Structural Consensus Controllability of Singular Multi-agent Linear Dynamic Systems

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Abstract: The analysis of control of linear multi-agent systems has recently emerged as an important domain that is receiving a lot of interest from a variety of research communities, and consensus plays a fundamental role in this field of study. We will show how using linear algebra techniques can be analyzed the consensus controllability problem for singular multi-agent systems, in which all agents have an identical linear dynamic mode that can be in any order.

Key–Words: Singular multi-agent systems, consensus, controllability, Structural consensus controllability.

1 Introduction

It is well known the great interest generated in many research communities about the study of control multi-agents system, as well as the growing interest in distributed control and coordination of networks consisting of multiple autonomous agents. There are many publications as for example ([6], [12], [14], [16]). It is due to the multi-agents appear in different fields as for example in consensus problem of communication networks ([12]), or formation control of mobile robots ([4]).

The consensus problem has been studied under different points of view, for example Jinhuan Wang, Daizhan Cheng and Xiaoming Hu in [14], analyze the case of multiagent systems in which all agents have an identical stable linear dynamics system, M.I. García-Planas in [6], generalize this result to the case where the dynamic of the agents are controllable.

The concept of controllability is fundamental in dynamic systems and it is studied under different approaches (see [2],[3],[5],[8], for example). The structural controllability is a generalization of the classical controllability concept for dynamical systems, and purely based on the graphic topologies among the entries of the matrices defining the system.

In this paper, we investigate the structural controllability of a class of multiagent singular systems consisting of k agents with dynamics

$$
\begin{aligned}
E\dot{x}^1 &= Ax^1 + Bu^1 \\
&\vdots \\
E\dot{x}^k &= Ax^k + Bu^k
\end{aligned}
$$

where $E, A \in M_n(\mathbb{C})$, $B \in M_{n\times 1}(\mathbb{C})$.

2 Preliminaries

2.1 Algebraic Graph theory

We consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of order k with the set of vertices $\mathcal{V} = \{1, \ldots, k\}$ and the set of edges $\mathcal{E} = \{(i,j) \mid i,j \in \mathcal{V}\} \subset \mathcal{V} \times \mathcal{V}$.

Given an edge (i,j) i is called the parent node and j is called the child node and j is in the neighbor of i, concretely we define the neighbor of i and we denote it by $\mathcal{N}_i$ to the set $\mathcal{N}_i = \{j \in \mathcal{V} \mid (i,j) \in \mathcal{E}\}$.

The graph is called undirected if verifies that $(i,j) \in \mathcal{E}$ if and only if $(j,i) \in \mathcal{E}$. The graph is called connected if there exists a path between any two vertices, otherwise is called disconnected.

Associated to the graph we consider a matrix $G = (g_{ij})$ called (unweighted) adjacency matrix defined as follows $g_{ii} = 0$, $g_{ij} = 1$ if $(i,j) \in \mathcal{E}$, and $g_{ij} = 0$ otherwise.

In a more general case we can consider that a weighted adjacency matrix is $G = (g_{ij})$ with $g_{ii} = 0$, $g_{ij} > 0$ if $(i,j) \in \mathcal{E}$, and $g_{ij} = 0$ otherwise).

The Laplacian matrix of the graph is

$$
\mathcal{L} = (l_{ij}) = \begin{cases} |\mathcal{N}_i| & \text{if } i = j \\ -1 & \text{if } j \in \mathcal{N}_i \\ 0 & \text{otherwise} \end{cases}
$$

Remark 1 *i) If the graph is undirected then the matrix $\mathcal{L}$ is symmetric, then there exist an orthogonal matrix P such that $P\mathcal{L}P^t = \mathcal{D}$.*

ii) If the graph is undirected then 0 is an eigenvalue of $\mathcal{L}$ and $\mathbf{1}_k = (1, \ldots, 1)^t$ is the associated eigenvector.

iii) If the graph is undirected and connected the eigenvalue 0 is simple.

For more details about graph theory see (D. West, 2007).

2.2 Kronecker product

Remember that $A = (a_{ij}) \in M_{n \times m}(\mathbb{C})$ and $B = (b_{ij}) \in M_{p \times q}(\mathbb{C})$ the Kronecker product is defined as follows.

Definition 2 *Let $A = (a_j^i) \in M_{n \times m}(\mathbb{C})$ and $B \in M_{p \times q}(\mathbb{C})$ be two matrices, the Kronecker product of A and B, write $A \otimes B$, is the matrix*

$$A \otimes B = \begin{pmatrix} a_1^1 B & a_2^1 B & \ldots & a_m^1 B \\ a_1^2 B & a_2^2 B & \ldots & a_m^2 B \\ \vdots & \vdots & & \vdots \\ a_1^n B & a_2^n B & \ldots & a_m^n B \end{pmatrix} \in M_{np \times mq}(\mathbb{C})$$

Among the properties that verifies the product of Kronecker we will make use of the following

1) $(A + B) \otimes C = (A \otimes C) + (B \otimes C)$

2) $A \otimes (B + C) = (A \otimes B) + (A \otimes C)$

3) $(A \otimes B) \otimes C = A \otimes (B \otimes C)$

4) If $A \in Gl(n; \mathbb{C})$ and $B \in Gl(p; \mathbb{C})$, then $A \otimes B \in Gl(np; \mathbb{C})$ and $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$

5) If the products AC and BD are possible, then $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$

See [10] for more information and properties.

3 Consensus

Roughly speaking, we can define the consensus as a collection of processes such that each process starts with an initial value, where each one is supposed to output the same value and there is a validity condition that relates outputs to inputs. More concretely, the consensus problem is a canonical problem that appears in the coordination of multi-agent systems. The objective is that Given initial values (scalar or vector) of agents, establish conditions under which through local interactions and computations, agents asymptotically agree upon a common value, that is to say: to reach a consensus.

We consider now, a multi-agent where the dynamic of each agent is given by the following dynamical systems

$$\begin{aligned} E\dot{x}^1 &= Ax^1 + Bu^1 \\ &\vdots \\ E\dot{x}^k &= Ax^k + Bu^k \end{aligned} \tag{1}$$

$x^i \in \mathbb{R}^n$, $u^i \in \mathbb{R}^m$, $1 \le i \le k$. Where matrices $E, A \in M_n(\mathbb{R})$ and $B \in M_{n \times m}(\mathbb{R})$.

The communication topology among agents is defined by means the undirected graph $\mathcal{G}$ with

i) Vertex set: $\mathcal{V} = \{1, \ldots, k\}$

ii) Edge set: $\mathcal{E} = \{(i, j) \mid i, j \in \mathcal{V}\} \subset \mathcal{V} \times \mathcal{V}$.

an in a similar way as before, we have the following.

Definition 3 *Consider the system 1. We say that the consensus is achieved using local information if there exists a state feedback*

$$u^i = K_i \sum_{j \in \mathcal{N}_i} (x^i - x^j), \ 1 \le i \le k$$

such that

$$\lim_{t \to \infty} \|x^i - x^j\| = 0, \ 1 \le i, j \le k.$$

$z^i = \sum_{j \in \mathcal{N}_i} (x^i - x^j), \ 1 \le i \le k.$

$$\begin{aligned} (I_k \otimes E)\dot{\mathcal{X}} &= (I_k \otimes A)\mathcal{X} + (I_k \otimes B)\mathcal{U} \\ \mathcal{Z} &= (\mathcal{L} \otimes I)\mathcal{X} \\ \mathcal{U} &= (I_k \otimes K)\mathcal{Z} \end{aligned}$$

Then, and taking into account that

$$\begin{aligned} (I_k \otimes B)(I_k \otimes K)(\mathcal{L} \otimes I_n)\mathcal{X} &= \\ (\mathcal{L} \otimes BK)\mathcal{X} = (\mathcal{L} \otimes B)(I_k \otimes K)\mathcal{X} \end{aligned}$$

The system is equivalent to

$$\begin{aligned} (I_k \otimes E)\dot{\mathcal{X}} &= (I_k \otimes A)\mathcal{X} + (\mathcal{L} \otimes B)\bar{\mathcal{U}} \\ \bar{\mathcal{U}} &= (I_k \otimes K)\mathcal{X} \end{aligned} \tag{2}$$

4 Controllability

The controllability of the system 2 can be analyzed using the generalized Hautus criteria

Proposition 4 *The system is controllable if and only if*

$$\text{rank} \begin{pmatrix} I_k \otimes E & \mathcal{L} \otimes B \end{pmatrix} = kn$$
$$\text{rank} \begin{pmatrix} s(I_k \otimes E) - (I_k \otimes A) & \mathcal{L} \otimes B \end{pmatrix} = kn$$

Proposition 5 *Let J be the Jordan reduced of the matrix $\mathcal{L}$ and P such that $\mathcal{L} = P^{-1}JP$. Then, the system 2 is controllable if and only if*

$$\text{rank} \begin{pmatrix} I_k \otimes E & J \otimes B \end{pmatrix} = kn$$
$$\text{rank} \begin{pmatrix} (s(I_k \otimes E) - (I_k \otimes A) & J \otimes B \end{pmatrix} = kn$$

Proof. Suppose that there exist S such that $P^{-1}JP = \mathcal{L}$ and

$$\text{rank} \begin{pmatrix} I_k \otimes E & P^{-1}JP \otimes B \end{pmatrix} =$$
$$\text{rank} \begin{pmatrix} P^1 I_k P \otimes E & P^{-1}JP \otimes B \end{pmatrix} =$$
$$\text{rank} \left((P^1 \otimes I)(I_k \otimes E)(P \otimes I) \quad (P^{-1} \otimes I)(J \otimes B)(P \otimes I) \right) =$$
$$\text{rank} \, (P^{-1} \otimes I) \begin{pmatrix} I \otimes E & J \otimes B \end{pmatrix} \begin{pmatrix} P \otimes I & \\ & P \otimes I \end{pmatrix} =$$
$$\text{rank} \begin{pmatrix} I \otimes E & J \otimes B \end{pmatrix}$$

In an analogous form:

$$\text{rank} \begin{pmatrix} s(I_k \otimes E) - (I_k \otimes A) & \mathcal{L} \otimes B \end{pmatrix} =$$
$$\text{rank} \, (P^{-1} \otimes I) \begin{pmatrix} s(I_k \otimes E) - (I_k \otimes A) & J \otimes B \end{pmatrix}$$
$$\begin{pmatrix} P \otimes I & \\ & P \otimes I \end{pmatrix} =$$
$$\text{rank} \begin{pmatrix} s(I_k \otimes E) - (I_k \otimes A) & J \otimes B \end{pmatrix}$$

$\square$

Corollary 6 *Suppose that the matrix $\mathcal{L}$ is diagonalizable with non-zero eigenvalues. Then, the system 2 is controllable if and only if each agent is controllable.*

Proof. Let $\lambda_i \neq 0$, $i = 1, \ldots n$ be the eigenvalues of $\mathcal{L}$. Then

$$\text{rank} \begin{pmatrix} I \otimes E & D \otimes B \end{pmatrix} =$$
$$\text{rank} \begin{pmatrix} E & & & & \lambda_1 B & & & \\ & E & & & & \lambda_2 B & & \\ & & \ddots & & & & \ddots & \\ & & & E & & & & \lambda_n B \end{pmatrix} =$$
$$\text{rank} \begin{pmatrix} E & & & & B & & & \\ & E & & & & B & & \\ & & \ddots & & & & \ddots & \\ & & & E & & & & B \end{pmatrix}$$

and

$$\text{rank} \begin{pmatrix} s(I \otimes E) - (I \otimes A) & D \otimes B \end{pmatrix} =$$
$$\text{rank} \begin{pmatrix} sE - A & & & \lambda_1 B & & \\ & \ddots & & & \ddots & \\ & & sE - A & & & \lambda_n B \end{pmatrix} =$$
$$\text{rank} \begin{pmatrix} sE - A & & & & B & & \\ & sE - A & & & & B & \\ & & \ddots & & & & \ddots \\ & & & sE - A & & & & B \end{pmatrix}$$

$\square$

Corollary 7 *If the graph is undirected the system is not controllable.*

Proof. If the graph is undirected, $\mathcal{L}$ is diagonalizable with zero as an eigenvalue. $\square$

In a more general case, the matrix $\mathcal{L}$ can be reduced to a Jordan form J:

$$J = \begin{pmatrix} J(\lambda_1) & & \\ & \ddots & \\ & & J(\lambda_r) \end{pmatrix}, \quad J(\lambda_i) = \begin{pmatrix} J_1(\lambda_i) & & \\ & \ddots & \\ & & J_{n_i}(\lambda_i) \end{pmatrix},$$

$$J_j(\lambda_i) = \begin{pmatrix} \lambda_i & & & \\ 1 & \lambda_i & & \\ & \ddots & \ddots & \\ & & 1 & \lambda_i \end{pmatrix}.$$

Corollary 8 *Suppose that the matrix $\mathcal{L}$ can be reduced to the Jordan form with non-zero eigenvalues $\lambda_1, \ldots, \lambda_r$. Then, the system 2 is controllable if and only if each agent is controllable.*
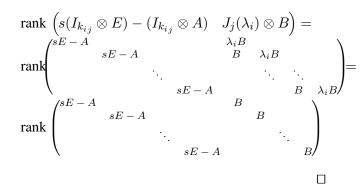
Proof. Let $\lambda_i \neq 0$, $i = 1, \ldots r$ be the eigenvalues of $\mathcal{L}$.

$$\text{rank} \begin{pmatrix} I_k \otimes E & J \otimes B \end{pmatrix} =$$
$$\sum_{i=1}^{r} \text{rank} \begin{pmatrix} I_{k_i} \otimes E & J(\lambda_i) \otimes B \end{pmatrix} =$$
$$\sum_{i=1}^{r} (\sum_{j=1}^{n_i} \text{rank} \begin{pmatrix} I_{k_{i_j}} \otimes E & J_j(\lambda_i) \otimes B \end{pmatrix}$$

with $k_1 + \ldots + k_r = k$, $k_{i_1} + \ldots k_{i_{n_i}} = k_i$.

$$\text{rank} \begin{pmatrix} I_{k_{i_j}} \otimes E & J_j(\lambda_i) \otimes B \end{pmatrix} =$$
$$\text{rank} \begin{pmatrix} E & & & & \lambda_i B & & & \\ & E & & & B & \lambda_i B & & \\ & & \ddots & & & \ddots & \ddots & \\ & & & E & & & B & \lambda_i B \end{pmatrix} =$$
$$\text{rank} \begin{pmatrix} E & & & & B & & \\ & E & & & & B & \\ & & \ddots & & & & \ddots \\ & & & E & & & & B \end{pmatrix}$$

and

$$\text{rank } \left(s(I_{k_{ij}} \otimes E) - (I_{k_{ij}} \otimes A) \quad J_j(\lambda_i) \otimes B \right) =$$

$$\text{rank} \begin{pmatrix} sE - A & & & & \lambda_i B \\ & sE - A & & & B & \lambda_i B \\ & & \ddots & & & \ddots & \ddots \\ & & & sE - A & & & B & \lambda_i B \end{pmatrix} =$$

$$\text{rank} \begin{pmatrix} sE - A & & & & B \\ & sE - A & & & & B \\ & & \ddots & & & & \ddots \\ & & & sE - A & & & & B \end{pmatrix}$$

$\square$

5 Structural controllability

A multiagent linear system is said to be structurally controllable if one can find a set of values for the non-zero entries of the matrices such that the corresponding multiagent linear system is controllable in the classical sense, that is to say to find a matrix $\mathcal{L}(\varepsilon)$ near $\mathcal{L}$ preserving structure in such a way that the new system is controllable.

In a more formal way we have the following definition.

Definition 9 *A singular system $E\dot{x} = Ax + Bu$ is structurally controllable if and only if $\forall \varepsilon > 0$, there exists a controllable system $\bar{E}\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t)$, of the same structure as $E\dot{x}(t) = Ax(t) + Bu(t)$ such that $\|\bar{E} - E\| < \varepsilon \|\bar{A} - A\| < \varepsilon$ and $\|\bar{B} - B\| < \varepsilon$.*

A singular system $E\dot{x}(t) = A_\sigma x(t) + B_\sigma u(t)$ has the same structure as another system $\bar{E}\dot{x}(t) = \bar{A}_\sigma x(t) + \bar{B}_\sigma u(t)$, of the same dimensions, if for every fixed zero entry of the triple of matrices (E, A, B), the corresponding entry of the triple of matrices $(\bar{E}, \bar{A}, \bar{B})$ is fixed zero and vice versa.

Remark 10 *If a singular system is controllable it is structural controllable, but the converse is false.*

5.1 Structural consensus controllability

In our particular setup we define structural controllability character of systems 2 in the following manner.

The system 2 is structural controllable if and only if, there exists a matrix $\bar{\mathcal{L}}$ with the same structure as $\mathcal{L}$ such that for all $\varepsilon > 0 \, \|\bar{\mathcal{L}} - \mathcal{L}\|$ the system

$$\begin{aligned} (I_k \otimes E)\dot{\mathcal{X}} &= (I_k \otimes A)\mathcal{X} + (\bar{\mathcal{L}} \otimes B)\bar{\mathcal{U}} \\ \bar{\mathcal{U}} &= (I_k \otimes K)\mathcal{X} \end{aligned} \tag{3}$$

is controllable.

It is obvious that if matrix L has a null eigenvalue, by the fact modify any nonzero term of the matrix, not a matrix of maximum range is achieved. It is even possible that this is not possible, as you can see in the following examples.

Example

i) Let $\mathcal{L}$ be the matrix

$$\begin{pmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

Considering

$$\mathcal{L}(\varepsilon) = \begin{pmatrix} 2 & -1 & -1 \\ -1-\varepsilon & 1+\varepsilon & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

the matrix $\mathcal{L}(\varepsilon)$ has not full rank but, but considering the perturbation

$$\mathcal{L}(\varepsilon) = \begin{pmatrix} 2+\varepsilon & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

the matrix $\mathcal{L}(\varepsilon)$ has full rank.

ii) Let $\mathcal{L}$ be the matrix

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

for all $\varepsilon_i < 0 \; i = 1, 2, 3$, the matrix

$$\mathcal{L}(\varepsilon) = \begin{pmatrix} 0 & 1+\varepsilon_1 & 1+\varepsilon_2 \\ 0 & 0 & 1+\varepsilon_3 \\ 0 & 0 & 0 \end{pmatrix}$$

the matrix $\mathcal{L}(\varepsilon)$ has not full rank.

We will analyze that (non-zero) elements of the matrix $\mathcal{L}$ must be modified in order to achieve a full rank matrix. When tackling the problem of how small perturbations of the system may lead to different equivalence classes (preserving canonical reduced form) a classical approach is to consider miniversal deformations, which provide all possible equivalence classes which can arise from small perturbations.

We recall here the definition of deformation and their characterization through versality (see [1], [7]).

Definition 11 *A deformation $\varphi(\lambda)$ of $x_0 \in \mathcal{M}$ is a smooth mapping*

$$\varphi : \mathcal{U}_0 \longrightarrow \mathcal{M}$$

such that $\mathcal{U}_0 \subseteq \mathbb{R}^l$ is an open neighborhood of the origin and $\varphi(0) = x_0$. The vector $\lambda = (\lambda_1, \ldots, \lambda_\ell) \in \mathcal{U}_0$ is called the parameter vector.

Let $\mathcal{G}$ be a Lie group acting over $\mathcal{M}$ via an action α, that is to say, for all $g \in \mathcal{G}$, $x \in \mathcal{M}$, $\alpha_x(g) = g \circ x \in \mathcal{M}$.

Definition 12 *A deformation $\varphi(\lambda)$ of x_0 is called versal if any deformation $\varphi'(\xi)$ of x_0, where $\xi = (\xi_1, \ldots, \xi_k) \in \mathcal{U}'_0 \subset \mathbb{R}^k$ is the parameter vector, can be represented in some neighborhood of the origin as*

$$\varphi'(\xi) = g(\xi) \circ \varphi(\phi(\xi)), \quad \xi \in \mathcal{U}''_0 \subset \mathcal{U}'_0, \quad (4)$$

where $\phi : \mathcal{U}''_0 \longrightarrow \mathbb{R}^\ell$ and $g : \mathcal{U}''_0 \longrightarrow \mathcal{G}$ are differentiable mappings such that $\phi(0) = 0$ and $g(0)$ is the identity element of $\mathcal{G}$.

When a versal deformation has the minimal number of parameters, it is called *miniversal*.

Locally, in $x \in \mathcal{M}$, $\mathcal{M}$ is isomorphic to the cartesian product of $\varphi(\mathcal{U})$ and a submanifold of $\mathcal{G}$. This can be stated as follows:

Theorem 13 ([1])

1. *A deformation $\varphi(\lambda)$ of x_0 is versal if, and only if, it is transversal to the orbit $\mathcal{O}(x_0)$ at x_0.*

2. *Minimal number of parameters of a versal deformation is equal to the codimension of the orbit of x_0 in $\mathcal{M}$, $d = \operatorname{codim} \mathcal{O}(x_0)$.*

Let $\{v_1, \ldots, v_d\}$ be a basis of any arbitrary complementary subspace $(T_{x_0}\mathcal{O}(x_0))^c$ to $T_{x_0}\mathcal{O}(x_0)$ (for example, $(T_{x_0}\mathcal{O}(x_0))^\perp$).

Corollary 14 *The deformation*

$$x : \mathcal{U}_0 \subset \mathbb{R}^d \longrightarrow M, \quad x(\lambda) = x_0 + \sum_{i=1}^{d} \lambda_i v_i \quad (5)$$

is a miniversal deformation.

For our particular set-up, the tangent space to the orbit of the matrix $\mathcal{L}$ is the set

$$\{\mathcal{L}P - P\mathcal{L}, \forall P \in M_k\}$$

and a complementary space is for example:

$$\{X \in M_k \mid \mathcal{L}X^* - X^*\mathcal{L} = 0\}.$$

So, we have the following corollary.

Corollary 15 *All possible deformations giveen different equivalence classes and preserving the structure of $\mathcal{L}$ is obtained intersecting a miniversal family with the variety of matrices having the same fixed zeros than the $\mathcal{L}$.*

Examples

i) We started showing an example for the case where the multiagents have identical mode

We consider 3 identical agents with the following dynamics of each agent

$$
\begin{aligned}
E\dot{x}^1 &= Ax^1 + Bu^1 \\
E\dot{x}^2 &= Ax^2 + Bu^2 \qquad\qquad (6) \\
E\dot{x}^3 &= Ax^3 + Bu^3
\end{aligned}
$$

with $E = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

The communication topology is defined by the graph $(\mathcal{V}, \mathcal{E})$:

$\mathcal{V} = \{1, 2, 3\}$

$\mathcal{E} = \{(i, j) \mid i, j \in \mathcal{V}\} = \{(1, 2), (1, 3)\} \subset \mathcal{V} \times \mathcal{V}$

and the adjacency matrix:

$$G = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

The neighbors of the parent nodes are $\mathcal{N}_1 = \{2, 3\}, \mathcal{N}_2 = \{1\}, \mathcal{N}_3 = \{1\}$.

The Laplacian matrix of the graph is

$$\mathcal{L} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

with eigenvalues $\lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 3$.

Taking into account that 0 is an eigenvalue of $\mathcal{L}$ the associated system 2 is not controllable.

The miniversal (orthogonal) deformation of the matrix $\mathcal{L}$ is given by

$$\mathcal{L} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_2 \\ \varepsilon_2 & \varepsilon_3 & \varepsilon_1 + \varepsilon_2 - \varepsilon_3 \\ \varepsilon_2 & \varepsilon_1 + \varepsilon_2 - \varepsilon_3 & \varepsilon_3 \end{pmatrix}$$

Intersecting the versal deformation with the variety defining the structure of matrix $\mathcal{L}$, we obtain

$$\mathcal{L} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_2 \\ \varepsilon_2 & \varepsilon_1 + \varepsilon_2 & 0 \\ \varepsilon_2 & 0 & \varepsilon_1 + \varepsilon_2 \end{pmatrix}.$$

So, taking $\varepsilon = \varepsilon_1$ and $\varepsilon_2 = 0$

$$\bar{\mathcal{L}} = \begin{pmatrix} 2 + \varepsilon & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

the new system is controllable, then the system s structurally controlable.

ii) Considering now, the matrix

$$\mathcal{L} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The miniversal (orthogonal) deformation of this matrix is defined as

$$\mathcal{L} + \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 \\ 0 & \varepsilon_1 & \varepsilon_2 \\ 0 & 0 & \varepsilon_1 \end{pmatrix}$$

That intersecting with the variety defining the structure of the matrix $\mathcal{L}$ we have

$$\mathcal{L} + \begin{pmatrix} 0 & \varepsilon_2 & \varepsilon_3 \\ 0 & 0 & \varepsilon_2 \\ 0 & 0 & 0 \end{pmatrix}.$$

6   Conclusions

In this paper, the structural control properties for multi-agent systems where all agents have an identical linear dynamic mode are analyzed.

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