New hybrid adaptive control for function projective synchronization of complex dynamical network with mixed time-varying and coupling delays

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Abstract: This paper investigates function projective synchronization (FPS) for complex dynamical network with mixed time-varying and hybrid coupling delays, which is composed of state coupling, time-varying delay coupling and distributed time-varying delay coupling. In contrast to previous results, the coupling configuration matrix need not be symmetric or irreducible. The designed controller ensures that the FPS of delayed complex dynamical network are proposed via hybrid error adaptive control, which contains error term, time-varying delay error term and distributed time-varying delay error term. Based on the construction of improved Lyapunov-Krasovskii functional is combined with Leibniz-Newton formula and the technique of dealing with some integral terms, we prove the stability of the closed-loop system and the convergence of the error system. Numerical example is included to show the effectiveness of the proposed hybrid adaptive control scheme.

Key–Words: hybrid adaptive control; function projective synchronization; complex dynamical network; mixed time-varying delay; mixed coupling delay

1 Introduction

Complex dynamical network, as an interesting subject, has been thoroughly investigated for decades. These networks show very complicated behavior and can be used to model and explain many complex systems in nature such as computer networks [5], the world wide web [6], food webs [7], cellular and metabolic networks [8], social networks [17], electrical power grids [10] etc.

The concept of chaos synchronization is making two or more chaotic systems oscillate in a synchronized manner. There are several schemes which can be used to achieve chaos synchronization of chaos complex network, for example time-delay feedback control [28], intermittent control [28], adaptive control [29, 30], active control [18], nonlinear feedback control [31–33], sampled-data control [33]. The traditional method to synchronize a complex network is to add a controller to each of the network nodes to tame the dynamics to approach a desired synchronization trajectory. Authors in [34], investigated adaptive control strategy for complex delayed dynamical networks with time-varying coupling strength and time-varying delayed. Shi et al. [27], introduced FPS for complex dynamical networks with state coupling, which it is not necessary for the coupling matrix to satisfy symmetric or nonnegative criteria. FPS is investigated via adaptive feedback control and pinning control with adaptive coupling strength.

In the last decade, the synchronization of complex dynamic networks has attracted much attention of researchers in this field [18, 22, 23, 27, 34]. Because the synchronization of complex dynamical networks can well explain many natural phenomena observed and is one of the important dynamical mechanisms for creating order in complex dynamical networks, the synchronization of coupled dynamical networks has come be a focal point in the study of nonlinear science. Wang and Chen introduced a uniform dynamical network model and also investigated its synchronization [37, 38]. They have shown that the synchronizability of a scale-free dynamical network is robust against random removal of nodes, and yet is fragile to specific removal of the most highly connected nodes [38]. Authors in [35, 36] investigated synchronization of general complex dynamical network models with coupling delays. Wang [39] introduced several synchronization criteria for both delay-independent and delay-dependent asymptotical stability. Li [40] investigated synchronization of complex networks with time-varying couplings, the stability criteria were obtained by using Lyapunov-Krasovskii function method and subspace projection method.

Function projective synchronization (FPS), which is the generalization of projective synchronization (PS), is one of the important synchronization methods that have been widely investigated to obtain faster communication with its proportional feature. FPS of general complex networks was investigated which means that the nodes of complex networks could be synchronize up to an equilibrium point or periodic orbit with a desired scaling function. FPS has attracted the interest of many researchers in various fields [15, 18, 20, 21, 23, 25]. Very recently, FPS has been investigated in a two-cell quantum-CNN chaotic oscillator, [25, 26]. In [21], the authors just considered the FPS in drive-response dynamical networks (DRDNs) with coupled partially linear chaotic systems by assuming that the node dynamics are identical and using a simple control law. Furthermore, In [15], investigated the problem of FPS in DRDNs with nonidentical nodes by the adaptive open-plus-closed-loop method. Ref. [22], investigated FPS in DRDNs with uncertain parameters and disturbances. In [23], a hybrid feedback control method was proposed for achieving FPS in CDNs with constant time delay and time-varying coupling delay. In [18], the authors studied projective synchronization by using active control approach. These synchronization methods or ideas can be applied to the synchronization of complex network.

This paper, inspired by the above discussions, we shall investigate function projective synchronization (FPS) for complex dynamical network with mixed time-varying and hybrid coupling delays, which is composed of state coupling, time-varying delay coupling and distributed time-varying delay coupling. In contrast to previous results, the coupling configuration matrix need not be symmetric or irreducible. The designed controller ensures that the FPS of delayed complex dynamical network are proposed via hybrid error adaptive control, which contains error term, time-varying delay error term and distributed time-varying delay error term. Based on the construction of improved Lyapunov-Krasovskii functional is combined with Leibniz-Newton formula and the technique of dealing with some integral terms, we prove the stability of the closed-loop system and the convergence of the error system. Numerical example is included to show the effectiveness of the proposed hybrid adaptive control scheme.

Notation $\mathbb{R}^n$ is the n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrices; I_n represents the n-dimensional identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{Re\lambda; \lambda \in \lambda(A)\}$; $C([0,t],\mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$-valued continuous functions on $[0,t]$; $L_2([0,t],\mathbb{R}^m)$ denotes the set of all the $\mathbb{R}^m$-valued square integrable functions on $[0,t]$; The notation $X \geq 0$ (respectively, $X > 0$) means that X is positive semidefinite (respectively, positive definite); $\mathrm{diag}(\cdots)$ denotes a b lock diagonal matrix; $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$ stands for $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

2 Problem statements and preliminaries

Consider a complex dynamical network consisting of N identical coupled nodes, with each node being an n-dimensional nonlinear dynamical system

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t), x_i(t-h(t)), \int_{t-k(t)}^{t} x_i(s)\,ds) \\ &\quad + c_1 \sum_{j=1}^{N} a_{ij} G_1 x_j(t) \\ &\quad + c_2 \sum_{j=1}^{N} b_{ij} G_2 x_j(t-h(t)) \qquad (1) \\ &\quad + c_3 \sum_{j=1}^{N} c_{ij} G_3 \int_{t-k(t)}^{t} x_j(s)\,ds + \mathcal{U}_i(t), \\ &\quad t \geq 0, \;\; i = 1,2,...,N, \\ x_i(t) &= \phi_i(t), t \in [-\tau_{\max}, 0], \tau_{\max} = \max\{h, k\}, \end{aligned}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), ..., x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of ith node; $\mathcal{U}_i(t) \in \mathbb{R}^m$ are the control input of the node i; the constant $c_1, c_2, c_3 > 0$ are the coupling strength; $G_1 = (g_{1ij})_{n\times n}$, $G_2 = (g_{2ij})_{n\times n}$, $G_3 = (g_{3ij})_{n\times n} \in \mathbb{R}^{n\times n}$ are a constant inner-coupling matrix; $A = (a_{ij})_{N\times N}$, $B = (b_{ij})_{N\times N}$, $C = (c_{ij})_{N\times N} \in \mathbb{R}^{N\times N}$ are the outer-coupling matrix of the network, in which a_{ij}, b_{ij} are defined as follows: if there are a connection between node i and node j $(j \neq i)$, then $a_{ij} > 0$, $b_{ij} > 0$, $c_{ij} > 0$; otherwise, $a_{ij} = a_{ji} = 0$, $b_{ij} = b_{ji} = 0$, $c_{ij} = c_{ji} = 0$ $(j \neq i)$, and the diagonal elements of matrix A, B and C are defined by

$$\begin{aligned} a_{ii} &= -\sum_{j=1,i\neq j}^{N} a_{ij} = -\sum_{j=1,i\neq j}^{N} a_{ji}, \\ b_{ii} &= -\sum_{j=1,i\neq j}^{N} b_{ij} = -\sum_{j=1,i\neq j}^{N} b_{ji}, \qquad (2) \\ c_{ii} &= -\sum_{j=1,i\neq j}^{N} c_{ij} = -\sum_{j=1,i\neq j}^{N} c_{ji}, i = 1,2,...,N. \end{aligned}$$

Definition 1. *The network (1) with time delay is said to achieve function projective synchronization if there exists a continuously differentiable scaling function matrix $\alpha(t)$ such that*

$$\lim_{t\to\infty} \|e_i(t)\| = \lim_{t\to\infty} \|x_i(t) - \alpha(t)s(t)\|, i = 1,2,\ldots,N$$

where $\|.\|$ stands for the Euclidean vector norm and $s(t) \in R^n$ can be an equilibrium point, or a (quasi-)periodic orbit, or an orbit of a chaotic attractor, which satisfies $\dot{s}(t) = f(s(t), s(t - h(t)), \int_{t-k_1(t)}^{t} s(\theta)\, d\theta)$.

To investigate the stability of the synchronized states (1), we set the synchronization error $e_i(t)$ in the form $e_i(t) = x_i(t) - \alpha(t)s(t)$, $i = 1,\ldots,N$, where $\alpha(t)$ is a n-order real diagonal matrix, i.e. $\alpha(t) =$diag$(\alpha(t)_1, \alpha(t)_2, \ldots, n)$ and $\alpha_i(t)$ is a continuously bounded differentiable function. Then, substituting it into complex dynamical network (1), it is easy to get the following:

$$\begin{aligned} \dot{e}_i(t) &= \dot{x}_i(t) - \dot{\alpha}(t)s(t) - \alpha(t)\dot{s}(t) \\ &= f(x_i(t), x_i(t-h(t)), \int_{t-k(t)}^{t} x_i(s)\, ds) \\ &\quad -\alpha(t) f(s(t), s(t-h(t)), \int_{t-k(t)}^{t} s(\theta)\, d\theta) \\ &\quad +c_1 \sum_{j=1}^{N} a_{ij} G_1 e_j(t) \qquad (3) \\ &\quad +c_2 \sum_{j=1}^{N} b_{ij} G_2 e_j(t-h(t)) \\ &\quad +c_3 \sum_{j=1}^{N} c_{ij} G_3 \int_{t-k(t)}^{t} e_j(s)\, ds \\ &\quad -\dot{\alpha}(t)s(t) + \mathcal{U}_i(t), \;\; i = 1,\ldots,N. \end{aligned}$$

The initial condition function $\phi_i(t)$ denotes a continuous vector-valued initial function of $t \in [-\tau_{\max}, 0]$, $\tau_{\max} = \max\{h, k\}$.

In the rest of this paper, we need the following assumption and some lemmas:

Assumption 2. *The time-varying delay functions $h(t)$ is differential function and $k(t)$ with satisfy the condition $0 \leq h(t) \leq h$, $0 \leq k(t) \leq k$ and $0 \leq \dot{h}(t) \leq h < 1$.*

Lemma 3. *[1] (Cauchy inequality) For any symmetric positive definite matrix $N \in M^{n\times n}$ and $x, y \in \mathbb{R}^n$ we have*

$$\pm 2x^T y \leq x^T N x + y^T N^{-1} y.$$

Lemma 4. *[1] For any constant symmetric matrix $M \in R^{m\times m}$, $M = M^T > 0$, $\gamma > 0$, vector function $\omega : [0,\gamma] \to \mathbb{R}^m$ such that the integrations concerned are well defined, then*

$$\begin{aligned} &\Big(\int_0^{\gamma} \omega^T(s) ds\Big)^T M \Big(\int_0^{\gamma} \omega(s) ds\Big) \\ \leq\;& \gamma \int_0^{\gamma} \omega^T(s) M \omega(s) ds. \end{aligned}$$

Lemma 5. *[2]. Let $c \in R$ and A, B, C, D, be matrices with appropriate dimensions. Then*

$$\begin{aligned} (i) &\quad c(A\otimes B) = (cA)\otimes B = A\otimes (cB), \\ (ii) &\quad (A\otimes B)^T = A^T \otimes B^T, \\ (iii) &\quad (A\otimes B)(C\otimes D) = (AC)\otimes (BD), \\ (iv) &\quad A\otimes B\otimes C = (A\otimes B)\otimes C = A\otimes (B\otimes C). \end{aligned}$$

3 Main results

In this section, we will give some sufficient condition for function projective synchronization of complex dynamical network with discrete and distributed time-varying delays and mixed- coupling delays (1) via hybrid adaptive control In order to stabilize the origin of delayed complex dynamical network (1) by means of the hybrid adaptive control $\mathcal{U}_i(t)$ such as

$$\mathcal{U}_i(t) = u_{i1}(t) + u_{i2}(t), i = 1,2,...,N, \qquad (4)$$

where

$$\begin{aligned} u_{i1}(t) &= \dot{\alpha}(t)s(t), \\ u_{i2}(t) &= -c_1 d_{i1}(t) G_1 e_i(t) - c_2 d_{i2}(t) G_2 e_i(t-h(t)) \\ &\quad -c_3 d_{i3}(t) G_3 \int_{t-k(t)}^{t} e_i(\theta)\, d\theta, \end{aligned}$$

and the updating laws are

$$\begin{aligned} \dot{d}_{i1}(t) &= q_{i1} e_i^T(t) G_1 e_i(t), \\ \dot{d}_{i2}(t) &= q_{i2} e_i^T(t) G_2 e_i(t-h(t)), \qquad (5) \\ \dot{d}_{i3}(t) &= q_{i3} e_i^T(t) G_3 \big[\int_{t-k(t)}^{t} e_i(\theta)\, d\theta\big], \end{aligned}$$

where q_{i1}, q_{i2} and q_{i3} are positive constants and $s(t)$ is a solution of an isolated node, satisfying $\dot{s}(t) = f(s(t), s(t-h(t)), \int_{t-k(t)}^{t} s(\theta)\,d\theta)$. The controller in (4), $u_{i1}(t)$ is the nonlinear feedback control and $u_{i2}(t)$ is the hybrid adaptive linear feedback control. Then, substituting it into complex dynamical network (6), it is easy to get the following:

$$\begin{aligned}
\dot{e}_i(t) &= \dot{x}_i(t) - \dot{\alpha}(t)s(t) - \alpha(t)\dot{s}(t) \\
&= f(x_i(t), x_i(t-h(t)), \int_{t-k(t)}^{t} x_i(s)\,ds) \\
&\quad -\alpha(t) f(s(t), s(t-h(t)), \int_{t-k(t)}^{t} s(\theta)\,d\theta) \\
&\quad +c_1 \sum_{j=1}^{N} a_{ij} G_1 e_j(t) \\
&\quad +c_2 \sum_{j=1}^{N} b_{ij} G_2 e_j(t-h(t)) \\
&\quad +c_3 \sum_{j=1}^{N} c_{ij} G_3 \int_{t-k(t)}^{t} e_j(s)\,ds \\
&\quad -c_1 d_{i1}(t) e_i(t) - c_2 d_{i2}(t) e_i(t-h(t)) \\
&\quad -c_3 d_{i3}(t) \int_{t-k(t)}^{t} e_i(\theta)\,d\theta, i = 1, \ldots, N, \quad (6) \\
\dot{d}_{i1}(t) &= q_{i1} e_i^T(t) G_1 e_i(t), i = 1, \ldots, N \\
\dot{d}_{i2}(t) &= q_{i2} e_i^T(t) G_2 e_i(t-h(t)), i = 1, \ldots, N \\
\dot{d}_{i3}(t) &= q_{i3} e_i^T(t) G_3 \big[\int_{t-k(t)}^{t} e_i(\theta)\,d\theta\big], i = 1, \ldots, N.
\end{aligned}$$

Let us set

1. $J(t) = f'(s(t), s(t-h(t)), \int_{t-k(t)}^{t} s(\xi)\,d\xi) \in R^{n\times n}$ is the Jacobian of $f(x(t), x(t-h(t)), \int_{t-k(t)}^{t} x(s)\,ds)$ at $s(t)$ with the derivative of $f(x(t), x(t-h(t)), \int_{t-k(t)}^{t} x(s)\,ds)$ respect to $x(t)$,

2. $J_h(t) = f'(s(t), s(t-h(t)), \int_{t-k(t)}^{t} s(\xi)\,d\xi) \in R^{n\times n}$ is the Jacobian of $f(x(t), x(t-h(t)), \int_{t-k(t)}^{t} x(s)\,ds)$ at $s(t-h(t))$ with the derivative of $f(x(t), x(t-h(t)), \int_{t-k(t)}^{t} x(s)\,ds)$ respect to $x(t-h(t))$,

3. $J_{k_1}(t) = f'(s(t), s(t-h(t)), \int_{t-k(t)}^{t} s(\xi)\,d\xi) \in R^{n\times n}$ is the Jacobian of $f(x(t), x(t-h(t)), \int_{t-k(t)}^{t} x(s)\,ds)$ at $\int_{t-k(t)}^{t} s(\xi)\,d\xi$ with the derivative of $f(x(t), x(t-h(t)), \int_{t-k(t)}^{t} x(s)\,ds)$ respect to $\int_{t-k(t)}^{t} x(s)\,ds$

and

$$\begin{aligned}
\delta &= \frac{1}{2\lambda_{min}(I_N \otimes G_2)} \big(\varepsilon_1 + c_2\varepsilon_2 + c_2 d_2^*\varepsilon_5\big), \\
\tau &= \frac{1}{2\lambda_{min}(I_N \otimes G_3)} \big(\varepsilon_2 + c_3\varepsilon_4 + 2c_3 d_3^*\varepsilon_6\big), \\
\eta &= \frac{1}{\lambda_{min}(I_N \otimes G_1)} \big(\bar{\lambda}(I_N \otimes J(t)) \\
&\quad +c_1\bar{\lambda}(A)\bar{\lambda}(G_1) + \frac{c_2}{2(1-\beta)}\bar{\lambda}(I_N \otimes G_2) \\
&\quad +\frac{c_3 k^2}{2}\bar{\lambda}(I_N \otimes G_3) + \frac{1}{2\varepsilon_1}\bar{\lambda}(I_N \otimes J_h(t)J_h^T(t)) \\
&\quad +\frac{1}{2\varepsilon_2}\bar{\lambda}(I_N \otimes J_k(t)J_k^T(t)) \\
&\quad +\frac{c_2}{2\varepsilon_3}\bar{\lambda}(BB^T)\bar{\lambda}(G_2G_2^T) + \frac{c_3}{2\varepsilon_4}\bar{\lambda}(CC^T)\bar{\lambda}(G_3G_3^T) \\
&\quad +\frac{c_2 d_2^*}{2\varepsilon_5}\bar{\lambda}(I_N \otimes G_2G_2^T) + \frac{c_3 d_3^*}{2\varepsilon_6}\bar{\lambda}(I_N \otimes G_3G_3^T)\big).
\end{aligned}$$

Theorem 6. *For some given synchronization scaling function $\alpha(t)$, the complex dynamical networks (1) with time-varying delay satisfying Assumption 2. and target system can realize function projective synchronization by the adaptive control law as shown in (4) if there exist positive constants ε_i, $i = 1, 2, \ldots, 5$ and by taking appropriate d_1^*, d_2^* and d_3^* such that*

$$\begin{aligned}
d_1^* - \frac{\eta}{c_1} &> 0, \quad (7) \\
d_2^* - \frac{1}{\varepsilon_5}\Big(\frac{\varepsilon_1}{c_2} + \varepsilon_2 - \lambda_{min}(I_N \otimes G_2)\Big) &> 0, \quad (8) \\
d_3^* - \frac{1}{\varepsilon_6}\Big(\frac{\varepsilon_2}{c_3} + \varepsilon_4 - \lambda_{min}(I_N \otimes G_3)\Big) &> 0. \quad (9)
\end{aligned}$$

Then the controlled complex dynamical networks (1) is function projective synchronization.

Proof. Since $f(.)$ is continuous differentiable, it is easy to know that the origin of the nonlinear system (6) is an asymptotically stable equilibrium point if it is an asymptotically stable equilibrium point of the following linear time-varying delays systems

$$\begin{aligned}
\dot{e}_i(t) &= J(t)e_i(t) + J_h(t)e_i(t-h(t)) \\
&\quad +J_{k_1}(t)\int_{t-k(t)}^{t} e_i(s)\,ds \\
&\quad +c_1 \sum_{j=1}^{N} a_{ij} G_1 e_j(t) \qquad (10) \\
&\quad +c_2 \sum_{j=1}^{N} b_{ij} G_2 e_j(t-h(t)) \\
&\quad +c_3 \sum_{j=1}^{N} c_{ij} G_3 \int_{t-k(t)}^{t} e_j(s)\,ds
\end{aligned}$$

$$
\begin{aligned}
& -c_1 d_{i1}(t)e_i(t) - c_2 d_{i2}(t)e_i(t-h(t)) \\
& -c_3 d_{i3}(t)\int_{t-k(t)}^{t} e_i(\theta)\,d\theta, i=1,\ldots,N, \\
\dot{d}_{i1}(t) &= q_{i1}e_i^T(t)G_1 e_i(t), i=1,\ldots,N \\
\dot{d}_{i2}(t) &= q_{i2}e_i^T(t)G_2 e_i(t-h(t)), i=1,\ldots,N \\
\dot{d}_{i3}(t) &= q_{i3}e_i^T(t)G_3\big[\int_{t-k(t)}^{t} e_i(\theta)\,d\theta\big], i=1,\ldots,N.
\end{aligned}
$$

Construct the the following Lyapunov-Krasovskii functional candidate:

$$
\begin{aligned}
V(t) &= \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{c_1}{q_{i1}}(d_{i1}(t)-d_1^*)^2 \\
&+ \frac{c_2}{2(1-\beta)}\sum_{i=1}^{N}\int_{t-h(t)}^{t} e_i^T(s)G_2 e_i(s)\,ds \\
&+ \frac{1}{2}\sum_{i=1}^{N}\frac{c_2}{q_{i2}}(d_{i2}(t)-d_2^*)^2 \\
&+ \frac{c_3 k}{2}\sum_{i=1}^{N}\int_{-k}^{0}\int_{t+s}^{t} e_i^T(\theta)G_3 e_i(\theta)\,d\theta ds \\
&+ \frac{1}{2}\sum_{i=1}^{N}\frac{c_3}{q_{i3}}(d_{i3}(t)-d_3^*)^2. \qquad (11)
\end{aligned}
$$

By taking the derivative of $V(t)$ along the trajectories of system (6), we have the following:

$$
\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^{N} e_i^T(t)J(t)e_i(t) \\
&+ \sum_{i=1}^{N} e_i^T(t)J_h(t)e_i(t-h(t)) \\
&+ \sum_{i=1}^{N} e_i^T(t)J_k(t)\int_{t-k(t)}^{t} e_i(s)\,ds \\
&+ c_1\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)a_{ij}G_1 e_j(t) \\
&+ c_2\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)b_{ij}G_2 e_j(t-h(t)) \\
&+ c_3\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^T(t)c_{ij}G_3\int_{t-h(t)}^{t} e_j(s)\,ds \\
&+ \frac{c_2}{2(1-\beta)}\sum_{i=1}^{N} e_i^T(t)G_2 e_i(t) \\
&- \frac{c_2}{2}\sum_{i=1}^{N} e_i^T(t-h(t))G_2 e_i(t-h(t)) \\
&- c_2 d_2^*\sum_{i=1}^{N} e_i^T(t)G_2 e_i(t-h(t)) \\
&- c_1 d_1^*\sum_{i=1}^{N} e_i^T(t)G_1 e_i(t) \\
&+ \frac{c_3 k^2}{2}\sum_{i=1}^{N} e_i^T(t)G_3 e_i(t) \\
&- \frac{c_3 k}{2}\sum_{i=1}^{N}\int_{t-k}^{t} e_i^T(s)G_3 e_i(s)\,ds \\
&- c_3 d_3^*\sum_{i=1}^{N} e_i^T(t)G_3\int_{t-k(t)}^{t} e_i(s)\,ds \qquad (12)
\end{aligned}
$$

Let $e(t) = (e_1(t),...,e_N(t)) \in R^{n\times N}$, $e(t-h(t)) = (e_1(t-h(t)),...,e_N(t-h(t))) \in R^{n\times N}$, $\int_{t-k(t)}^{t} e(s)\,ds = \int_{t-k(t)}^{t}(e_1(s),e_2(s),...,e_N(s))\,ds \in R^{n\times N}$. we have

$$
\begin{aligned}
\dot{V}(t) \leq\ & e^T(t)(I_N\otimes J(t))e(t) \\
&+ e^T(t)(I_N\otimes J_h(t))e(t-h(t)) \\
&+ c_1 e^T(t)(A\otimes G_1)e(t) \\
&+ c_2 e^T(t)(B\otimes G_2)e(t-h(t)) \\
&- c_1 d_1^* e^T(t)(I_N\otimes G_1)e(t) \\
&+ c_3 e^T(t)(C\otimes G_3)\int_{t-k(t)}^{t} e(s)\,ds \\
&- c_2 d_2^* e^T(t)(I_N\otimes G_2)e(t-h(t)) \\
&- c_3 d_3^* e^T(t)(I_N\otimes G_3)\int_{t-k(t)}^{t} e(s)\,ds \\
&+ \frac{c_2}{2(1-\beta)}e^T(t)(I_N\otimes G_2)e(t) \\
&- \frac{c_2}{2}e^T(t-h(t))(I_N\otimes G_2)e(t-h(t)) \\
&+ \frac{c_3 k^2}{2}e^T(t)(I_N\otimes G_3)e(t) \\
&+ e^T(t)(I_N\otimes J_k(t))\int_{t-k(t)}^{t} e(s)\,ds \\
&- \frac{c_3 k}{2}\int_{t-k}^{t} e^T(s)(I_N\otimes G_3)e(s)\,ds. \qquad (13)
\end{aligned}
$$

Applying Lemma 3., Lemma 4. and Lemma5. gives

$$
\begin{aligned}
& e^T(t)(I_N\otimes J_h(t))e(t-h(t)) \\
\leq\ & \frac{1}{2\varepsilon_1}e^T(t)(I_N\otimes J_h(t)J_h^T(t))e(t) \\
& + \frac{\varepsilon_1}{2}e^T(t-h(t))e(t-h(t)), \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
& e^T(t)(I_N \otimes J_k(t)) \int_{t-k(t)}^{t} e(s)\, ds \\
\leq\ & \frac{1}{2\varepsilon_2} e^T(t)(I_N \otimes J_k(t) J_k^T(t)) e(t) \qquad (15)\\
& + \frac{\varepsilon_2}{2}\Big(\int_{t-k(i)}^{t} e^T(s)\, ds\Big)^T \Big(\int_{t-k(i)}^{t} e^T(s)\, ds\Big) \\
& c_2 e^T(t)(B \otimes G_2) e(t-h(t)) \\
\leq\ & \frac{c_2}{2\varepsilon_3} e^T(t)(BB^T \otimes G_2 G_2^T) e(t) \\
& + \frac{c_2\varepsilon_3}{2} e^T(t-h(t)) e(t-h(t)), \qquad (16)\\
& c_3 e^T(t)(C \otimes G_3) \int_{t-k(t)}^{t} e(s)\, ds \\
\leq\ & \frac{c_3}{2\varepsilon_4} e^T(t)(CC^T \otimes G_3 G_3^T) e(t) \qquad (17)\\
& + \frac{c_3\varepsilon_4}{2}\Big(\int_{t-k(i)}^{t} e^T(s)\, ds\Big)^T \Big(\int_{t-k(i)}^{t} e^T(s)\, ds\Big) \\
& - c_2 d_2^* e^T(t)(I_N \otimes G_2) e(t-h(t)) \\
\leq\ & \frac{c_2 d_2^*}{2\varepsilon_5} e^T(t)(I_N \otimes G_2 G_2^T) e(t) \\
& + \frac{c_2 d_2^* \varepsilon_5}{2} e^T(t-h(t)) e(t-h(t)), \qquad (18)\\
& - c_3 d_3^* e^T(t)(I_N \otimes G_3) \int_{t-k(t)}^{t} e(s)\, ds \\
\leq\ & \frac{c_3 d_3^*}{2\varepsilon_6} e^T(t)(I_N \otimes G_3 G_3^T) e(t) \qquad (19)\\
& + \frac{c_3 d_3^* \varepsilon_6}{2}\Big(\int_{t-k(i)}^{t} e^T(s)\, ds\Big)^T \Big(\int_{t-k(i)}^{t} e^T(s)\, ds\Big).
\end{aligned}
$$

Hence, according to(13) -(14) we have

$$
\begin{aligned}
\dot{V}(t)\ \leq\ & e^T(t)\big(I_N \otimes J(t) + c_1(A \otimes G_1) \\
& - c_1 d_1^*(I_N \otimes G_1)) + \frac{c_2}{2(1-\beta)}(I_N \otimes G_2) \\
& + \frac{c_3 k^2}{2}(I_N \otimes G_3) + \frac{1}{2\varepsilon_1}(I_N \otimes J_h(t) J_h^T(t)) \\
& + \frac{1}{2\varepsilon_2}(I_N \otimes J_k(t) J_k^T(t)) \\
& + \frac{c_2}{2\varepsilon_3}(BB^T \otimes G_2 G_2^T) \\
& + \frac{c_3}{2\varepsilon_4}(CC^T \otimes G_3 G_3^T) \\
& + \frac{c_2 d_2^*}{2\varepsilon_5}(I_N \otimes G_2 G_2^T) \\
& + \frac{c_2 d_3^*}{2\varepsilon_6}(I_N \otimes G_3 G_3^T)\big) e(t) \\
& + \frac{c_2}{2\varepsilon_3}(BB^T \otimes G_2 G_2^T) \\
& - e^T(t-h(t))\big(\frac{c_2}{2}(I_N \otimes G_2) \\
& + \delta(I_N \otimes G_2)\big) e(t-h(t)) \\
& - \Big(\int_{t-k}^{t} e(s)\, ds\Big)^T \big(\frac{c_3}{2}(I_N \otimes G_3) \\
& - \tau(I_N \otimes G_3)\big)\Big(\int_{t-k}^{t} e(s)\, ds\Big) \\
\leq\ & (\eta - c_1 d_1^*) e^T(t)(I_N \otimes G_1) e(t) \\
& - e^T(t-h(t))((\frac{c_2}{2} - \delta) \\
& (I_N \otimes G_2)) e(t-h(t)) \\
& - \Big(\int_{t-k}^{t} e(s)\, ds\Big)^T ((\frac{c_3}{2} - \tau)(I_N \otimes G_3) \\
& \Big(\int_{t-k}^{t} e(s)\, ds\Big). \qquad (20)
\end{aligned}
$$

It is obviously that there exists sufficiently large positive constant d_1^*, d_2^* and d_3^* such that

$$
\begin{aligned}
d_1^* - \frac{\eta}{c_1} &> 0, \qquad (21)\\
d_2^* - \frac{1}{\varepsilon_5}(\frac{\varepsilon_1}{c_2} + \varepsilon_2 - \lambda_{min}(I_N \otimes G_2)) &> 0, \qquad (22)\\
d_3^* - \frac{1}{\varepsilon_6}(\frac{\varepsilon_2}{c_3} + \varepsilon_4 - \lambda_{min}(I_N \otimes G_3)) &> 0. \qquad (23)
\end{aligned}
$$

We can choose d_1^*, d_2^* d_3^* satisfying (21), (22) and (23), respectively. Since G_1, G_2 and G_3 are positive definite diagonal matrix, we know that $\dot{V}(t) \leq 0$ and $\dot{V}(t) = 0$ if and only if $\xi(t) = 0$. Hence, the set $\mathcal{W} = \{\xi(t) = 0,\ d_{1i} = d_1^*,\ d_{2i} = d_2^*,\ d_{3i} = d_3^*\}$ is the invariant set contained in $\mathcal{W}_1 = \{\xi(t) = 0 \ : \dot{V}(t) = 0\}$ for system (6). According to LaSalle invariance principle [3] and Lyapunov stability theory, for any initial condition, every solution of system (6) approaches $\mathcal{W}$ as $t \longrightarrow \infty$, which indicates that $\|e_i(t)\| \longrightarrow 0,\ \ i = 1, 2, \dots N$, this means that the function projective synchronization between the delayed complex dynamical networks (1) and the reference node $s(t)$ is achieved under hybrid adaptive control (4). The proof is this completed. □

4 Numerical Example

In this section, we present example to illustrate the effectiveness and the reduced conservatism of our result.

Example 4.1 We first consider the perturbed Chua's circuit system with mixed time-varying delays is used as uncoupled node in the network (1) to show the effectiveness of the proposed control scheme. The perturbed Chua's circuit system with mixed time-varying delays is given by [12]

$$\begin{aligned} \dot{x}_1(t) &= p\Big(x_2(t-h(t)) - \frac{1}{7}\Big(2x_1^3(t) - x_1(t)\Big)\Big) \\ \dot{x}_2(t) &= x_1(t) - sx_2(t) + x_3(t-h(t)) \qquad (24) \\ \dot{x}_3(t) &= qx_2(t) + r\int_{t-k_1(t)}^{t} x_1^2(s)\, ds \end{aligned}$$

where p, q, r and s are real positive constants. It is well known that the system (24) exhibits chaotic behavior with the parameters p, q, r and s are chosen as $p = 7$, $q = -\frac{100}{7}$, $r = 0.07$ and $s = 1.5$, the initial condition function $\phi(t) = [0.65\cos t, 0.3\cos t, -0.2\cos t]^T$, the time-varying delay functions $h(t) = 0.1 + 0.1\sin^2 t$ and $k(t) = 0.1\cos^2 t$ is shown in Figure 1. It is stable at the equilibrium point $s(t) = 0$, $s(t-h(t)) = 0$, $\int_{t-k_1(t)}^{t} s(\theta)\, d\theta = 0$ and Jacobian matrices are

$$J(t) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1.5 & 0 \\ 0 & -\frac{100}{7} & 0 \end{bmatrix}, \quad J_h(t) = \begin{bmatrix} 0 & 7 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$J_{k_1}(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

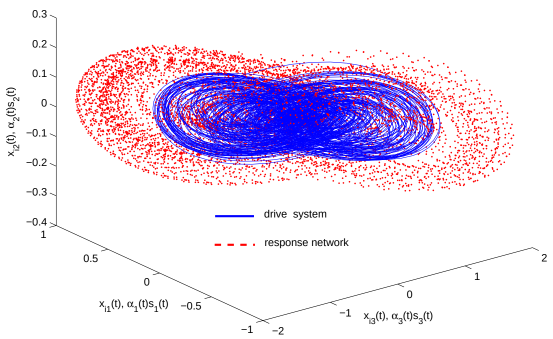

Figure 1: Chaotic behavior of the perturbed Chua's circuit system with mixed time-varying delays (24)

The parameters are selected as follows: the coupling strength $c_1 = 0.4$, $c_2 = 0.3$, $c_3 = 0.5$, the time-varying scaling function matrix $\alpha(t)$ =diag$(0.6\sin(\frac{2\pi}{15}), 0.7\sin(\frac{2\pi}{15}), 0.75\sin(\frac{2\pi}{15}))$, the inner-coupling matrix are

$$G_1 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and The coupling configuration matrices are given respectively as follows:

$$A = \begin{bmatrix} -5 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & -3 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -4 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -3 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & -4 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & -3 \end{bmatrix},$$

$$B = \begin{bmatrix} -4 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & -3 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -4 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & -3 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -3 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & -2 \end{bmatrix},$$

$$C = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -3 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -3 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -2 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}.$$

Solution: From the conditions (7)-(9) of Theorem 6 and there exist positive constants $\varepsilon_1 = 0.86$, $\varepsilon_2 = 0.75$, $\varepsilon_3 = 0.90$, $\varepsilon_4 = 1.20$, $\varepsilon_5 = 1.10$, $\varepsilon_6 = 0.70$, one can check that the last three conditions in Theorem 6. are satisfied. From the conditions of Theorem 6, we can obtain $d_1^* > 15.5354$, $d_2^* > 2.7727$ and $d_3^* > 2.9643$

The numerical simulations are carried out using the explicit Runge-Kutta-like method (dde45), interpolation and extrapolation by spline of the third order. Figure 2. shows the function projective synchronization errors between the states of isolate node $\alpha(t)s(t)$ and node $x_i(t)$, where $e_{ij}(t) = x_{ij}(t) - \alpha_j(t)s_j(t)$ for $i = 1, ..., 8, j = 1, 2, 3$ without hybrid adaptive control. Figure 3. shows the function projective synchronization errors between the states of isolate node $\alpha(t)s(t)$ and node $x_i(t)$, where $e_{ij}(t) = x_{ij}(t) - \alpha_j(t)s_j(t)$ for $i = 1, ..., 8, j = 1, 2, 3$ with hybrid adaptive control. We see that the synchronization errors converge to zero under the above conditions.

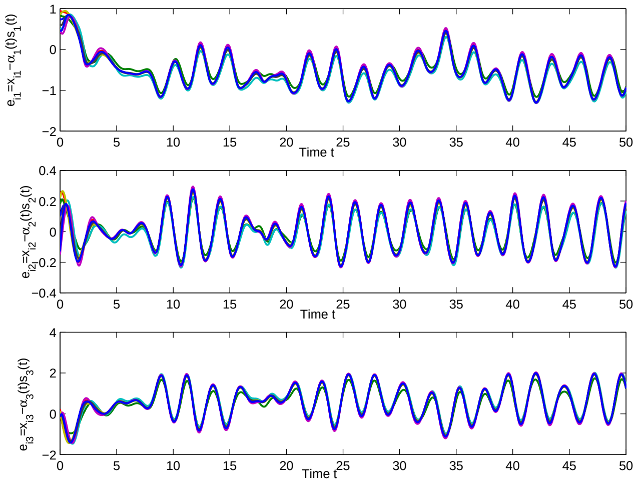

Figure 2: Shows the function projective synchronization errors between the states of isolate node $\alpha(t)s(t)$ and node $x_i(t)$, where $e_{ij}(t) = x_{ij}(t) - \alpha_j(t)s_j(t)$ for $i = 1, ..., 8, j = 1, 2, 3$ without adaptive controller

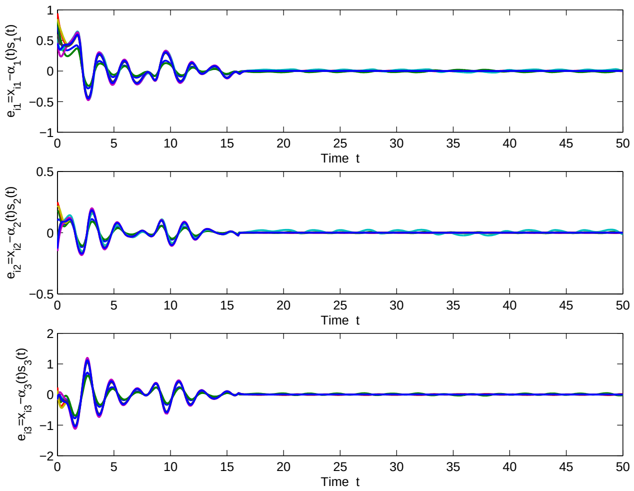

Figure 3: Shows the function projective synchronization errors between the states of isolate node $\alpha(t)s(t)$ and node $x_i(t)$, where $e_{ij}(t) = x_{ij}(t) - \alpha_j(t)s_j(t)$ for $i = 1, ..., 8, j = 1, 2, 3$ with adaptive controller

5 Conclusion

This paper investigated function projective synchronization (FPS) for complex dynamical network with mixed time-varying and hybrid coupling delays, which is composed of state coupling, time-varying delay coupling and distributed time-varying delay coupling. In contrast to previous results, the coupling configuration matrix need not be symmetric or irreducible. The designed controller ensures that the FPS of delayed complex dynamical network are proposed via hybrid error adaptive control, which contains error term, time-varying delay error term and distributed time-varying delay error term. By using new methods to deal with asymmetric coupling matrix and a new class of Lyapunov-Krasovskii functional, improved PFS criteria are obtained. Simulation results have been given to illustrate the effectiveness of the proposed method.

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