# A Statistical Measure of Educational Disparities Due to Zero Tolerance Policies 

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#### Abstract

We use the disproportion statistic to quantify the differences among ethnic groups and school settings. Patterns of disciplinary actions in four school settings, Non-Title I Non-Charter, Non-Title I Charter, Title I NonCharter, and Title I Charter schools are examined. Analysis is based on the most recent data provided by the U. S. Office of Civil Rights. The importance of this research is that it is one of the first studies to show that not only are there differences in disparities due to zero tolerance policies among ethnic groups, but there exists statistically significant differences in disparities among school settings as well.


Key-Words: Disparity, Disproportion Statistic, Zero Tolerance

## 1 Introduction

The term disparities indicates unfair or unjust differences. A statistical index of disparity is often used to compare the degree of racial / ethnic disparities among groups in such fields as healthcare [6][1][5], and education [2]. We present a new statistical measure of disparity not typically used in the field of education, the disproportion statistic. This rigorous measure is suitable for educational disparities because it deliberately does not weigh each subgroup by its fraction of the overall population, but averages the absolute differences between the proportion of members of each subgroup who receive the particular type of treatment and the proportion in the total population. To formulate the disproportion statistic, we apply a modification of the mean deviation of group rates to disparities in education [4]. We also theoretically derive its mean square error.

In schools, zero tolerance refers to the concept that certain types of disciplinary offenses will not be tolerated and will automatically result in suspension or expulsion. We focus on zero tolerance policy disparities among Blacks, Whites, Hispanics, Na-
tive Americans, Pacific Americans, and Asians. The school settings we focus on are Non-Title I NonCharter schools, Non-Title I Charter schools, Title I Non-Charter schools, and Title I Charter schools. Non-Charter schools are traditional public schools. A Charter school is a publicly funded independent school. In order for a school to be classified as Title I, at least $40 \%$ of the students must be low income as defined by the U. S. Department of Education.

This paper is organized as follows: Section 2 presents an application of the disproportion statistic; Section 3 evaluates the estimation quality of the disproportion statistic by theoretically deriving its mean square error; Conclusions are provided in Section 4.

## 2 Disproportion Statistic Application

To determine if disparity is consistent over the four populations (school settings), we determine the disproportion statistic $d$ for the four populations: NonTitle I Non-Charter, Non-Title I Charter, Title I NonCharter, and Title I Charter schools. The dispropor-
tion statistic is the fraction of the entire population that did not receive the most disciplinary placements relative to the group who did receive the most placements [3].

We also determined the statistic $d_{i}, i=1 \ldots 6$ for the six subgroups: Blacks, Whites, Hispanics, Native Americans, Pacific Americans, and Asians. Each statistic is scaled by a factor $10^{2}$ since values are close to zero. The overall disproportion statistic $d$ is determined for each of the four school settings, and the $i^{t h}$ disproportion statistic $d_{i}$, is determined for each of the six ethnic groups within each of the four school settings.

Each table that follows represents one of the four school settings. Specifically, tables contain the sample size of each of the six ethnic groups, the number of students in their particular ethnic group who were expelled due to zero tolerance, and each ethnic group's corresponding disproportion statistic $d_{i}, i=1 \ldots 6$. Additionally, the overall $d$ statistic is calculated for each table.

The overall $d$ statistic for Table 1 is $d=0.0848$. This value will be compared to $d$ values for the remaining three tables, and will be used to determine if there are disparities among school settings. Pairwise comparisons of $d$ are provided in Table 5. The overall $d$ statistic for each table is the sum of the $d_{i}$, $i=1, \ldots,(k-1)$ for each ethnic group.

Table 1: Non-Title I Non-Charter schools total students enrolled, total students expelled due to zero tolerance policies, and the disproportion statistic $d_{i}$ for each ethnic group. For the group that has the highest level of disciplinary placements, $d_{i}=0$. The overall $d$ statistic for the table is the sum of the $d_{i}$, $i=1, \ldots,(k-1)$ for each ethnic group:

| Ethnicity | Enrolled | Expelled | Disp., $d_{i}$ |
| :--- | ---: | ---: | :---: |
| Black | 1150984 | 1362 | 0.0078 |
| White | 5546608 | 4380 | 0.0631 |
| Hispanic | 1328719 | 1763 | 0.0068 |
| Native Amer. | 84937 | 150 | 0.0000 |
| Pacific Amer. | 36199 | 45 | 0.0002 |
| Asian | 431582 | 177 | 0.0068 |

Figure 1 depicts disparity of disciplinary placements based on zero tolerance policies received by Native Americans, who received the highest level of disciplinary placements, as compared to the five ethnic groups who received less than Native Americans in Non-Title 1 Non-Charter schools.

Table 2 contains data for Non-Title I Charter


Figure 1: Dot chart displays disparity of disciplinary placements based on zero tolerance policies received by Native Americans, who received the highest level of disciplinary placements, as compared to the five other ethnic groups who received less than Native Americans in Non-Title 1 Non-Charter schools.

Table 2: Non-Title I Charter schools total students enrolled, total students expelled due to zero tolerance policies, and the disproportion statistic $d_{i}$ for each ethnic group. For the group that has the highest level of disciplinary placements, $d_{i}=0$. The overall $d$ statistic for the table is the sum of the $d_{i}, i=1, \ldots,(k-1)$ for each ethnic group:

| Ethnicity | Enrolled | Expelled | Disp. $\left(\times 10^{2}\right)$ |
| :--- | ---: | ---: | :---: |
| Black | 20299 | 22 | 0.0509 |
| White | 51515 | 54 | 0.1310 |
| Hispanic | 24530 | 12 | 0.0758 |
| Native Amer. | 1647 | 6 | 0.0000 |
| Pacific Amer. | 384 | 0 | 0.0012 |
| Asian | 3640 | 0 | 0.0130 |

schools. The scaled $d$ statistic for Table 2 is $d=$ 0.2722 . Figure 2 demonstrates the disparity of disciplinary placements based on zero tolerance policies received by Native Americans, who received the highest level of disciplinary placements, as compared to the five other ethnic groups who received less than Na tive Americans in Non-Title 1 Charter schools.


Figure 2: Dot chart displays disparity of disciplinary placements based on zero tolerance policies received by Native Americans, who received the highest level of disciplinary placements, as compared to the five other ethnic groups who received less than Native Americans in Non-Title 1 Charter schools.

Table 3: Title I Non-Charter schools total students enrolled, total students expelled due to zero tolerance policies, and the disproportion statistic $d_{i}$ for each ethnic group. For the group that has the highest level of disciplinary placements, $d_{i}=0$. The overall $d$ statistic for the table is the sum of the $d_{i}, i=1, \ldots,(k-1)$ for each ethnic group:

| Ethnicity | Enrolled | Expelled | Disp. $\left(\times 10^{2}\right)$ |
| :--- | ---: | ---: | :---: |
| Black | 678278 | 1477 | 0.0092 |
| White | 922809 | 1024 | 0.0447 |
| Hispanic | 1237654 | 2486 | 0.0236 |
| Native Amer. | 49808 | 129 | 0.0000 |
| Pacific Amer. | 19583 | 20 | 0.0010 |
| Asian | 146045 | 67 | 0.0102 |

The scaled $d$ statistic for Table 3 is $d=0.0886$. Figure 3 shows disparity of disciplinary placements based on zero tolerance policies received by Native Americans, who received the highest level of disciplinary placements, as compared to the five other ethnic groups who received less than Native Americans in Title 1 Non-Charter schools.

Disparity of Discipline for Native Americans


Figure 3: Dot chart displays disparity of disciplinary placements based on zero tolerance policies received by Native Americans, who received the highest level of disciplinary placements, as compared to the five other ethnic groups who received less than Native Americans in Title 1 Non-Charter schools.

Table 4: Title I Charter schools total students enrolled, total students expelled due to zero tolerance policies, and the disproportion statistic $d_{i}$ for each ethnic group. For the group that has the highest level of disciplinary placements, $d_{i}=0$. The overall $d$ statistic for the table is the sum of the $d_{i}, i=1, \ldots,(k-1)$ for each ethnic group:

| Ethnicity | Enrolled | Expelled | Disp. $\left(\times 10^{2}\right)$ |
| :--- | ---: | ---: | :---: |
| Black | 45526 | 101 | 0.0000 |
| White | 28632 | 28 | 0.0293 |
| Hispanic | 41492 | 58 | 0.0281 |
| Native Amer. | 1879 | 4 | 0.0001 |
| Pacific Amer. | 332 | 0 | 0.0006 |
| Asian | 3102 | 2 | 0.0040 |

Table 4 contains data for Title I Charter schools. The scaled $d$ statistic for Table 4 is $d=0.0623$. Figure 4 highlights the disparity of disciplinary placements based on zero tolerance policies received by Blacks, who received the highest level of disciplinary placements, as compared to the five other ethnic groups who received less than Blacks in Title 1 Char-
ter schools.


Figure 4: Dot chart displays disparity of disciplinary placements based on zero tolerance policies received by Blacks, who received the highest level of disciplinary placements, as compared to the five other ethnic groups who received less than Blacks in Title 1 Charter schools.

Table 5: Pairwise Comparisons of disparity between schools, p -value, and statistical significance.

| Comparison | p -value | significant |
| :---: | :--- | :---: |
| $\mathrm{d}_{\text {table } 1}, d_{\text {table } 2}$ | $2.000 \times 10^{-1}$ | yes |
| $\mathrm{d}_{\text {table } 1}, d_{\text {table } 3}$ | $0.002 \times 10^{0}$ | yes |
| $\mathrm{d}_{\text {table } 1}, d_{\text {table } 4}$ | $2.000 \times 10^{-16}$ | yes |
| $\mathrm{d}_{\text {table } 2}, d_{\text {table } 3}$ | $2.000 \times 10^{-16}$ | yes |
| $\mathrm{d}_{\text {table } 2}, d_{\text {table } 4}$ | $2.000 \times 10^{-16}$ | yes |
| $\mathrm{d}_{\text {table } 3}, d_{\text {table } 4}$ | $2.000 \times 10^{-16}$ | yes |

To test the disproportion statistics, $d$ from the four tables, we perform a test of equality for multiple proportions. The null hypothesis is all proportions are equal, and the alternative is at least one proportion is different. The test statistic is $d$. The p -value for the given data set is $3.432 \times 10^{-24}$. Therefore, reject the null hypothesis at the $\alpha=.01$ level of significance. Conclude that not all school systems are equal with respect to disparity. Therefore, it is important to determine which school systems are different. We use the $R$ function pairwise.prop.test to calculate pairwise comparisons between pairs of the four $d$ statistics,
then use the Holm procedure to adjust the p -values for multiple testing. The function pairwise.prop.test enables us to simultaneously test pairs of proportions. From Table 5 conclude that all pairs of school settings have statistically significant differences with respect to disparity due to zero tolerance policies.

## 3 Mean Square Error of Disproportion Statistic

Since we can usually apply more than one disparity measure in a particular situation, a difficulty that may arise is the task of choosing the best estimator. Therefore, we need some criteria such as the mean square error to evaluate the quality of an estimator. The mean square error of an estimator $\hat{\theta}$ of a parameter $\theta$ is defined as the expected value

$$
\begin{equation*}
\operatorname{MSE}(\hat{\theta})=E\left[(\hat{\theta}-\theta)^{2}\right] \tag{1}
\end{equation*}
$$

Theorem 1 Let d be the disproportion statistic. Since $d$ is defined to be a proportion, the Mean Square Error of the estimator $\operatorname{MSE}(\hat{d})=\left(\frac{p q}{n}\right)\left(\frac{N-n}{N-1}\right)$. Where $p$ represents the fraction of the entire population that did not receive the most disciplinary placements due to zero tolerance policies relative to the group who did receive the most placements, and $q=1-p$. The variable $N$ represents the population size (total students enrolled in a school setting). The variable $n$ represents subgroup size.
Proof: Let $d \in R$, and let $\hat{d}$ be an unbiased estimator of $d$. The population mean of $d$ is $\mu$. From the definition of mean square error,

$$
\begin{aligned}
\operatorname{MSE}(\hat{d}) & =E\left[(\hat{d}-d)^{2}\right] \\
& =E\left[(\hat{d}-\mu+\mu-d)^{2}\right] \\
& =E\left[(\hat{d}-\mu)^{2}+2(\hat{d}-\mu)(\mu-d)+(\mu-d)^{2}\right] \\
& =E\left[(\hat{d}-\mu)^{2}\right]+E[2(\hat{d}-\mu)(\mu-d)]+E\left[(\mu-d)^{2}\right] \\
& =\operatorname{Var}[\hat{d}]+2(\mu-d) E[(\hat{d}-\mu)]+(\mu-d)^{2} \\
& =\operatorname{Var}[\hat{d}] \\
& =\left(\frac{p q}{n}\right)\left(\frac{N-n}{N-1}\right)
\end{aligned}
$$

from the fact that $E[(\hat{d}-\mu)]=0$, and the sample proportion is an unbiased estimator of the population proportion.

## 4 Conclusions

Creating a fair and positive school environment and devising creative approaches to discipline are strategies being considered to replace the rigid punishments
associated with zero tolerance policies. Prior research shows that there are disparities among ethnic groups regarding the enforcement of zero tolerance policies. The importance of this research is that for one of the first times it is shown that there are statistically significant differences in disparities due to zero tolerance policies among school settings, as well.

To quantify these differences, we compute the disproportion statistic for each ethnic group and for each of the four school settings, Non-Title I NonCharter, Non-Title I Charter, Title I Non-Charter, and Title I Charter schools. Findings show disciplinary disparities based on zero tolerance policies are largest among Native Americans and Blacks, who receive the greatest level of disciplinary placements, and that of Whites who receive the least. Findings also show that all pairs of school settings have statistically significant differences with respect to disparity due to zero tolerance policies.

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