Statistical Characteristic of Ratio and Product of Rician Random Variables and its Application in Analysis of Wireless Communication Systems

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Abstract: - In this paper, the product of two Rician random variables, the ratio of two Rician random variables, the ratio of Rician random variable and product of two Rician random variables, and the ratio of product of two Rician random variables and Rician random variable are considered. The expressions for the level crossing rate of product of two Rician random processes, ratio of two Rician random processes, ratio of Rician random process and product of two Rician random processes, and ratio of product of two Rician random processes and Rician random process are derived. We use those results for calculation the average fade duration of wireless communication system operating over Rician multipath fading channels. The influence of Rician factor K on the level crossing rate is analyzed and discussed.

Key-Words: - Rician random variable, product, ratio, wireless communication systems

1 Introduction

The ratios and products of random variables have applications in performance analysis of wireless communication systems operating over multipath fading channels in the presence of cochannel interference subjected to short term fading. There are more published works in technical literature considering the first and the second order statistics of ratios and products of random variables and its application in calculation the outage probability, the bit error probability, channel capacity and average level crossing rate of wireless radio systems.

In paper [1], the level crossing rate of wireless relay communication system with N sections operating over Rayleigh short term fading channel is analyzed. Product of N Rayleigh random variables is considered.

The product of two Nakagami-*m* random variables is studied in [2], and the level crossing rate of relay system with two sections in the presence of Nakagami-*m* multipath fading is evaluated. The first order statistics of ratio of random variable and product of two random variables are considered in [3], and the ratio of product of two random variables and random variable is analyzed in [4].

In [5], probability density function (PDF) of ratio of product of two Rayleigh random variables and Rayleigh variable in closed form is derived. Resulting PDF can be used for analysis of wireless telecommunication systems located in urban and crowded environments. Under these conditions, signal used for information transfer is exposed to interference. When fading is present, the shadowing effect is also present. The distributions of ratios of random variables are of interest in many areas of the sciences. In [6], the authors present the joint probability density function and PDF of maximum of ratios and for the cases R/r, where R and r are Rayleigh, Rician, Nakagami-m and Weibull distributed random variables. All random variables are correlated.

Rician distribution can be used to describe signal envelope variation in line of sight multipath fading channels [7]. This distribution has parameter K which can be calculated as the ratio of dominant component power and scattering components powers. For parameter K is zero, Rician multipath fading channel becomes Rayleigh multipath fading channel, and for K goes to infinity, Rician fading channel becomes no fading channel.

In this paper, the random variable is formed as product of two Rician random variables and denoted as Rice*Rice. This distribution can be used in performance analysis of wireless communication systems with two sections operating over Rician multipath fading channel. Under determined conditions, signal at the output of relay system with two sections can be expressed as product of signals at sections.

The random variable derived as ratio of two Rician random variables is marked as Rice/Rice and has application in performance analysis of wireless communication systems in the presence of Rician small scale fading and Rician cochannel interference.

The random variable calculated as ratio of random variable and product of two random variables is tagged as Rice/Rice*Rice, and has implementation in analysis of properties of wireless communication systems disturbed by Rician multipath fading and cochannel interference originated from relay system with two sections.

The random variable obtained as ratio of two Rician random variables and Rician random variable is denoted as Rice*Rice/Rice. It has application in performance analysis of wireless relay communication systems with two sections which operate over Rician multipath fading channel in the presence of Rician cochannel interference.

In this paper, the expressions for the level crossing rates for Rice*Rice random process, Rice/Rice random process, Rice/Rice*Rice random process and Rice*Rice/Rice random process are derived. The obtained results can be used for calculation of the level crossing rate of random process as Rayleigh*Rayleigh, Rayleigh/Rayleigh, Rayleigh/Rayleigh*Rayleigh random process and Rayleigh*Rayleigh/Rayleigh random process. Also, the expressions for the level crossing rate of Rayleigh*Rice random process, Rayleigh/Rice random process, Rayleigh/Rice*Rice random process and Rayleigh*Rayleigh/Rice random process can be evaluated. The obtained expressions for the level crossing rate are useful in calculating the average fade duration of wireless communication systems which operate in channel with Rice*Rice multipath fading. The level crossing rate is evaluated as average value of the first derivative of random process and average fade can be calculated as the ratio of the outage probability and level crossing rate. The obtained results can be used in performance analysis of wireless communication system working over Rician multipath fading channels.

2 Level Crossing Rate of Ratio of Two Rician Random Variables

The ratio of two Rician random variables is [8]:

$$z = \frac{x}{y}, \quad x = z \cdot y \tag{1}$$

where x and y follow Rician distribution [6]:

$$p_x(x) = \frac{2(K_1+1)x}{e^{K_1}\Omega_1} e^{-\frac{(K_1+1)x^2}{\Omega_1}} I_0\left(2\sqrt{\frac{K_1(K_1+1)}{\Omega_1}}x\right)$$

After some mathematical manipulations this expression becomes:

$$p_x(x) = \frac{2(K_1+1)}{e^{K_1}\Omega_1} \sum_{i_1=0}^{\infty} \left(\frac{K_1(K_1+1)}{\Omega_1}\right)^{i_1} \frac{1}{(i_1!)^2} \cdot$$

$$\cdot x^{2i_1+1} e^{-\frac{K_1+1}{\Omega_1}x^2}, \; x \geq 0 \tag{2}$$

Similarly, for variable y is valid:

$$p_y(y) = \frac{2(K_2+1)}{e^{K_2}\Omega_2} \sum_{i_2=0}^{\infty} \left(\frac{K_2(K_2+1)}{\Omega_2}\right)^{i_2} \frac{1}{(i_2!)^2} \cdot$$

$$\cdot y^{2i_2+1} e^{-\frac{K_2+1}{\Omega_2}y^2}, \; y \geq 0 \tag{3}$$

The first derivative of z is:

$$\dot{z} = \frac{\dot{x}}{y} - \frac{x\dot{y}}{y^2}. \tag{4}$$

The first derivative of Rician random process is Gaussian random process. The linear combination of Gaussian random variables follows Gaussian distribution. Thus, the first derivative of ratio of two Rician random variables has conditional Gaussian distribution with zero mean and variance:

$$\sigma_{\dot{z}}^2 = \frac{1}{y^2}\sigma_{\dot{x}}^2 + \frac{x^2}{y^4}\sigma_{\dot{y}}^2 \tag{5}$$

where

$$\sigma_{\dot{x}}^2 = \pi^2 f_m^2 \Omega_1, \quad \sigma_{\dot{y}}^2 = \pi^2 f_m^2 \Omega_2,$$

and f_m is maximal Dopler frequency.

After substituting, the expression for variance becomes:

$$\sigma_{\dot{z}}^2 = \frac{\pi^2 f_m^2}{y^2}\left(\Omega_1 + z^2\Omega_2\right) \tag{6}$$

The joint probability density function of z and $\dot{z}$ is:

$$p_{z\dot{z}}(z\dot{z}) = \int_0^{\infty} dy\, y\, p_x(zy) \cdot p_y(y) \frac{\sigma_{\dot{z}}}{\sqrt{2\pi}} p_{\dot{z}}(z/zy) \cdot \tag{7}$$

The level crossing rate of z is:

$$N_z = \int_0^\infty d\dot{z}\, \dot{z}\, p_{z\dot{z}}(z\dot{z}) =$$

$$= \int_0^\infty dy\, y\, p_x(zy) \cdot p_y(y) \frac{\sigma_{\dot{z}}}{\sqrt{2\pi}} =$$

$$= \frac{\pi f_m}{\sqrt{2\pi}} \frac{2(K_1+1)}{e^{K_1}\Omega_1} \frac{2(K_2+1)}{e^{K_2}\Omega_2} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \left(\frac{K_1(K_1+1)}{\Omega_1} \right)^{i_1} \frac{1}{(i_1!)^2} \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\frac{K_2(K_2+1)}{\Omega_1} \right)^{i_2} \frac{1}{(i_2!)^2} \cdot$$

$$\cdot z^{2i_1+1} \sqrt{\Omega_1 + z^2\Omega_2} \int_0^\infty dy\, y^{2i_1+1+2i_2+2} \cdot$$

$$\cdot e^{-y^2\left(\frac{z^2(K_1+1)}{\Omega_1} + \frac{(K_2+1)}{\Omega_2} \right)} =$$

$$= \pi f_m \frac{4(K_1+1)(K_2+1)}{e^{K_1+K_2}\Omega_1\Omega_2} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \left(\frac{K_1(K_1+1)}{\Omega_1} \right)^{i_1} \frac{1}{(i_1!)^2} \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\frac{K_2(K_2+1)}{\Omega_2} \right)^{i_2} \frac{1}{(i_2!)^2} \cdot$$

$$\cdot z^{2i_1+1} \sqrt{\Omega_1 + z^2\Omega_2} \cdot$$

$$\frac{1}{2} \cdot \frac{(\Omega_1\Omega_2)^{i_1+i_2+3/2}}{\Omega_1\left((K_2+1) + \Omega_2(K_1+1)z^2\right)^{i_1+i_2+3/2}} \tag{8}$$

Previous expression is level crossing rate of Rician/Riacian random process.

3 Level Crossing Rate of Product of Two Rician Random Variables

Product of two Rician random variables is:

$$z = x \cdot y, \quad x = \frac{z}{y}, \tag{9}$$

The first derivative of z is:

$$\dot{z} = \dot{x}y + x\dot{y} \tag{10}$$

Random variable $\dot{z}$ has conditional Gaussian distribution. The main of $\dot{z}$ is zero. The variance of $\dot{z}$ is:

$$\sigma_{\dot{z}}^2 = \sqrt{y^2\sigma_{\dot{x}}^2 + x^2\sigma_{\dot{y}}^2} = \pi f_m \sqrt{y^2\Omega_1 + x^2\Omega_2} =$$

$$= \frac{\pi f_m}{y} \sqrt{y^4\Omega_1 + z^2\Omega_2} \tag{11}$$

The joint probability density function of z, $\dot{z}$ and y is:

$$p_{z\dot{z}y}(z\dot{z}y) = \frac{1}{y} p_x\left(\frac{z}{y}\right) \cdot p_y(y)\, p_{\dot{z}}(\dot{z}/zy) \tag{12}$$

The level crossing rate of z is:

$$N_z = \int_0^\infty d\dot{z}\, \dot{z}\, p_{z\dot{z}}(z\dot{z}) =$$

$$= \int_0^\infty dy\, \frac{1}{y} \cdot p_x\left(\frac{z}{y}\right) \cdot p_y(y) \frac{\sigma_{\dot{z}}}{\sqrt{2\pi}} =$$

$$= \frac{\pi f_m}{\sqrt{2\pi}} \frac{(K_1+1)(K_2+1)}{e^{K_1}e^{K_2}\Omega_1\Omega_2} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \left(\frac{K_1(K_1+1)}{\Omega_1} \right)^{i_1} \frac{1}{(i_1!)^2} \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\frac{K_2(K_2+1)}{\Omega_2} \right)^{i_2} \frac{1}{(i_2!)^2} \cdot$$

$$\cdot z^{2i_1+1} \cdot \int_0^\infty dy \sqrt{y^4\Omega_1 + z^2\Omega_2} \cdot y^{-2-2r_1+2r_2} \cdot e^{-\frac{K_1+1}{\Omega_1}\frac{z^2}{y^2} - \frac{K_2+1}{\Omega_2}y^2} \tag{13}$$

Previous integral can be solved by using Laplace approximation theorem:

$$\int_0^\infty dx\, g(x) \cdot e^{-\lambda f(x)} = \left(\frac{\pi}{\lambda}\right)^{1/2} \frac{g(x_0)}{f(x_0)} e^{-\lambda f(x_0)}$$

where x_0 is solution of $f'(x_0) = 0$.

4 The Ratio of Rician Random Variable and Product of Two Rician Random Variables

The ratio of Rician random variable and product of two Rician random variables is:

$$w = \frac{x}{yz}, \quad x \geq 0,\ y > 0,\ z > 0 \tag{14}$$

where x, y and z follow Rician distribution given by (2), (3) and (15):

$$p_z(z)=\frac{2(K_3+1)}{e^{K_3}\Omega_3}\sum_{i_3=0}^{\infty}\left(\frac{K_3(K_3+1)}{\Omega_3}\right)^{i_3}\frac{1}{(i_3!)^2}\cdot$$

$$\cdot z^{2i_3+1}e^{-\frac{K_3+1}{\Omega_3}z^2},\ z\geq 0. \tag{15}$$

The first derivative of w is:

$$\dot{w}=\frac{\dot{x}}{yz}-\frac{x}{y^2z}\dot{y}-\frac{x}{yz^2}\dot{z}. \tag{16}$$

Random variable $\dot{w}$ follows conditional Gaussian distribution with zero main and variance:

$$\sigma_{\dot{w}}^2=\frac{1}{y^2z^2}\sigma_{\dot{x}}^2+\frac{x^2}{y^4z^2}\sigma_{\dot{y}}^2+\frac{x^2}{y^2z^4}\sigma_{\dot{z}}^2 \tag{17}$$

where :

$$\sigma_{\dot{x}}^2=\pi^2 f_m^2\frac{\Omega_1}{K_1+1},\ \sigma_{\dot{y}}^2=\pi^2 f_m^2\frac{\Omega_2}{K_2+1},$$

$$\sigma_{\dot{z}}^2=\pi^2 f_m^2\frac{\Omega_3}{K_3+1}.$$

After substituting, the variance of $\dot{w}$ becomes:

$$\sigma_{\dot{w}}^2=\pi^2 f_m^2\left(\frac{1}{y^2z^2}\frac{\Omega_1}{K_1+1}+\frac{w^2}{y^2}\frac{\Omega_2}{K_2+1}+\frac{w^2}{z^2}\frac{\Omega_3}{K_3+1}\right) \tag{18}$$

The level crossing rate of ratio of Rician random variable and product of two Rician random variables is:

$$N_w=\int_0^\infty dy\int_0^\infty dz\, yz\cdot p_x(wyz)\cdot p_y(y)\cdot p_z(z)\frac{\sigma_{\dot{w}}}{\sqrt{2\pi}}=$$

$$=\frac{\pi f_m}{\sqrt{2\pi}}\frac{2(K_1+1)}{e^{K_1}\Omega_1}\cdot\frac{2(K_2+1)}{e^{K_2}\Omega_2}\cdot\frac{2(K_3+1)}{e^{K_3}\Omega_3}$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{K_1(K_1+1)}{\Omega_1}\right)^{i_1}\frac{1}{(i_1!)^2}\cdot$$

$$\cdot\sum_{i_2=0}^{\infty}\left(\frac{K_2(K_2+1)}{\Omega_2}\right)^{i_2}\frac{1}{(i_2!)^2}\cdot$$

$$\cdot\sum_{i_3=0}^{\infty}\left(\frac{K_3(K_3+1)}{\Omega_3}\right)^{i_3}\frac{1}{(i_3!)^2}\cdot$$

$$\cdot\int_0^\infty dy\int_0^\infty dz\; y^{3+2r_1+2r_2}\cdot z^{3+2r_1+2r_3}\cdot$$

$$\cdot e^{-\frac{K_1+1}{\Omega_1}w^2y^2z^2-\frac{K_2+1}{\Omega_2}y^2-\frac{K_3+1}{\Omega_3}z^2}\cdot$$

$$\cdot\sqrt{\frac{1}{y^2z^2}\frac{\Omega_1}{K_1+1}+\frac{w^2}{y^2}\frac{\Omega_2}{K_2+1}+\frac{w^2}{z^2}\frac{\Omega_3}{K_3+1}}. \tag{19}$$

Previously presented integral can be solved by using Laplace approximation formula for two fold integrals:

$$\int_0^\infty dx\int_0^\infty dy\; g(x,y)\cdot e^{-\lambda f(x,y)}=\frac{\pi}{\lambda}\frac{g(x_0,y_0)}{B(x_0,y_0)}e^{-\lambda f(x_0,y_0)}, \tag{20}$$

where x_0 and y_0 are solutions of equations:

$$\frac{\partial f(x_0,y_0)}{\partial x_0}=0$$

and

$$\frac{\partial f(x_0,y_0)}{\partial y_0}=0.$$

and

$$B=\begin{vmatrix}\frac{\partial^2 f(x_0,y_0)}{\partial x_0^2} & \frac{\partial^2 f(x_0,y_0)}{\partial x_0\partial y_0}\\ \frac{\partial f(x_0,y_0)}{\partial y_0\partial x_0} & \frac{\partial^2 f(x_0,y_0)}{\partial y_0^2}\end{vmatrix}.$$

5 The Ratio of Product of Two Rician Random Variables and Rician Random Variable

The ratio of product of two Rician random variables and Rician random variable is:

$$w=\frac{xy}{z},\ \ x=\frac{wz}{y} \tag{21}$$

The first derivative of w is:

$$\dot{w}=\frac{\dot{x}y}{z}+\frac{x\dot{y}}{z}-\frac{xy\dot{z}}{z^2}. \tag{22}$$

The random variable $\dot{w}$ has conditional Gaussian distribution with zero main and variance:

$$\sigma_{\dot{w}}^2=\frac{y^2}{z^2}\sigma_{\dot{x}}^2+\frac{x^2}{z^2}\sigma_{\dot{y}}^2+\frac{x^2y^2}{z^4}\sigma_{\dot{z}}^2=$$

$$=\pi^2 f_m^2\left(\frac{y^2}{z^2}\frac{\Omega_1}{K_1+1}+\frac{w^2}{y^2}\frac{\Omega_2}{K_2+1}+\frac{w^2}{z^2}\frac{\Omega_3}{K_3+1}\right) \tag{23}$$

The level crossing rate of w is:

$$
\begin{aligned}
N_w &= \int_0^\infty dy \int_0^\infty dz \frac{z}{y} \cdot p_x\left(\frac{wz}{y}\right) \cdot p_y(y) \cdot p_z(z) \frac{\sigma_{\dot{w}}}{\sqrt{2\pi}} = \\
&= \frac{\pi f_m}{\sqrt{2\pi}} \frac{2(K_1+1)}{e^{K_1}\Omega_1} \cdot \frac{2(K_2+1)}{e^{K_2}\Omega_2} \cdot \frac{2(K_3+1)}{e^{K_3}\Omega_3} \\
&\cdot \sum_{i_1=0}^{\infty} \left(\frac{K_1(K_1+1)}{\Omega_1}\right)^{i_1} \frac{1}{(i_1!)^2} \cdot \\
&\cdot \sum_{i_2=0}^{\infty} \left(\frac{K_2(K_2+1)}{\Omega_2}\right)^{i_2} \frac{1}{(i_2!)^2} \cdot \\
&\cdot \sum_{i_3=0}^{\infty} \left(\frac{K_3(K_3+1)}{\Omega_3}\right)^{i_3} \frac{1}{(i_3!)^2} \cdot \\
&\cdot \int_0^\infty dy \int_0^\infty dz \; y^{-1-2r_1+2r_2} \cdot z^{3+2r_1+2r_3} \cdot \\
&\cdot e^{-\frac{K_1+1}{\Omega_1}\frac{w^2 z^2}{y^2} - \frac{K_2+1}{\Omega_2} y^2 - \frac{K_3+1}{\Omega_3} z^2} \cdot \\
&\cdot \sqrt{\frac{y^2}{z^2}\frac{\Omega_1}{K_1+1} + \frac{w^2}{y^2}\frac{\Omega_2}{K_2+1} + \frac{w^2}{z^2}\frac{\Omega_3}{K_3+1}} \qquad (24)
\end{aligned}
$$

Previously written integral can be solved by using Laplace approximation formula (20) for two fold integrals.

6 Numerical Results

In Fig. 1, the histogram of the ratio of two Rician random variables is shown. The abscissa of the histogram is the amplitude of Rician random process; the ordinate is the number of samples in the interval of abscissa.

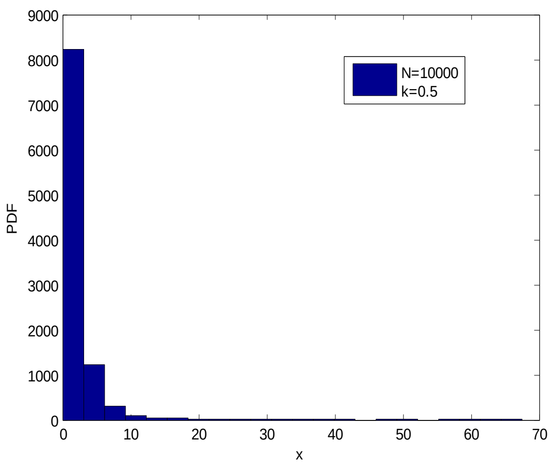

Fig. 1. Histogram of the ratio of two Rician random variables

The outage probability for the ratio of two Rician random variables is presented in Fig. 2.

The level crossing rate for the ratio of two Rician random variables is given in Fig. 3.

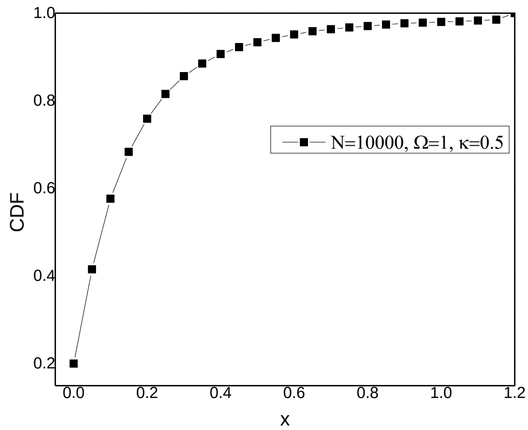

Fig.2. Outage probability for the ratio of two Rician random variables

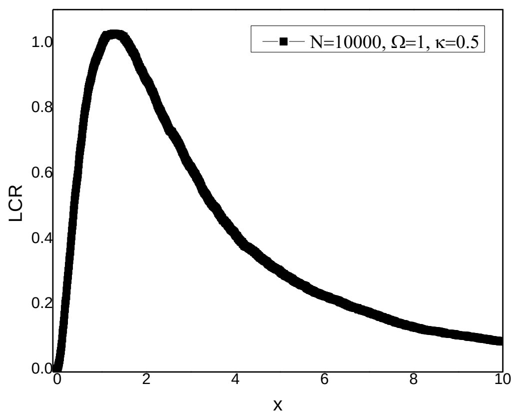

Fig.3. Level crossing rate for the ratio of two Rician random variables

The histogram of the product of two Rician random variables is drawn in Fig. 4. The outage probability for the product of two Rician random variables is introduced in Fig. 5. The level crossing rate for this product of two Rician random variables is plotted in Fig. 6.

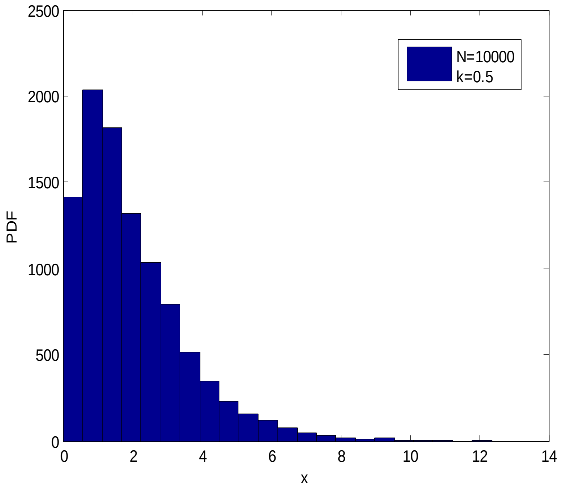

Fig. 4. Histogram of the product of two Rician random variables

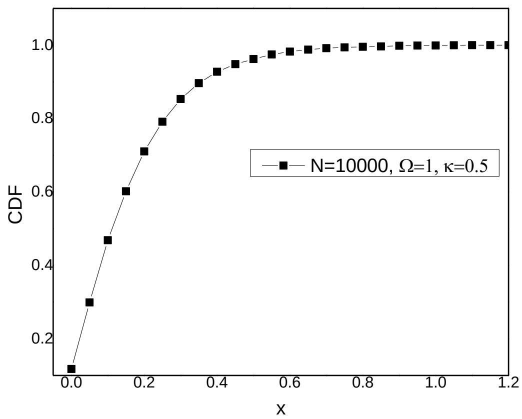

Fig.5. Outage probability for the product of two Rician random variables

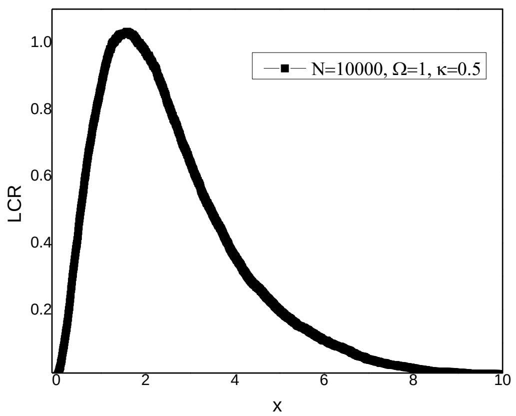

Fig.6. Level crossing rate for the product of two Rician random variables

The histogram of the ratio of Rician random variable and product of two Rician random variables is presented in Fig. 7.

The outage probability for the ratio of Rician random variable and product of two Rician random variables is given in Fig. 8.

The level crossing rate for this ratio of Rician random variable and product of two Rician random variables is shown in Fig. 9.

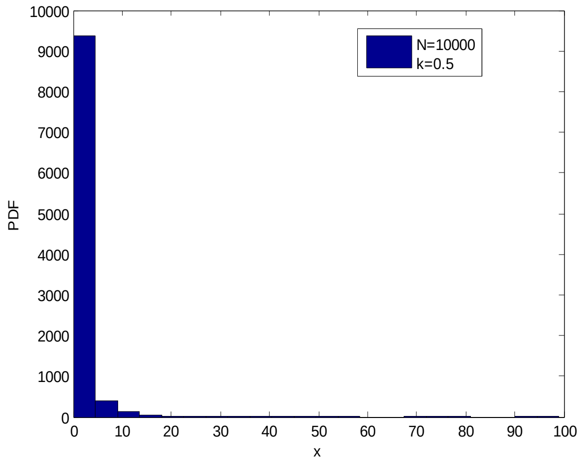

Fig.7. Histogram of the ratio of Rician random variable and product of two Rician random variables

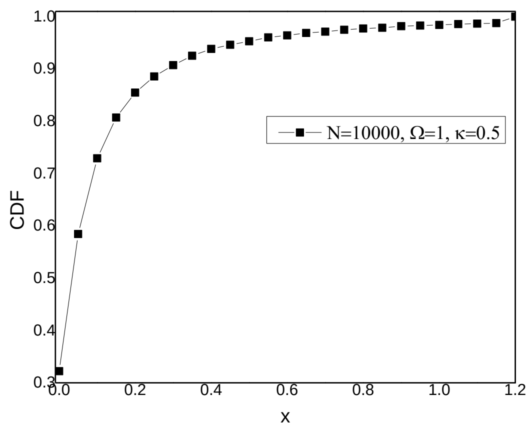

Fig.8. Outage probability for the ratio of Rician random variable and product of two Rician random variables

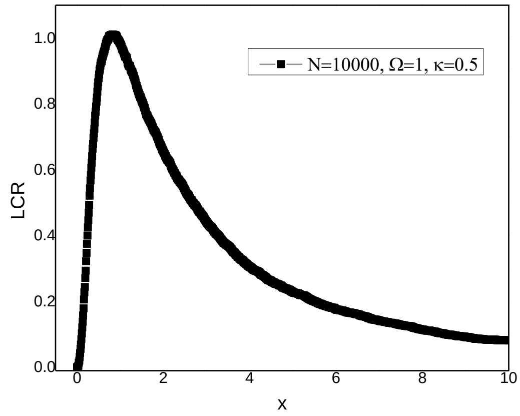

Fig.9. Level crossing rate of the ratio of Rician random variable and product of two Rician random variables

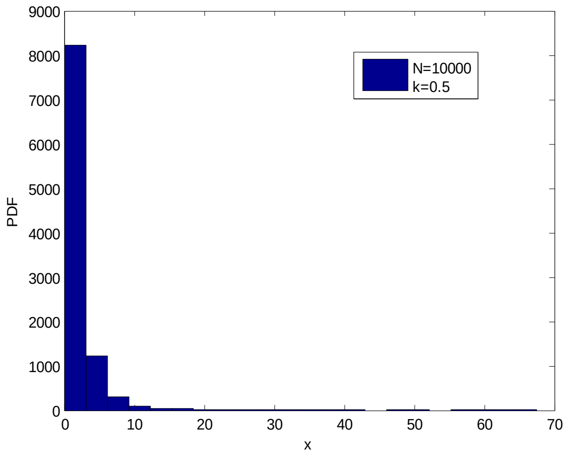

Fig.10. Histogram of the ratio of product of two Rician random variables and Rician random variable

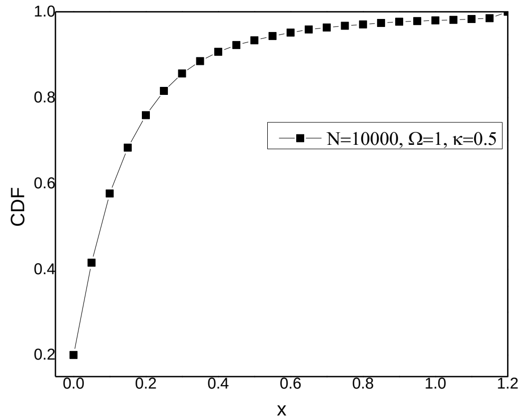

Fig.11. Outage probability for the ratio of product of two Rician random variables and Rician random variable

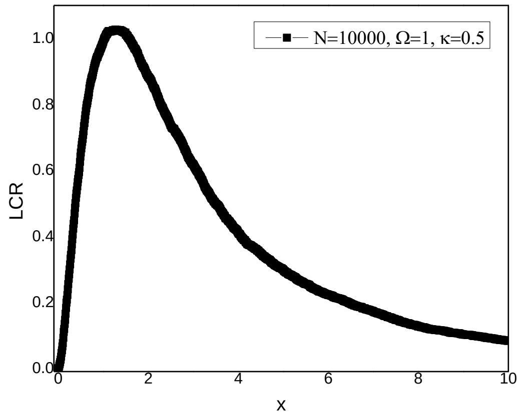

Fig.12. Level crossing rate of the ratio of product of two Rician random variables and Rician random variable

The histogram of the ratio of product of two Rician random variables and Rician random variable is presented in Fig. 10.

The outage probability for the ratio of product of two Rician random variables and Rician random variable is pointed out in Fig. 11.

The level crossing rate for this ratio of Rician random variable and product of two Rician random variables is shown in Fig. 12.

These results help designers of wireless telecommunications system to pick out the best parameters of the system in the presence of Rician fading.

7 Conclusion

In this paper, Rice*Rice random variable is defined and can be calculated as product of two Rician random variables. The ratio of two Rician random variables is denoted as Rice/Rice and random variable derived as the ratio of Rician random variables and product of two Rician random variables is marked as Rice/Rice*Rice. Random variable labeled as Rice* Rice/Rice can be calculated as product of two Rician random variables and Rician random variable.

The formula for the level crossing rate of Rice*Rice random process, Rice/Rice random process, Rice*Rice/Rice random process and Rice /Rice* Rice random process are evaluated in this paper. The expressions for other distributions as Rayleigh*Rayleigh, Rayleigh/Rayleigh, Rayleigh* Rayleigh/Rayleigh and Rayleigh/Rayleigh*Rayleigh can be calculated by using derived terms for level crossing rates.

The formula for level crossing rate of Rice*Rice can be used for calculation the average fade duration of wireless relay system operating over Rician multipath fading channel. The expression for level crossing rate of Rice/Rice can be used for calculation the average fade duration of wireless communication system working over Rician short term fading channel in the presence of cochannel interference affected to Rician multipath fading. The expression derived for level crossing rate of Rice*Rice/Rice can be utilized in performance analysis of wireless relay communication system with two sections working over Rician multipath fading channel in the presence of Rician cochannel interference. Finally, formula for level crossing rate of Rice/Rice*Rice random process can be employed for calculation the average fade duration of communication system that runs over Rician multipath fading environment in the presence of cochannel interference originated from wireless relay system with two sections.

References:

[1] N. Zlatanov, Z. Hadzi-Velkov, G. K. Karagiannidis, Level Crossing Rate and Average Fade Duration of the Double Nakagami-m Random Process and Application in MIMO Keyhole Fading Channels, *IEEE Communications Letters*, Vol. 12, Issue 11, November 2008, pp. 822 – 824.

[2] Z. Hadzi-Velkov, N. Zlatanov, G. K. Karagiannidis, On the second order statistics of the multihop Rayleigh fading channel, *IEEE Transactions on Communications*, Vol. 57, Issue: 6, June 2009, pp. 1815 – 1823

[3] A. Matović, E. Mekić, N. Sekulović, M. Stefanović, M. Matović, Č Stefanović, The Distribution of the Ratio of the Products of Two Independent - Variates and Its Application in the Performance Analysis of Relaying Communication Systems, *Mathematical Problems in Engineering*, Vol. 2013, Article ID 147106, 6 pages, 2013.

[4] E. Mekić, M. Stefanović, P. Spalević, N. Sekulović, A. Stanković, "Statistical analysis of ratio of random variables and its application in performance analysis of multi-hop wireless transmissions", *Mathematical Problems in Engineering*, Vol. 2012, Article ID 841092

[5] A. Stanković, N. Sekulović, M. Stefanović, E. Mekić, S. Zdravković, "Statistical characteristic of ratio of product of two random Rayleigh variables and Rayleigh random variable and its application in performance analysis of wireless communication systems", Proc. of Infoteh-Jahorina, Istočno Sarajevo, BIH, 16-28 March 2011, vol. 10, Ref. B-I-3, pp.79-81.

[6] D. H. Pavlović, N. M. Sekulović, G. V. Milovanović, A. S. Panajotović, M. Č. Stefanović, Z. J. Popović, Statistics for Ratios of Rayleigh, Rician, Nakagami-*m*, and Weibull Distributed Random Variables, *Mathematical Problems in Engineering*, Vol. 2013, Article ID 252804, http://dx.doi.org/10.1155/2013/252804

[7] M. K. Simon, M. S. Alouini, *Digital Communication over Fading Channels*, USA: John Wiley & Sons. 2000.

[8] W.C.Y. Lee, *Mobile communications engineering*, Mc-Graw-Hill, NewYork, USA, 2003.