Stochastic Inverse Analysis of Fatigue Cracks based on Linear Fracture Mechanics

ZDENĚK KALA
Department of Structural Mechanics
Brno University of Technology, Faculty of Civil Engineering
Veveří Str. 95, Brno
CZECH REPUBLIC
kala.z@fce.vutbr.cz; http://www.vutbr.cz

Abstract: - For steel structures and bridges subjected to fatigue loading, it is possible to determine the probabilities for basic phenomena that are related to the growth of fatigue cracks. Emerging new methods of probabilistic reliability assessment consider the effects of possible defects in the form of initiation cracks, which are the main cause of the propagation of fatigue cracks. The strongly nonlinear dependence between the initial crack size and fatigue resistance can lead to unrealistic probabilistic models if the types of probability density functions are selected inappropriately. The article discusses the uncertainties present in determining the variables in the calculation, which must be logically related to the probabilistic model of fatigue resistance. The aim of the present paper is to provide a methodology of inverse stochastic analysis, which is suitable for the verification of probabilistic models of fatigue crack propagation.

Key-Words: - Fatigue, fracture, mechanics, Paris-Erdogan, crack, stochastic, skewness, steel, bridge, structure

1 Introduction

In Eurocode 3, the widespread method of design and assessment of fatigue is based on the detail category specified Wöhler curve (S-N curve) and the Palmgren–Miner cumulative damage rule [1]. Wöhler curves permits a limited lifetime to failure, which is problematically determined, based on constant amplitude and the expected number of load cycles. The methodology has been gradually developed into procedures that describe real conditions and facilitate the work of designers; however, it is not sufficiently universal [2]. Wöhler curves are only available for selected structural details given by the classification tables in design codes, such as Eurocode 3 [3], BS 5400 [4] and AASHTO [5].

The traditional Wöhler (S-N) method cannot be used to determine the effect of a specific defect on the fatigue life. Linear elastic fracture mechanics presents a tool for the analysis of fatigue crack propagation of numerous cracked structural details [6, 7]. Approaches based on linear elastic fracture mechanics provide information on crack size and the growth rate of cracks under actual service loads [2].

An important input quantity for the analysis of fatigue degradation is stress history, which can be generated using deformation measurements in combination with FE models [8, 9]. The fatigue crack propagation life-span of each structural detail and critical connection can be predicted using the standard Paris-Erdogan crack growth model [10]. The prediction of the lifetime of fatigue cracks requires stochastic models that consider the uncertainty of all parameters, which by their nature are random variables, see e.g. [11, 12]. Monte Carlo numerical simulation methods are effective, but are not the only tools for the analysis of fatigue degradation and lifetime of structural steel constructions and steel bridges, see e.g. [13, 14]. Results of probabilistic studies are mainly used to determine inspection times and to analyse their results, which in the absence of cracks, lead to the conditional probability of their occurrence.

2 Linear Fracture Mechanics

Linear fracture mechanics has been the subject of research for many years, especially in the field of mechanical engineering and is gradually being applied and modified for the design of load bearing building structures. Commonly applied linear elastic fracture mechanics analyses the propagation of an initial crack of magnitude a in dependence on the number of fatigue cycles N. Fatigue crack growth is generally described by Paris's rule which is expressed by Paris and Erdogan [10].

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where m and C are Paris-Erdogan (material-related) law parameters and the range of stress intensity factor ΔK can be determined by [15].

$$\Delta K = \Delta\sigma\sqrt{\pi a}F(a) \tag{2}$$

where $F(a)$ is the geometric factor (calibration function) describing the course of crack propagation with respect to the geometry of the sample and $\Delta\sigma$ is the quasi–constant stress range.

$$\int_{a_0}^{a_{cr}} \frac{da}{\left[F(a)\cdot\sqrt{\pi\cdot a}\right]^m} = C\cdot N_F\cdot\Delta\sigma^m \tag{3}$$

where N_F is the total number of cycles at crack growth from a_0 to a_{cr}. The quasi–constant stress range $\Delta\sigma$ = 50 MPa is considered. C, m are material constants according (6)

$$\log(C) = c_1 + c_2 m \tag{4}$$

where c_1, c_2 can be considered for steel grade S235 as c_1 = -11.141 and c_2 = -0.507 [16]. $F(a)$ is the calibration function evaluated for pure bending in the form [17]:

$$F(a) = 1.114 - 1.8975(a/W) + 2.752(a/W)^2 - 1.1323(a/W)^3 \tag{5}$$

where a is crack length and W is specimen width in the direction of crack propagation.

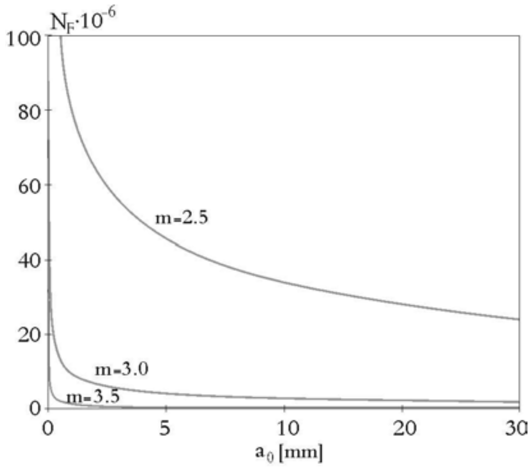

Fig.1: Fatigue resistance N_F vs a_0, W=400, a_{cr}=175

An example of the dependence between N_F and a_0 is shown in Fig.1. With regards to the strongly non-linear dependence between N_F and a_0, it is more practical to work with the logarithms of these variables. An example of the dependence between logarithms N_F and a_0 is shown in Fig.2.

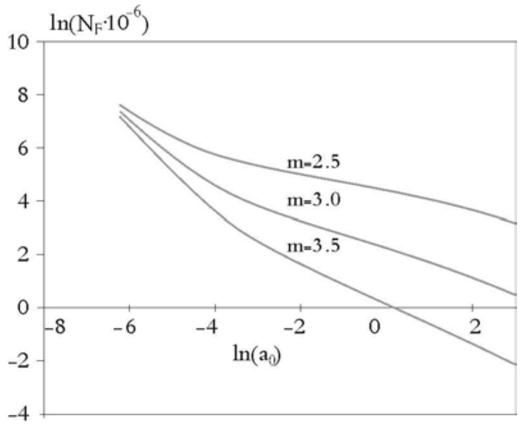

Fig.2: ln(N_F) vs ln(a_0) for W=400, a_{cr}=175

3 Probabilistic Analysis

The input random variables of the probabilistic model are listed in the Table 1. Initial crack size a_0 has a log-normal probability density function (pdf), the other random variables have Gauss pdf. The Latin Hypercube Sampling (LHS) method [18, 19] based on repeated random sampling is used to obtain the numerical results.

Table 1: Input random variables

Random variables		Mean value	Standard Deviation
Initial crack size	a_0	0.526 mm	0.504 mm
Critical crack size	a_{cr}	175 mm	14 mm
Specimen width	W	400 mm	20 mm
Parameter	m	3	0.03

The fatigue resistance N_F is the output random variable, whose statistical characteristics and pdf are examined. The mean value of N_F is m_{NF}=16.71E6 and standard deviation is m_{NF}=7.62E6, which are the statistical results, obtained using one million runs of the LHS method. The Chi-square goodness-of-fit test does not reject the hypothesis that N_F has a log-normal pdf. Practically, it means that the data fit the log-normal pdf very well, but it does not necessarily imply a hundred percent fit. If N_F really has a log-normal pdf, then we can use this pdf (with parameters m_{NF}, m_{NF}) to simulate the random realizations of N_F and subsequently use inverse analysis to obtain random variable a_0, which has statistical characteristics listed in Table 1. It may be added that the above-described statistical model of initial crack size a_0 has skewness of 3.7, which is a parameter that will also be monitored.

Let us try to study a_0 using inverse analysis. Let us consider N_F as a random variable whose mean value and standard deviation are listed above with theoretical consideration of several different types of parametric pdfs.

The aim of the study is to perform an inverse analysis and obtain the histogram of a_0, whose pdf will be subsequently examined. The theoretically presumed pdfs introduced for N_F are listed in Table 2 and are also depicted in Fig.3 and Fig.4. The second chosen pdf type is the Hermite pdf, which has four parameters [20]. The third and fourth parameters of this pdf are skewness and kurtosis, which are considered to have values of 0.6 and 3.

Table 2: Variants for N_F random pdf

Var.	Pdf	Mean Value	Standard Deviation
1	Log-normal	16.71E6	7.62E6
2	Hermite	16.71E6	7.62E6
3	Truncated Gauss	16.71E6	7.62E6
4	Decreasing Triangular	16.71E6	7.62E6
5	Growing Triangular	21.71E6	7.62E6

Statistical analysis of a_0 is evaluated using ten thousand simulation runs of the LHS method. The results are shown in Fig. 5 to Fig. 14.

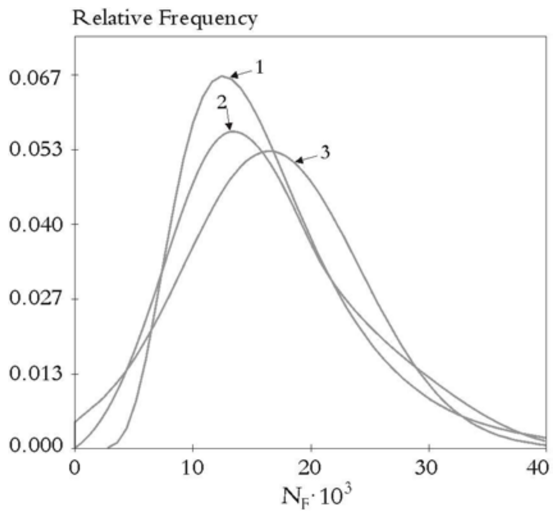

Fig.3: Pdfs of N_F - variants 1, 2, 3

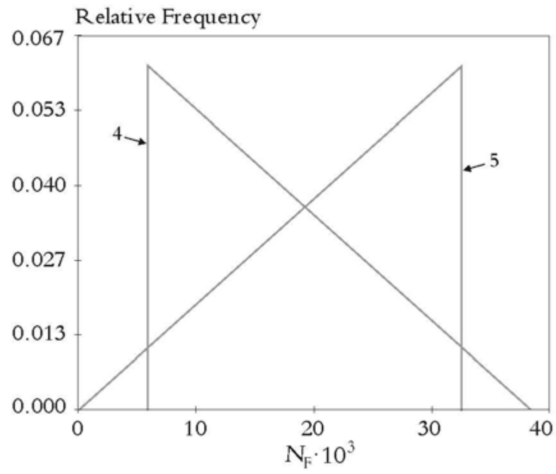

Fig.4: Pdfs of N_F - variants 4, 5

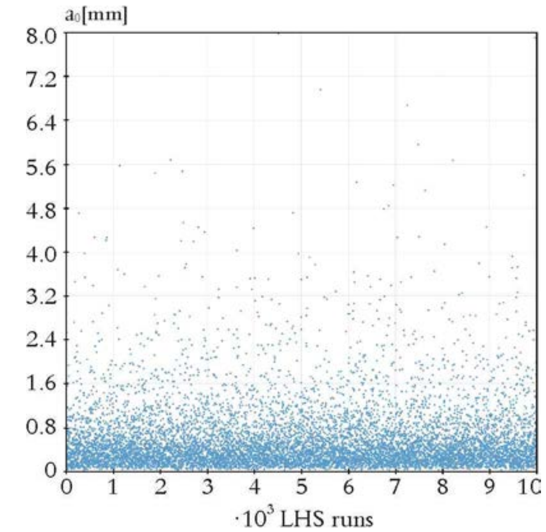

Fig.5: Observations of a_0 for Variant 1

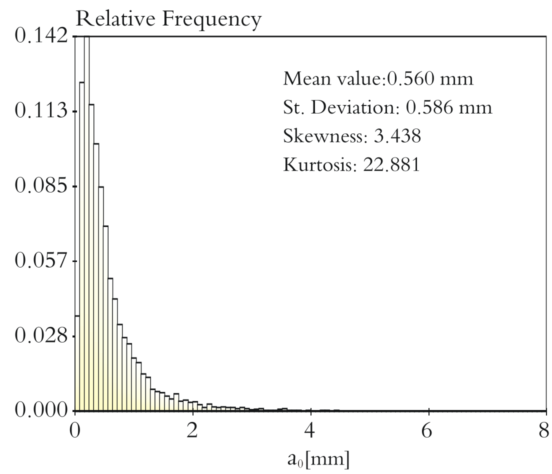

Fig.6: Histogram of a_0 for Variant 1

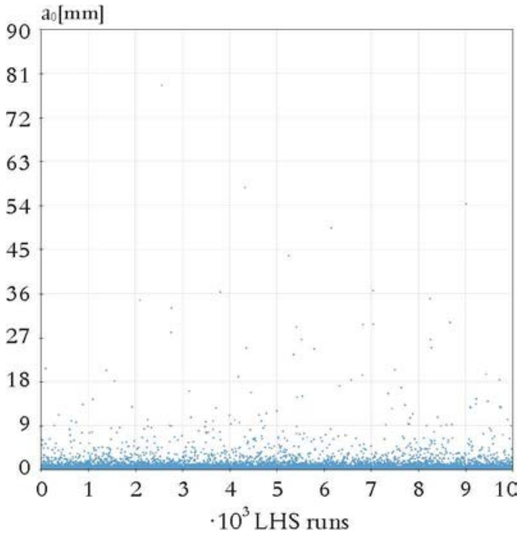

Fig.7: Observations of a_0 for Variant 2

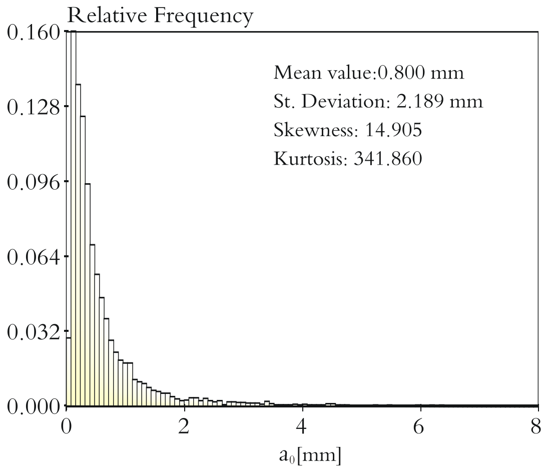

Fig.8: Histogram of a_0 for Variant 2

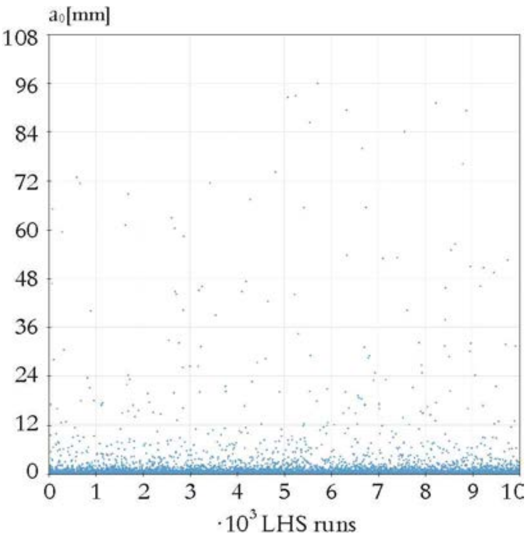

Fig.9: Observations of a_0 for Variant 3

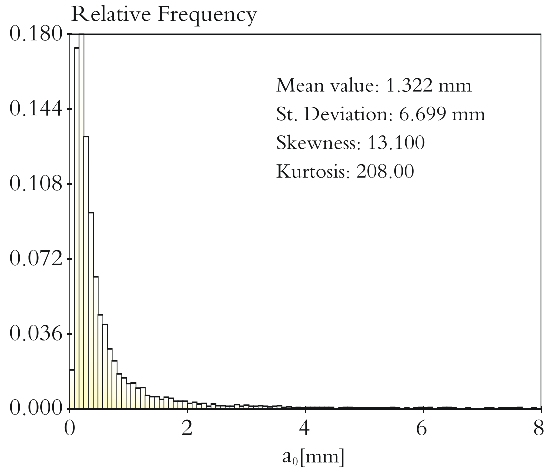

Fig.10: Histogram of a_0 for Variant 3

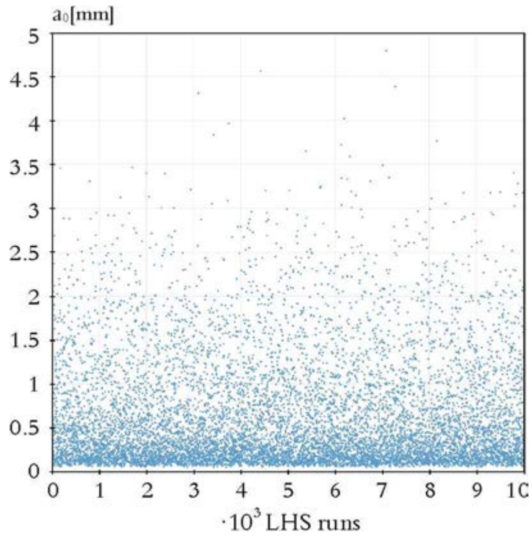

Fig.11: Observations of a_0 for Variant 4

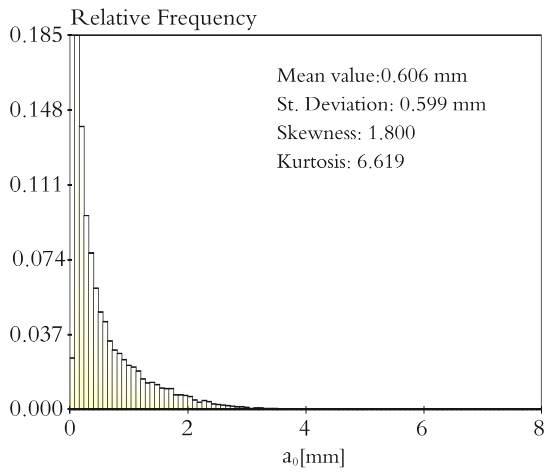

Fig.12: Histogram of a_0 for Variant 4

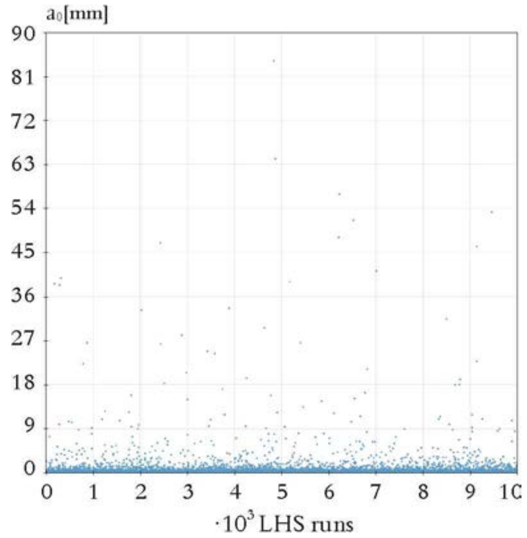

Fig.13: Observations of a_0 for Variant 5

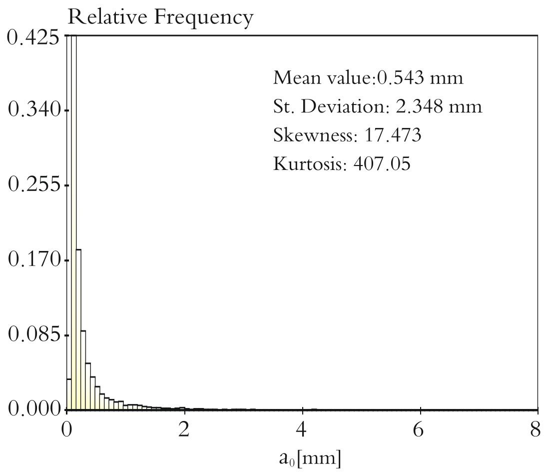

Fig.14: Histogram of a_0 for Variant 5

From the graphs shown above, it can be observed that the selection of inappropriate types of probability density functions of fatigue resistance lead to remote observations of the size of the initial crack. Observations of the initial crack must be proportionate to the frequency of the occurrence of real failures, which are observed during inspections of steel structures or bridges subjected to cyclical loads, see e.g. [12, 21, 22].

4 Conclusion

Inverse analysis a_0 is an important part of the verification of stochastic models because it provides information on crack propagation, which is needed to plan regular inspections. The inverse analysis performed in this article showed that if the fatigue resistance N_F has a log-normal pdf then a_0 has a log-normal pdf. However, the statistical characteristics a_0 correspond to the original values listed in Table 1 only approximately. Even so, the log-normal pdf is, out of all the pdf types introduced for N_F, the most suitable distribution that can be accepted with a high probability in stochastic models of structural elements subjected to fatigue damage. The unsuitable pdfs include Hermite, Truncated Gauss and Growing Triangular pdfs, which lead to the occurrence of large (unreal) frequencies of observations of high values of a_0. The Decreasing Triangular pdf, whose random realization maximum of a_0 determined in this article has a value of 4.8mm, is also worth considering in probabilistic studies. Practically, it is necessary to adopt for a_0 such a pdf, whose probability density function is zero in the vicinity of zero values of a_0 and at the same time decreases very rapidly when observing higher values. The log-normal pdf satisfies these requirements, but is not necessarily the only suitable pdf.

Acknowledgement

The article was elaborated within the framework of project GAČR 17-01589S.

References:

[1] M.A. Miner, Cumulative Damage in Fatigue, *Journal of Applied Mechanics*, *Transactions ASME*, Vol.12, No.3, 1945, pp. A159–A164.

[2] H. Zhou, G. Shi, Y. Wang, H. Chen, G. De Roeck, Fatigue Evaluation of a Composite Railway Bridge Based on Fracture Mechanics Through Global–local Dynamic Analysis, *Journal of Constructional Steel Research*, Vol.122, 2016, pp. 1–13.

[3] EN 1993-1-9 Eurocode 3, Design of Steel Structures – Part 1–9: Fatigue, European Committee for Standardization (CEN), Brussels, 2005.

[4] BS 5400, Steel, Concrete and Composite Bridges – Part 10: Code of Practice for Fatigue, British Standards Institute (BSI), London, 1980.

[5] American Association of State Highway Transportation Officials (AASHTO), LRFD Bridge Design Specifications, third ed. AASHTO, Washington, DC, 2004.

[6] S.T. Rolfe, J.M. Barsom, *Fracture and Fatigue Control in Structures: Applications of Fracture Mechanics*, ASTM International, 1977, p. 562.

[7] T.L. Anderson, *Fracture Mechanics: Fundamentals and Applications*, third ed. CRC Press, 2004, p. 640.

[8] M. Hušek, J. Kala, P. Král, F. Hokeš, Effect of the Support Domain Size in SPH Fracture Simulations, *International Journal of Mechanics*,Vol.10, 2016, pp. 396–402.

[9] S. Seitl, T. Thienpont, W. De Corte, Fatigue Crack Behaviour: Comparing Three-point Bend Test and Wedge Splitting Test Data on Vibrated Concrete using Paris' law, *Frattura ed Integrità Strutturale*, Vol.39, 2017, pp. 110–117.

[10] P.C. Paris, F. Erdogan, A Critical Analysis of Crack Propagation Laws, *Journal of Basic Engineering*, Vol.85, No.4, 1963, pp. 528–533.

[11] M. Krejsa, P. Janas, V. Krejsa, Z. Kala, S. Seitl, DOProC-based Reliability Assessment of Steel Structures Exposed to Fatigue, *Perspectives in Science*, Vol.7, 2016, pp. 228–235.

[12] M. Krejsa, Z. Kala, S. Seitl, Inspection Based Probabilistic Modeling of Fatigue Crack Progression, *Procedia Engineering*, Vol.142, 2016, pp. 146–153.

[13] Z. Kala, Fuzzy Probability Analysis of the Fatigue Resistance of Steel Structural Members under Bending, *Journal of Civil Engineering and Management*, Vol.14, No.1, 2008, pp. 67–72.

[14] K. Frydrýšek, L. Václavek, Stochastic Computer Approach Applied in the Reliability Assessment of Engineering Structures, *Advances in Intelligent Systems and Computing*, Vol.451, 2016, pp. 121–129.

[15] D. Broek, *Elementary Engineering Fracture Mechanics*, Martinus Nijhoff Publishers, 1988.

[16] D. Kocanda, V. Hutsaylyuk, T. Slezak, J. Torzewski, H. Nykyforchyn, V. Kyryliv, Fatigue Crack Growth Rates of S235 and S355 Steels After Friction Stir Processing, *Materials Sciences Forum*, Vol.726, 2012, pp. 203–210.

[17] G.V. Guinea, J.Y. Pastor, J. Planas and M. Elices, Stress Intensity Factor, Compliance and CMOD for a General Three-point-bend Beam, *International Journal of Fracture*, Vol.89, 1998, pp. 103–116.

[18] R. C. Iman, W. J. Conover, Small Sample Sensitivity Analysis Techniques for Computer Models with an Application to Risk Assessment, *Communications in Statistics – Theory and Methods*, Vol.9, No.17, 1980, pp. 1749–1842.

[19] M. D. McKey, R. J. Beckman, W. J. Conover, A Comparison of the Three Methods of Selecting Values of Input Variables in the Analysis of Output from a Computer Code, *Technometrics*, Vol.21, 1979, pp. 239–245.

[20] Z. Kala, Global Interval Sensitivity Analysis of Hermite Probability Density Function Percentiles, *International Journal of Mathematical Models and Methods in Applied Sciences*, Vol.10, 2016, pp. 373–380.

[21] M. Krejsa, Probabilistic Failure Analysis of Steel Structures Exposed to Fatigue, *Key Engineering Materials*, Vol. 577-578, 2014, pp. 101-104.

[22] M. Krejsa, Probabilistic Reliability Assessment of Steel Structures Exposed to Fatigue, *Safety, Reliability and Risk Analysis: Beyond the Horizon - Proceedings of the European Safety and Reliability Conference, ESREL 2013*, 2014, pp. 2671-2679.