to estimate the states of the system. The state estimation performed through this available data is not sufficient to control whole power system. So in order to measure accurate voltage and current, Synchrophasors devices are introduced into the power systems. Synchrophasors or phasor measurement units (PMUs) measure phasors associated current and voltage with reference to time provided by the Coordinated Universal Time (UTC) such as Global Position Systems (GPS) [1]. Allocation and installation of PMUs on every bus of system is not feasible as it leads to more cost. Placement of PMUs in network should be in such way that system does not lose observability and ensure sensitive measurements at bus locations that frequently prone to load changes. Placement of PMUs at sensitive locations can measure accurate states of the system and can increases redundancy of the system.

George *et al.* [2] formulated a binary semi definite programming (BSDP) with binary decision variables to minimize the linear objective function. In this procedure of state estimation, system is made numerically observable through finding rank of the Jacobian matrix. Castro and Muller [3] proposed a new genetic algorithm for PMU placement considering observability and security issues. Observability is preserved in case of single loss of PMU in this procedure. Qiao *et al.* [4] presented a greedy algorithm that is suitable for any PMUs placement budget. The proposed theoretic criterion, i.e., mutual information between power system states and PMU measurements, not only improves observability of system but also reduces the uncertainty of power system states with PMU measurements. Mazhari *et al.* [5] presented a Cellular Learning Automata (CLA) optimization for multi-objective PMU placement considering observability, redundancy and contingencies. A generalized observability function has been presented to allocate PMUs in presence of non synchronous measurements. Chakrabarti *et al.* [6] presented integer quadratic programming approach to make system completely observable under normal operating conditions and at single line or single PMU outages. The programming approach minimizes the number of PMUs with maximum redundancy. Manousakis and Korres [7] presented non linear quadratic optimization problem considering continuous decision variables (0,1) for optimal placement of PMUs and solved the problem using weighted least squares approach. Many authors [8-12] proposed evolutionary approaches for optimization and also modeled optimal allocation of PMUs in linear case but quadratic equations related to cost functions and non linear observability constraints cannot be solved in linear case.

In this paper, pattern search approach is used to minimize the cost quadratic problem subjected to non linear constraints for optimal allocation of PMUs considering sensitive buses along with zero injection buses. The PMUs are placed at sensitive buses which are indentified based on voltage stability index with complete observability. The method is tested with different test cases and results are presented.

The rest of the paper is organized as follows: Section 2 presents problem formulation; Section 3 deals with PSA for optimal allocation of PMUs; Section 4 covers optimal allocation of PMUs considering sensitive buses; Section 5 provides results and analysis; and Section 6 gives conclusions

2 Problem Formulation

The objective function of the problem is formulated as minimization of cost quadratic function subjected to non linear constraints, to allocate PMUs in the network with highest preference given to sensitive buses to increase redundancy with complete observability of the system. The objective function is written as follows:

$$J(z) = Min \sum_{\mathrm{k}=1}^{\mathrm{n}} z_k^T \mathrm{c_k} z_k \qquad (1)$$

Subjected to

$$b(z) = 0, \qquad 0 \le z \le 1 \qquad (2)$$

where Z_k is PMU placement variable of bus-k. $C = diag(c_1, c_2, ...c_n)$ is PMU cost vector or weight vector, n is number of buses, $b(z)$ is a vector of non linear bus observability constraints whose entries for bus k are defined as

$$b_k(z) = (1 - z_k) \prod_{j \in g_k} (1 - z_j) = 0 \quad \forall\ k \in \beta \qquad (3)$$

where β is bus set, g_k is vector of buses incident to bus-*k* the above equation shows that at least one PMU should be located and installed at any one of the buses *k* and *j* to make the bus *k* observable.

The above problem is quadratic unconstrained optimization problem in which the PMU allocation cost at each bus is same for all buses. Equality constraints (3) ensure that optimal values will be either $z_k = 1$ or $z_k = 0$ and the cost quadratic equation (1) implies that $0 \le z_k \le 1$. For instance consider an objective function for IEEE 14 bus system to which the above optimization problem can be formulated as

$$J(z) = Min \sum_{\mathrm{k}=1}^{14} z_k^T \mathrm{c_k} z_k \qquad (4)$$

Subjected to nonlinear observability constraints

$$b(z) = \begin{cases} bus-1 = (1-z_1)(1-z_2)(1-z_5) = 0 \\ bus-2 = (1-z_1)(1-z_2)(1-z_3)(1-z_4)(1-z_5) = 0 \\ bus-3 = (1-z_2)(1-z_3)(1-z_4) = 0 \\ bus-4 = (1-z_2)(1-z_3)(1-z_4)(1-z_5)(1-z_7)(1-z_9) = 0 \\ bus-5 = (1-z_1)(1-z_2)(1-z_4)(1-z_5)(1-z_6) = 0 \\ bus-6 = (1-z_5)(1-z_6)(1-z_{11})(1-z_{12})(1-z_{13}) = 0 \\ bus-7 = (1-z_4)(1-z_7)(1-z_8)(1-z_9) = 0 \\ bus-8 = (1-z_7)(1-z_8) = 0 \\ bus-9 = (1-z_4)(1-z_7)(1-z_9)(1-z_{10})(1-z_{14}) = 0 \\ bus-10 = (1-z_9)(1-z_{10})(1-z_{11}) = 0 \\ bus-11 = (1-z_6)(1-z_{10})(1-z_{11}) = 0 \\ bus-12 = (1-z_6)(1-z_{13})(1-z_{12}) = 0 \\ bus-13 = (1-z_6)(1-z_{12})(1-z_{13})(1-z_{14}) = 0 \\ bus-14 = (1-z_9)(1-z_{13})(1-z_{14}) = 0 \end{cases}$$

3 Pattern Search Algorithm for Optimal Allocation of PMUs

Torczon and lewisy [13-15] used positive-basis techniques to prove the convergence of pattern search method for special class of functions such as non linear unconstrained optimization methods and bound constrained optimization methods. This method is applicable to some class of functions and not applicable to all classes. Charles and Dennis [16] proposed a generalized pattern search process which is applicable to both linear and non linear, constrained and unconstrained optimization methods. This process is applicable to bound constrained problems only.

3.1 PSA procedure

Pattern search algorithm starts with computation of sequence of points to find optimal point. The algorithm starts by creating set of points called mesh around a given point. This point is initial starting point which is obtained from previous step of the algorithm. The current point is added to set of points to form a mesh. If the current point in the mesh is able to improve the function value, the new point becomes current point at the next iteration.

For instance, assume the initial point z_0 as starting point. At first iteration, the mesh size considered is 1, the pattern vectors are constructed as [0 1], [1 0], [-1 0], [0 -1] which are known as direction vectors.

Add the constructed direction vectors to initial point z_0 to compute mesh points as follows:

$$z_0 + [1 \quad 0], z_0 + [0 \quad 1],$$
$$z_0 + [-1 \quad 0], z_0 + [0 \quad -1]$$

The PSA polls the mesh points until it finds optimal value than function value of z_0. If it computes the optimal value then poll is successful and PSA sets this point as z_1. After the poll is successful, PSA moves to second iteration and the mesh size is multiplied with expansion factor. Expansion factor considered here is 2.

At second iteration mesh contains the following points

$$z_1 + 2*[1 \quad 0], \quad z_1 + 2*[0 \quad 1],$$
$$z_1 + 2*[-1 \quad 0], \quad z_1 + 2*[0 \quad -1]$$

The PSA polls the mesh points until it finds minimum value than functional value of z_1. If it finds the minimum value the poll is successful then algorithm sets the point as z_2. After the poll success, the current mesh size is multiplied by expansion factor to get mesh size 4.

At iteration 3 mesh size is 4, if none of mesh points have optimal functional value when compared to the value of z_2, so the poll is not satisfied. In this situation the process does not change current point. i.e., at next iteration PSA multiplies 0.5 (contraction factor) to current size of the mesh. So that size of mesh obtained is minimum value at next iteration. Then PSA polls with small value of mesh size.

$$z_2 + 0.5*[1 \quad 0], \quad z_2 + 0.5*[0 \quad 1],$$
$$z_2 + 0.5*[-1 \quad 0], \quad z_2 + 0.5*[0 \quad -1]$$

At next iteration, if size of the mesh is minimum value, then PSA polls with minimum mesh size. After performing more number of experiments on pattern search it was found that 2 and 0.5 are best expansion and contraction factors. The flow chart of PSA for non linear optimization is shown in Fig. 1.

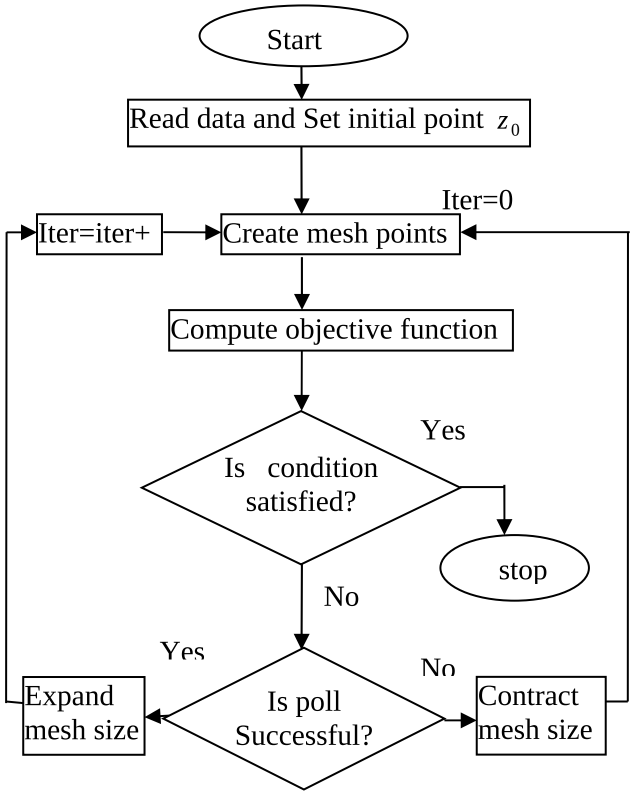


Fig.1. Flow chart of pattern search and polling method

The PSA algorithm is repeated until the following criteria is met

1. Algorithm runs until the size of mesh is less than the predefined tolerance value.
2. PSA repeats until objective functional value evaluated reaches preset number of functional evaluations.
3. PSA stops when the distance from the point of one satisfied poll to the point of next satisfied poll is less than the tolerance value and if it exceeds predefined maximum iterations.

3.2 Non Linear Constraints

In this paper to solve non linear constraint PMU optimization problem an Augmented Lagrangian Pattern Search (ALPS) is used to allocate the optimal number of PMUs in the network [15]. A sub problem is developed combining the objective function with non linear constraints using penalty parameters and Lagrangian. Sequences of such sub problems are optimized using PSA such that nonlinear constraints are satisfied. The number of sub objective functions evaluated per iteration could be higher when using nonlinear constraints.

The tolerance value considered is 10^{-7}, which is distance from current point to the boundary of linearly constrained region. This simplifies in to a new sub problem formulation and evaluation of the problem. These steps are iterated until stopping criteria are met.

4 Optimal Allocation of PMUs considering Sensitive constraints

4.1 Detection of Sensitive Buses

Buses which are affected due to heavy load changes are considered as sensitive buses.

Table 1 Voltage Sensitive Indices of 14-Bus System

Bus no	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14
VSI	0	0	0	0.006	0.005	0	0.005	0	**0.009**	0.008	0.005	0.003	0.004	**0.011**

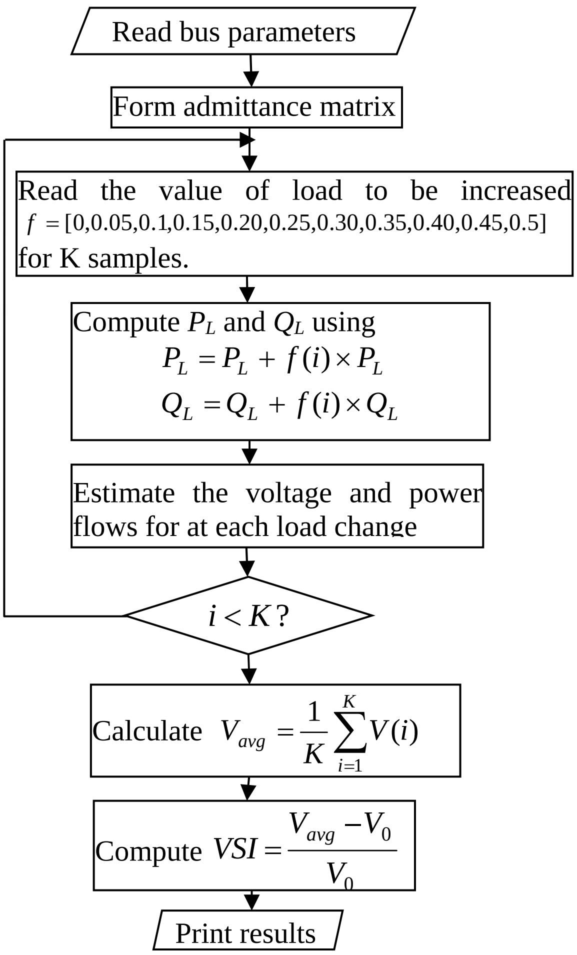


Fig.2. Flowchart to estimate sensitive buses

In this paper voltage sensitive index is formulated with load flows, at increase of load value from 5 to 50 percent. The procedure to calculate the VSI for bus networks is shown in Fig. 2.

$$V_{avg} = \frac{1}{K}\sum_{i=1}^{K} V(i) \quad (6)$$

$$VSI = \frac{V_{avg} - V_0}{V_0} \quad (7)$$

VSI can be defined as ratio of difference of average voltages of all increasing loads to nominal voltage and nominal voltage. The index with highest value is considered as most sensitive bus of the system. For instance consider 14-bus system to which the procedure of VSI formulation is applied. VSI computed data at each bus is shown in Table 1.

From the above data obtained, bus-14 and bus-9 are highest sensitive buses obtained.

4.2 Zero Injection Bus constraint modeling

In a power system, some buses are left without any power injections, those buses in which current flow is nearly equal to zero are known as zero injection buses. With removal of such ZI bus, non linear constraints in the optimization process can benefit the optimal location of PMUs for complete observability with increase of redundancy at each bus in the network. A few buses left with single incident branches in bus network, such buses are called as radial buses. If radial buses are connected to zero injection buses, the radial buses and ZI buses are considered to remove from observability of the system.

In 14 bus network, bus 7 is a zero injection bus and only one radial bus (bus 8) emanates from it. The constraints associated with bus 7 and 8 i.e., $(1-z_7),(1-z_8)$ are removed from the observability condition to minimize the PMUs. The nonlinear constraints are reduced with the rule of subsets. Given two sets A and B, where set A is a subset of set B, Then $A+B=B$ [17]. Applying set rule to observability constraints of buses.

(bus-1 +bus-2) = bus2, (bus-3 +bus-4) = bus4, (bus-8 +bus7) = bus-7, (bus-12 +bus13) = bus-13

Optimization problem can be formulated as

$$J(z) = Min\sum_{k=1}^{14} z_k^T c_k z_k \qquad (8)$$

Subject to nonlinear observability constraints

$$b(z) = \begin{cases} bus-2 = (1-z_1)(1-z_2)(1-z_3)(1-z_4)(1-z_5) = 0 \\ bus-4 = (1-z_2)(1-z_3)(1-z_4)(1-z_5)(1-z_7)(1-z_9) = 0 \\ bus-5 = (1-z_1)(1-z_2)(1-z_4)(1-z_5)(1-z_6) = 0 \\ bus-6 = (1-z_5)(1-z_6)(1-z_{11})(1-z_{12})(1-z_{13}) = 0 \\ bus-7 = (1-z_4)(1-z_9) = 0 \\ bus-9 = (1-z_4)(1-z_9)(1-z_{10})(1-z_{14}) = 0 \\ bus-10 = (1-z_9)(1-z_{10})(1-z_{11}) = 0 \\ bus-11 = (1-z_6)(1-z_{10})(1-z_{11}) = 0 \\ bus-13 = (1-z_6)(1-z_{12})(1-z_{13})(1-z_{14}) = 0 \\ bus-14 = (1-z_9)(1-z_{13})(1-z_{14}) = 0 \end{cases} \qquad (9)$$

With application of proposed PSA approach to the nonlinear observability constraints the optimal location of PMUs obtained is (4, 6, 9) or (2, 6, 9).

4.3 Optimal PMU Allocation considering Sensitive constraints

In sensitive constrained pattern search programming formulation, for instance in 14-bus system, let us consider bus 14, 9 are most sensitive buses as described in section (a). To locate PMUs at sensitive buses the locations suggested are 14 and 9 in 14- bus system. Substituting $z_{14} = 1, z_9 = 1$ in the non linear observability constraints in (5), buses 4, 7, 9, 10, 13, 14 are satisfied. The problem of optimization is subjected to remaining constraints as follows

The sensitive constrained optimization problem is written as

$$J(z) = Min\sum_{k=1}^{14} z_k^T c_k z_k \qquad (10)$$

Subjected to observability constraints

$$b(z) = \begin{cases} bus-1 = (1-z_1)(1-z_2)(1-z_5) = 0 \\ bus-2 = (1-z_1)(1-z_2)(1-z_3)(1-z_4)(1-z_5) = 0 \\ bus-3 = (1-z_2)(1-z_3)(1-z_4) = 0 \\ bus-5 = (1-z_1)(1-z_2)(1-z_4)(1-z_5)(1-z_6) = 0 \\ bus-6 = (1-z_5)(1-z_6)(1-z_{11})(1-z_{12})(1-z_{13}) = 0 \\ bus-8 = (1-z_7)(1-z_8) = 0 \\ bus-11 = (1-z_6)(1-z_{10})(1-z_{11}) = 0 \\ bus-12 = (1-z_6)(1-z_{13})(1-z_{12}) = 0 \end{cases} \qquad (11)$$

With application of pattern search programming the optimal PMU locations obtained at 2, 6, 7, 9, and 14 forming complete observability of the system.

4.4 Sensitive Constrained Optimal PMU Allocation with Zero Injection Modeling

With removal of non linear zero injection constraints and application of set rule to the equation (11) the problem subjected to observability constraints resolved are as follows
The problem is formulated as

$$J(z) = Min\sum_{k=1}^{14} z_k^T c_k z_k \qquad (12)$$

Subjected to

$$b(z) = 0, \qquad 0 \le z \le 1 \qquad (13)$$

$$A_{eq}\,(1 - z_k) = 0 \qquad (14)$$

$$b(z) = \begin{cases} bus-2 = (1-z_1)(1-z_2)(1-z_3)(1-z_4)(1-z_5) = 0 \\ bus-5 = (1-z_1)(1-z_2)(1-z_4)(1-z_5)(1-z_6) = 0 \\ bus-6 = (1-z_5)(1-z_6)(1-z_{11})(1-z_{12})(1-z_{13}) = 0 \\ bus-11 = (1-z_6)(1-z_{10})(1-z_{11}) = 0 \end{cases} \qquad (15)$$

where z_k is PMU placement variable of bus-k. $C = diag(c_1, c_2, ...c_n)$ is PMU cost vector or weight vector, n is number of buses, A_{eq} is vector of sensitive buses whose entries are all equal one.

With application of pattern search approach to the problem the optimal number of PMUs obtained to be located at 2, 6, 9 and 14. The system is complete observable with removal of radial buses.

4.5 Observability of the system

In power system state estimation, observability of system is obtained by two methods, one is through finding rank of the Jacobian matrix formulated in state estimation process other is through topological observability of system. Here we preferred topological observability of system. Through this process each bus of system is checked with Bus Observability Index (BOI) to find the system is complete observable or not. The limitation of BOI is considered as maximum number of branches connected plus one [18].

$$\Re_k \le \chi_k + 1 \qquad (16)$$

For bus-*k*, *BOI* ($\Re$) gives the number of PMUs allocated to measure the bus. Sum of BOI at all buses of the system gives Complete System Observability Index (CSORI) and it is given as

$$CSORI = \sum_{k=1}^{n} \Re_k \tag{17}$$

Maximum redundancy of the bus can be formulated as

$$Max \sum_{k=1}^{n} bz_k \tag{18}$$

Subjected to following constraints

$$\sum_{k=1}^{n} z_k = \mu_0 \tag{19}$$

where μ_0 is minimum number of PMUs obtained for complete system observability, b is bus connectivity matrix, z_k is binary decision variable.

$$z_k = \begin{cases} 1 & \text{if PMU is allocated at bus } k \\ 0 & \text{otherwise} \end{cases}$$

$$b_{i,j} = \begin{cases} 1 & \text{if } i = j \text{ or connected to each other} \\ 0 & \text{otherwise} \end{cases}$$

PMUs are allocated at sensitive buses that can increase the BOI and redundancy of the system.

5 Results and Analysis

To allocate PMUs and check complete observability, four different test cases such as 14-, 24-, 30- and 57-bus test systems are considered to analyze the applicability of the proposed PSA. The single line diagrams of 14- bus system are shown in Fig. 3. The optimal PMU allocation problem with PSA is programmed in MATALB and it is run on Intel(R) core(TM), i3 processor at 2.20 GHz with 4 GB of RAM. PSA is modelled with sensitive constrained optimization for allocation of PMUs in the network.

The initial starting point z_0 considered is as 0.5 for the search process in optimization method. When computing cost function, the value of weight is normally taken as 1 for all diagonal elements of matrix for n-bus system.

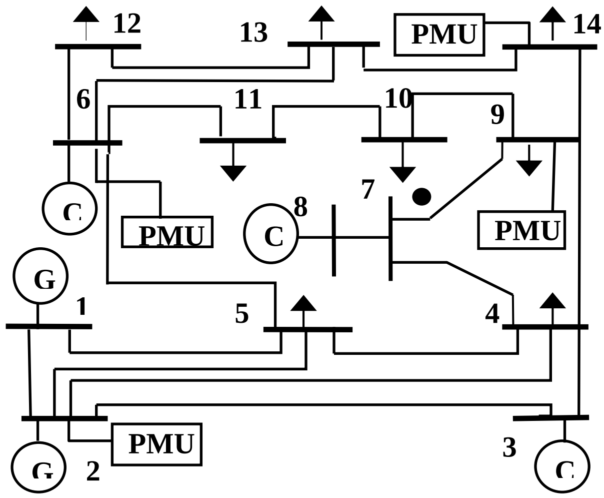


Fig.3. Single line diagram of 14-bus network with PMU locations

. The weight for PMU installation can be varied from 0 to 2 depending on cost, installation, and manufacture criteria of PMUs. As weight considered increases, the cost function value of PMUs increases. Table 2 shows sensitive buses arranged in descending order from higher sensitive to lower one and ZI bus of respective IEEE 14-, 24-, 30-, 57 test systems.

Table 2 Sensitive and ZI Buses

IEEE test systems	Sensitive buses	ZIB buses
14 bus	14,9	7
24 bus	22,21	11,12,16,17
30 bus	30,26,24,19	6,9,22,25,27,28
57 bus	31,33,29,32	4,7,11,21,22,24,26,34 ,36,37,39,40,45,46,48

For 14-bus system, BOI is computed at each bus to estimate number of times bus is observed to increase redundancy and to achieve full observability of the system. BOI with and without sensitive constrained modeling for each bus in 14- bus system is shown in Table 3.

Table 3. Bus Observabilty Index (BOI) with and without Sensitive buses

Bus no	1	2	3	4	5	6	7	8	9	10	11	12	13	14
BOI Without Sensitive buses	1	1	1	3	2	1	2	1	2	1	1	1	1	1
BOI With Sensitive Buses	1	1	1	3	2	1	2	1	3	1	1	1	2	2

Table 4. PMU Locations with and without ZI bus constraint Modeling

IEEE test systems	PMU locations without ZI Modeling		PMU locations with ZI Modeling	
	No. of PMUs	PMU locations	No. of PMUs	PMU Locations
14 bus	4	2,6,7,9,	3	2,6,9,
24 bus	9	1,2,7,11,12,15,17,19,21	8	2,5,9,13,14,15,18,19
30 bus	10	1,5,8,9,10,12,15,18,25,27	8	1,7,10,12,15,18,22,29
57 bus	18	1,4,9,10,15,20,23,27,29, 30,32,36,38,39,41,46,49,53	17	1,6,9,10,14,15,18,23,29, 30,32,35,38,41,49,53,56

Table 5. Sensitive Constrained PMU Locations with and without ZI bus constraint Modeling

IEEE test systems	PMU locations without ZI Modeling		PMU locations with ZI Modeling	
	No. of PMUs	PMU locations	No. of PMUs	PMU Locations
14 bus	5	2,6,7,9,14	4	2,6,9,14
24 bus	9	1,2,3,7,9,11,19,21,22	8	2,.3,5,9,14,19,21,22
30 bus	10	1,5,8,9,10,12,19,24,26,30	8	1,7,10,12,19,24,26,30
57 bus	20	1,6,9,10,15,19,22,24,26,29,3 1,32,33,36,38,39,41,46,49,53	17	1,6,9,10,15,18,25,29,31,32,3 3,35,38,41,49,53,56

Table 6. Complete System Observability Redundancy Index (CSORI)

IEEE test systems	without ZI Bus modeling	With ZI Bus modeling	Proposed sensitive constrained PMU locations without ZI Bus modeling	Proposed sensitive constrained PMU locations with ZI Bus modeling
14 bus	19	15	22	18
24 bus	35	30	35	30
30 bus	43	34	38	31
57 bus	76	75	79	77

Table 4 presents the PMU locations with and without considering ZI bus constraint modeling

From table 4, it is observed that number of PMUs is less with inclusion of ZI bus constraint modeling than without ZI bus constraint modeling. With application of sensitive constrained pattern search approach for optimal allocation of PMUs in the networks, PMUs are allocated with maximum redundancy and complete observability of the system. Table 5 presents PMU location at sensitive buses considering with and without ZI bus constraint modeling. From table 5, it is observed that number of PMUs with sensitive constrained locations is less with inclusion ZI bus constraint modeling than without ZI bus constraint modeling.

Total redundancy of system is improved with allocation of PMUs at sensitive buses and optimum redundancy is achieved with ZI bus constraint modeling. Performance of sensitive constrained optimization (PSA) process can be estimated through redundancy index (17). Complete System

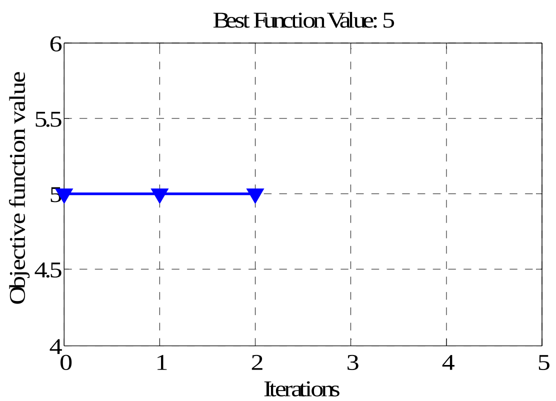
Observability Redundancy Index (CSORI) is computed at sensitive constrained locations with and without ZI bus constraint modeling for different test case systems are shown in Table 6. From the table 6, it is observed that CSORI is less with ZI bus constraint modeling compared to without ZI bus constraint modeling. The computation time required for obtaining optimum value of objective function and number of iteration taken to converge to optimum value in each test case systems is shown in Table 7. Mesh size converges at 9.3325x10^{-7} for every system. From the table it is observed that for large power systems, time taken for converge is more compared to small systems and time consumed for computation is very less compared to other methods. The advantage of pattern search algorithm is, it converges with in less number of iterations.

Table 7. Time consumed for each test case systems

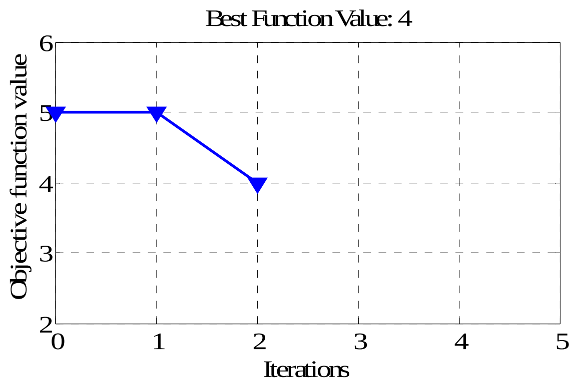
IEEE test cases	Time (sec)
14 bus	0.421608
24 bus	0.878450
30 bus	1.168460
57 bus	387.684419

For instance consider 14 and 30 bus systems to which cost function is evaluated with PSA approach. The mesh points in Fig. 4(a) shows the optimal values of PMUs considering sensitive buses without zero injection modeling. Fig. 4(b) shows the minimum cost function value of PMUs considering sensitive buses with zero injection modeling. From Fig. 4(a) and (b), it is observed that number of iterations to converge optimal cost value is very less in number.

The mesh points in Fig.5(a) shows optimal best cost function value of PMUs considering sensitive buses without zero injection modeling for 30 bus system.

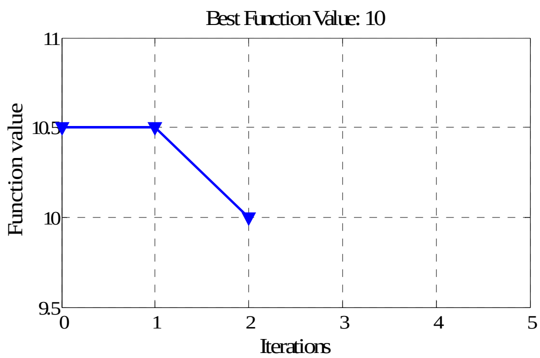


a) Cost function without ZI Modeling

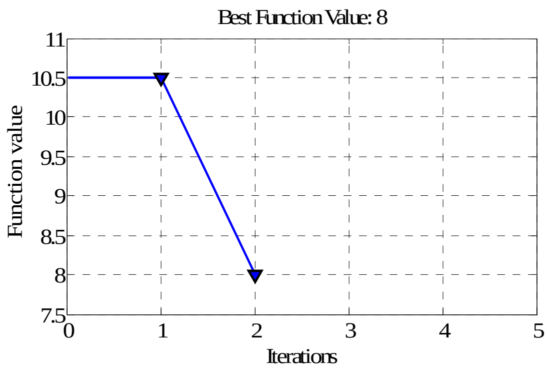


b) Cost function with ZI Modeling

Fig.4. Cost functions considering sensitive buses without and with ZI bus modeling for 14-Bus System (a, b)



a) Cost function without ZI Modeling



b) Cost function with ZI Modeling

Fig.5. Cost functions considering sensitive buses without and with ZI Bus modelling for 30-Bus System (a, b)

Fig.5(b) presents minimum cost function value of PMUs considering sensitive buses with ZI bus modeling. From Fig 5(a) and (b), it is observed that optimal function value is attained faster with less time and number of iterations. The proposed PSA method is compared with other mathematical models that obtained complete observability is shown in Table 8. Generally for linear problems the methods used are integer linear programming method (ILP) proposed in [19-20] to optimally allocate PMUs. To solve this problem subjected to non linear constraints the methods used till now is WLS method proposed by [7].

Table 8. Comparison of number of PMU Allocated with Complete Observability

Methods	14 bus system	24 bus system	30bus system	57bus system
Integer quadratic [6]	4	-	10	17
WLS[7]	4#	-	10#	17#
Generalized ILP [19]	4	-	10	17
Pattern search method	4	9	10	18
Pattern search with ZIB modeling	3	8	8	17
Sensitive constrained pattern search	5	10	10	20
Sensitive constrained pattern search with ZIB modeling	4*	8*	8*	17*

* Optimal number of PMUs allocated at sensitive buses.
\# Optimal number of PMUs obtained solely with PMU measurements

The proposed PSA method when compared with these mathematical methods gives better solutions for non linear type of problems at very less number of iterations with complete observability compared to other heuristic methods [8-12]. All the methods under literature survey optimized the number of PMUs without considering sensitive buses in their allocations and that can lose observability and decrease in redundancy of buses which result in inaccurate states of system. Many papers published (literature survey) minimized PMUs with ZI bus constraint modeling but complete observability of system is not obtained. With sensitive constrained allocation and zero injection modeling in this paper, redundancy is gained which ensured the observability of each bus. Redundant numbers of PMUs are attained with complete observability.

Conclusion

This paper presented a pattern search approach for non linear constrained optimal allocation of PMUs at sensitive buses by giving priority to improve redundancy at each bus with complete observability of the system. A new VSI is proposed by performing load flows at increasing loads through which sensitive buses are identified. The proposed PSA method with ZI Bus constraint modeling and without ZI bus constraint modeling is compared to show the effectiveness of method. BOI with and without sensitive buses are presented to show the redundancy increase with optimal allocation of PMUs. With application of ZI bus constraint modeling to sensitive constrained optimization, the number of PMUs for allocation is minimized which decreased cost for installation of PMUs. The performance of complete observability of the system is analyzed through Complete System Observability Redundancy Index (CSORI).

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