Introducing Smart Symmetries with application to gravity related radiation

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Abstract—Local one-parameter ϵ point symmetries do not always lead to readily integrable results. To address this here, a secondary infinitesimal parameter ω is introduced, leading to what we term smart symmetries. These enable the transformation of the integrals into limit expressions, much like Cauchys contour integration. As proof of validity, we use these type of symmetries to develop a procedure for determining frequencies for detecting any material, depending on the elements constituting it, including metals such as Copper, Silver, Gold and Platinum. Here, we give the frequency for Platinum.

Keywords—Differential equations, Gravitational waves, Symmetry analysis.

I. Introduction

ANYONE who has ever attempted to use group invariant solutions in practical applications, can attest how difficult it is to do so, primarily because of the unintegrable results the pure Lie approach usually leads to.

In here, we introduce a parameter ω in addition to ϵ. The parameter ϵ maintains the role it has always been used for. The new parameter ω is used to evaluate the integrals, possible through limits and continuity principles.

II. The theoretical basis

There are a number of symmetry type in use today. Local one-parameter symmetries are the most popular; hence the choice we made to ground our theory around them.

A. Local One-Parameter Type Point Transformations

To begin, let

$$\bar{\mathbf{x}} = \chi\left(\mathbf{x}; \omega, \epsilon\right) \tag{2.1}$$

be a family of invertible transformations of points $\mathbf{x} = (x^1, \cdots, x^N) \in \mathbb{R}^N$ into points $\bar{\mathbf{x}} = (\bar{x}^1, \cdots, \bar{x}^N) \in \mathbb{R}^{\mathbf{N}}$, with ω and ϵ in $\mathbb{R}$, subject to the conditions

$$\bar{\mathbf{x}}|_{\epsilon=0} = \mathbf{x}. \tag{2.2}$$

That is,

$$\chi\left(\mathbf{x}; \omega, \epsilon\right)\Big|_{\epsilon=0} = \mathbf{x}. \tag{2.3}$$

These become the regular local one-parameter transformation when $\omega = 0$. The symmetries that follow from these new new symmetries differs from the original local one-parameter symmetries by this parameter as discussed in texts like [1] and [2], and the rules that gorven them holds so long as the condition $\omega = 0$ holds, as outlined below.

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1) Invariant Functions in $\mathbb{R}^2$:

In $\mathbb{R}^2$, we have $\chi = (\phi; \psi)$, while $\bar{\mathbf{x}} = (\bar{x}, \bar{y})$ and $\bar{x} = (x; y)$, so that

$$\bar{x} = \phi\left(x, y, \epsilon\right) \tag{2.4}$$

and

$$\bar{y} = \psi\left(x, y, \epsilon\right) \tag{2.5}$$

Expanding (2.4) and (2.5) about $\epsilon = 0$, in some neighborhood of $\epsilon = 0$, gives

$$\bar{x} = x + \epsilon\left(\frac{\partial\phi}{\partial\epsilon}\Big|_{\epsilon=0}\right) + O\left(\epsilon^2\right) \tag{2.6}$$

and

$$\bar{y} = y + \epsilon\left(\frac{\partial\psi}{\partial\epsilon}\Big|_{\epsilon=0}\right) + O\left(\epsilon^2\right). \tag{2.7}$$

Letting

$$\xi\left(x, y\right) = \frac{\partial\phi}{\partial\epsilon}\Big|_{\epsilon=0}, \tag{2.8}$$

and

$$\eta\left(x, y\right) = \frac{\partial\psi}{\partial\epsilon}\Big|_{\epsilon=0}, \tag{2.9}$$

reduces the expansions to

$$\bar{x} = x + \epsilon\xi\left(x, y\right) + O\left(\epsilon^2\right) \tag{2.10}$$

and

$$\bar{y} = y + \epsilon\eta\left(x, y\right) + O\left(\epsilon^2\right), \tag{2.11}$$

or simply

$$\bar{x} = x + \epsilon\xi\left(x, y\right) \tag{2.12}$$

and

$$\bar{y} = y + \epsilon\eta\left(x, y\right). \tag{2.13}$$

2) The group generator:

The local one-parameter point transformations in (2.12) and (2.13) can be rewritten in the form

$$\bar{x} = x + \epsilon(\xi\left(x, y\right); \eta\left(x, y\right)) \cdot \nabla x, \tag{2.14}$$

or

$$\bar{y} = y + \epsilon(\xi\left(x, y\right); \eta\left(x, y\right)) \cdot \nabla y, \tag{2.15}$$

so that

$$\bar{x} = \left(1 + \epsilon(\xi\left(x, y\right); \eta\left(x, y\right)) \cdot \nabla\right) x \tag{2.16}$$

and

$$\bar{y} = \left(1 + \epsilon(\xi\left(x, y\right); \eta\left(x, y\right)) \cdot \nabla\right) y. \tag{2.17}$$

An operator,

$$G = (\xi(x,y);\eta(x,y)) \cdot \nabla, \tag{2.18}$$

or

$$G = \xi(x,y)\frac{\partial}{\partial x} + \eta(x,y)\frac{\partial}{\partial y}, \tag{2.19}$$

can then be introduced, so that (2.12) and (2.13) assume the form

$$(\bar{x};\bar{y}) = (1+\epsilon G)(x;y). \tag{2.20}$$

3) Prolongations formulas :

In determining the prolongations, it is convenient to use the operator of total differentiation

$$D = \frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} + y''\frac{\partial}{\partial y'} + \cdots, \tag{2.21}$$

where

$$y' = \frac{dy}{dx}, \quad y'' = \frac{d^2y}{dx^2}, \quad \cdots. \tag{2.22}$$

The derivatives of the transformed point is then

$$\bar{y}' = \frac{d\bar{y}}{d\bar{x}}. \tag{2.23}$$

Since

$$\bar{x} = x + \epsilon\xi \ \text{ and } \ \bar{y} = y + \epsilon\eta, \tag{2.24}$$

then

$$\bar{y}' = \frac{dy + \epsilon d\eta}{dx + \epsilon d\xi}. \tag{2.25}$$

That is,

$$\bar{y}' = \frac{dy/dx + \epsilon d\eta/dx}{dx/dx + \epsilon d\xi/dx}. \tag{2.26}$$

Now introducing the operator D:

$$\bar{y}' = \frac{y' + \epsilon D(\eta)}{1+\epsilon D(\xi)} = \frac{(y'+\epsilon D(\eta))(1-\epsilon D(\xi))}{1-\epsilon^2(D(\xi))^2}. \tag{2.27}$$

Hence

$$\bar{y}' = \frac{y' - \epsilon(D(\eta) - y'D(\xi)) - \epsilon^2(D(\xi))}{1-\epsilon^2(D(\xi))^2}. \tag{2.28}$$

That is,

$$\bar{y}' = y' + \epsilon(D(\eta) - y'D(\xi)), \tag{2.29}$$

or

$$\bar{y}' = y' + \epsilon\zeta^1, \tag{2.30}$$

with

$$\zeta^1 = D(\eta) - y'D(\xi). \tag{2.31}$$

It expands into

$$\zeta^1 = \eta_x + (\eta_y - \xi_x)y' - y'^2\xi_y. \tag{2.32}$$

The first prolongation of G is then

$$G^{[1]} = \xi(x,y)\frac{\partial}{\partial x} + \eta(x,y)\frac{\partial}{\partial y} + \zeta^1\frac{\partial}{\partial y'}. \tag{2.33}$$

For the second prolongation, we have

$$\bar{y}'' = \frac{y'' + \epsilon D(\zeta^1)}{1+\epsilon D(\xi)} \approx y'' + \epsilon\zeta^2, \tag{2.34}$$

with

$$\zeta^2 = D(\zeta^1) - y''D(\xi). \tag{2.35}$$

This expands into

$$\begin{aligned}\zeta^2 &= \eta_{xx} + (2\eta_{xy} - \xi_{xx})y' + (\eta_{yy} - 2\xi_{xy})y'^2 \\ &- y'^3\xi_{yy} + (\eta_y - 2\xi_x - 3y'\xi_y)y''.\end{aligned} \tag{2.36}$$

The second prolongation of G is then

$$G^{[2]} = \xi(x,y)\frac{\partial}{\partial x} + \eta(x,y)\frac{\partial}{\partial y} + \zeta^1\frac{\partial}{\partial y'} + \zeta^2\frac{\partial}{\partial y''}. \tag{2.37}$$

Most applications involve up to second order derivatives.

4) Lie equations :

One-parameter groups are obtained by their generators by means of *Lie's theorem*:

Theorem 1: Given the infinitesimal transformations (2.1) or its symbol G, the corresponding one-parameter group G is obtained by solution of the Lie equations

$$\frac{d\bar{x}^i}{da} = \xi^i(\bar{x},\bar{u}), \quad \frac{d\bar{u}^a}{da} = \eta^a(\bar{x},\bar{u}),$$

subject to the initial conditions

$$\bar{x}^i|_{a=0} = x^i, \quad \bar{u}^a|_{a=0} = u^a.$$

B. Our integration procedure

Anyone who has ever used Lie's group symmetrical methods to solve differential equations [3], knows the danger that afflicts this approach. One could end with beautifully looking solutions, but an impractical because they be based on unsubstantiated assumptions. The reason for such assumptions being easy integration. Some such examples can be found in [4] and [5], and a lot more in [6].

To avert this for one parameter local point transformations

$$\bar{\mathbf{x}} = \phi(\mathbf{x},\epsilon), \tag{2.38}$$

we introduce the parameter ω, in addition to ϵ, so that the transformation assumes the form

$$\bar{\mathbf{x}} = \phi(\mathbf{x},\epsilon,\omega). \tag{2.39}$$

This is not to be confused with the two-parameters local point transformations. The parameter ω does is not involved in determining the symmetry generators. Instead, it is allowed to enter the infinitesimals of the generators. The result is that integrals evaluated through the simple formula

$$\int_a^b f(x)dx = \left[\sum_{i=1}^{N}\xi_i f(x)\xi_i^{-1}\Big|_{\omega=0}\right]_a^b,$$

where $\xi_i^{-1} = \xi_i^{-1}(\bar{x},\omega)$ is the inverse function of the infinitesimal $\xi_i = \xi_i(x,\omega)$, obtained by solving the Lie equations.

Theorem 2: Suppose $\phi : [a,b] \mapsto R$ is a continuous and differentiable function, and is a solution of the differential equation $f(x,\phi(x),\phi'(x),\phi''(x)) = 0$ on $[\alpha,\beta] \subseteq [a,b]$. Then it holds on $[\alpha,\beta] \cup R \setminus [\alpha,\beta] \subseteq [a,b]$ if $\phi^{(m)}(\alpha)\phi^{(n)}(\alpha)\phi^{(m+1)}(\alpha)\phi^{(n+1)}(\alpha) > 0$. If these conditions hold, then

$$\phi^{(n)}(\xi)\phi^{(m+1)}(\xi) - \phi^{(n+1)}(\xi)\phi^{(m)}(\xi) = 0. \tag{2.40}$$

A special case of this theorem, useful here, with $m = 0$ and $n = 1$ assumes the form

Theorem 3: Suppose $\phi : [a,b] \mapsto R$ is a continuous and differentiable function, and is a solution of the differential equation $f(x,\phi(x),\phi'(x),\phi''(x)) = 0$ on $[\alpha,\beta] \subseteq [a,b]$. Then it holds on $[\alpha,\beta] \cup R \setminus [\alpha,\beta] \subseteq [a,b]$ if $\phi(\alpha)\phi'(\alpha)\phi''(\alpha)\phi^{(3)}(\alpha) > 0$. If these conditions hold, then

$$\phi''(\xi)\phi'(\xi) - \phi^{(3)}(\xi)\phi(\xi) = 0. \tag{2.41}$$

Its proof is as follows:

Suppose $[\phi(\alpha) - \phi(\beta)]\phi'(\alpha) < 0$. Then this implies that either $\phi(\alpha) - \phi(\beta) < 0$ and $\phi'(a) > 0$ or $\phi(\alpha) - \phi(\beta) > 0$ and $\phi'(\alpha) < 0$. The first case implies that $\phi(x)$ has a maximum on (α,β), meaning there exists $\xi \in (\alpha,\beta)$ such that $\phi(\xi) = 0$. The second case implies $\phi(x)$ has a minimum on (α,β), meaning there exists $\xi \in (\alpha,\beta)$ such that $\phi(\xi) = 0$. That is, $[\phi(\alpha) - \phi(\beta)]\phi'(\alpha) < 0$ implies there exists $\xi \in (\alpha,\beta)$ such that $\phi(\xi) = 0$. Similarly, $[\phi''(\alpha) - \phi''(\beta)]\phi^{(3)}(\alpha) < 0$ implies there exists $\eta \in (\alpha,\beta)$ such that $\phi''(\eta) = 0$. If then ϕ and ϕ'' have a common root in (α,β), then $\eta = \xi$. If β is chosen to coincide with ξ, we have $\phi(\alpha)\phi'(\alpha) < 0$ implying $\phi(\xi) = 0$ and $\phi''(\alpha)\phi^{(3)}(\alpha) < 0$ implying $\phi''(\xi) = 0$. Or simply $\phi(\alpha)\phi'(\alpha)\phi''(\alpha)\phi^{(3)}(\alpha) > 0$ implying $\phi(\xi) = 0$ and $\phi''(\xi) = 0$. By L'Hospital's rule, this means $\phi''(\xi)\phi'(\xi) - \phi^{(3)}(\xi)\phi(\xi) = 0$. Since ϕ is a continuous and differentiable on (a,b), it follows then from the Taylor series expansion that ϕ and ϕ'' have infinite number of roots on (a,b), meaning the result in (2.41) extends to $[\alpha,\beta] \cup R \setminus [\alpha,\beta] \subseteq [a,b]$, which completes the proof. For a more detailed applications of the above see [7], [8], [9], [10], [11] and [12].

III. Gravity related radiation

We are of the view that while Newton's mechanics has been successfully applied to a wide variety of areas, it is this *wide variety* that makes it difficult to understand G through it. Newton's mechanics can be used to accurately predict the motions of satellites. For us, this mechanics does not really explain how they move. For a solid object in motion without spin, Newton's mechanics is usually used to describe the average motion. The interest is not on the motion of the individual atoms. While this is understandable, it would not be wise to account for the motion of the individual atoms in a satellite or a moving truck, but the sacrifice is huge. The true picture gets forgotten, and is replaced by Newton's averages. It then becomes impossible to explain concepts like G.

A. Formulation of the model

We are not really interested in exactly how this motion happens. There are just too many atoms in an object to account for each. We just want enough information that would allow a formula for G to emerge. Applying Newton's mechanics to a single atom does not really help much. Applying it to the charges in the atom does make a difference; G emerges. The simplest atom is the element Hydrogen. We only have two objects to work with. It is possible to go beyond this level, but that would generate too much information, more than what we bargained for. We would be overwhelmed.

1) The classical approach:

Let us suppose that the positions of two atoms, p_1 and p_2, in an object are given by the points (X_1,Y_1,Z_1) and (X_2,Y_2,Z_2) with respect to some reference point. Let us also suppose that when the object is in motion with velocity $(v,0,0)$ with respect to the same reference point, then the same atoms assume the positions (x_1,y_1,z_1) and (x_2,y_2,z_2). In Newtonian mechanics, the distance between the atoms is not affected by this. That is,

$$\begin{aligned} &||(X_1,Y_1,Z_1) - (X_2,Y_2,Z_2)|| \\ &= ||(x_1,y_1,z_1) - (x_2,y_2,z_2)||. \end{aligned} \tag{3.42}$$

We do not agree with this. The general belief is that this is the case when $v << c$.

2) Relativistic approach:

Einstein's relativistic mechanics is punted as a viable alternative. According to this theory, there should be length contraction. That is,

$$\begin{aligned} &||(X_1,Y_1,Z_1) - (X_2,Y_2,Z_2)|| \\ &> ||(x_1,y_1,z_1) - (x_2,y_2,z_2)||, \end{aligned} \tag{3.43}$$

for $v \neq 0$. And this led to $E = mc^2$, and many benefits for mankind. We partially agree with this view.

3) Our view:

Our belief is that the positions of the atoms p_1 and p_2 should be referenced with respect to the galactic center. The position of the reference point does not matter in Newton and Einstein's mechanics. It does here. Any other reference point will lead to inaccurate results. It then becomes possible to understand Newton's gravitational law in terms of electrodynamic variables, and deduce a formula for G. The distance between the atoms oscillates between (3.43) and

$$\begin{aligned} &||(X_1,Y_1,Z_1) - (X_2,Y_2,Z_2)|| \\ &< ||(x_1,y_1,z_1) - (x_2,y_2,z_2)||. \end{aligned} \tag{3.44}$$

The value of $||(x_1,y_1,z_1) - (x_2,y_2,z_2)||$ is not necessarily what Einstein calculated it to be. This view is not really new, except for the galactic center priority. The continuum hypothesis explains this very well, just that it is unheard to have it applied to an object in motion, or not having a balancing force. All that we have to do is replace the material elements of Continuum mechanic with atoms.

B. Newton's mass based formula for gravity

In his universal gravitation law, Newton maintained that every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportinal to the square of the distance between them. That is,

$$\mathbf{F}_N \;=\; -G_N\,\frac{m_1\;m_2}{s^2}\;\hat{\mathbf{d}}_1. \tag{3.45}$$

This is called the universal gravitation law, and G is the parameter whose formula we are interested in. Applied to two Hydrogen atoms of mass $m_H = m_p + m_e$ each, separated by the distance r, we have

$$\mathbf{F}_N \;=\; -G_N\,\left(\frac{m_H}{s}\right)^2\;\hat{\mathbf{d}}_1. \tag{3.46}$$

This was viewed as a two body problem, in that there are two bodies, one has mass m_1 and the other m_2, much like the Earth and the Moon. Our quest here is to understand the constant G in the law, and this two body does not help us much in this regard. For us, this should be seen as a many body problem, in that the Earth has infinitely many atoms, and so does the Moon. This complicated picture can be simplified by considering the gravitational force between two hydrogen atoms. This is a four body problem, in that each atom has two subatomic particles, each with its own mass. The atom on its own is a two body problem.

The two body problem analysis can be found in many texts, including [13] and [14]. We briefly outline it below for easy reference.

1) The two-body problem for a Hydrogen atom :

Let us consider a system consisting of two bodies of mass m_p and m_e at distances $\mathbf{r}_p$ and $\mathbf{r}_e$ from the galactic centre O. Let $\mathbf{F}_p$ and $\mathbf{F}_e$ be the external forces acting on m_p and m_e, respectively, while $\mathbf{F}_{pe}$ is the internal force acting on body m_p due to m_e, and $\mathbf{F}_{ep}$ the internal force acting on body m_e due to m_p.

According to Newton's second law, the motionof the two bodies may be written as

$$m_p\frac{d^2\mathbf{r}_p}{dt^2} = \mathbf{F}_p + \mathbf{F}_{pe}, \tag{3.47}$$

and

$$m_e\frac{d^2\mathbf{r}_e}{dt^2} = \mathbf{F}_e + \mathbf{F}_{ep}. \tag{3.48}$$

The centre of mass coordinate $\mathbf{R}$ is given by

$$\mathbf{R} = \frac{m_p\mathbf{r}_p + m_e\mathbf{r}_e}{m_p + m_e}, \tag{3.49}$$

and the relative coordinate $\mathbf{r}$ is given by

$$\mathbf{r} = \mathbf{r}_p - \mathbf{r}_e. \tag{3.50}$$

The inverse transformation are given by

$$\mathbf{r}_e = \mathbf{R} + \frac{m_e}{m_p + m_e}\mathbf{r}, \tag{3.51}$$

and

$$\mathbf{r}_p = \mathbf{R} - \frac{m_p}{m_p + m_e}\mathbf{r}. \tag{3.52}$$

To transform the equations from $\mathbf{r}_p$ and $\mathbf{r}_e$ to $\mathbf{R}$, we add (3.47) and (3.48); that is

$$m_p\frac{d^2\mathbf{r}_p}{dt^2} + m_e\frac{d^2\mathbf{r}_e}{dt^2} = \mathbf{F}_p + \mathbf{F}_{pe} + \mathbf{F}_e + \mathbf{F}_{ep}. \tag{3.53}$$

According to Newton's third law

$$\mathbf{F}_{pe} + \mathbf{F}_{ep} = 0; \tag{3.54}$$

hence,

$$\frac{d^2(m_p\mathbf{r}_p + m_e\mathbf{r}_e)}{dt^2} = \mathbf{F}_p + \mathbf{F}_e. \tag{3.55}$$

Using the result in (3.49), the preceding equation may be written as

$$(m_p + m_e)\frac{d^2\mathbf{R}}{dt^2} = \mathbf{F}. \tag{3.56}$$

Hence,

$$M\frac{d^2\mathbf{R}}{dt^2} = \mathbf{F}. \tag{3.57}$$

where $\mathbf{F} = \mathbf{F}_p + \mathbf{F}_e$, and $M = m_p + m_e$. This is the first of the equations we are interested in. To get the second, we multiply (3.47) by m_e and (3.48) by m_p and subtract:

$$\begin{aligned} &m_em_p\frac{d^2\mathbf{r}_p}{dt^2} - m_pm_e\frac{d^2\mathbf{r}_e}{dt^2} = \\ &m_e\mathbf{F}_p + m_e\mathbf{F}_{pe} - m_p\mathbf{F}_e - m_p\mathbf{F}_{ep}. \end{aligned} \tag{3.58}$$

It can be rewritten as This is the first of the equations we are interested in. To get the second, we multiply (3.47) by m_e and (3.47) by m_p and subtract:

$$\begin{aligned} &m_em_p\frac{d^2(\mathbf{r}_p - \mathbf{r}_e)}{dt^2} = \\ &m_e\mathbf{F}_p - m_p\mathbf{F}_e + (m_e + m_p)\mathbf{F}_{pe}. \end{aligned} \tag{3.59}$$

Dividing by $m_e + m_p$ and using the result in (3.50):

$$\begin{aligned} &\frac{m_em_p}{m_e + m_p}\frac{d^2\mathbf{r}}{dt^2} = \\ &\frac{1}{m_e + m_p}\left(m_e\mathbf{F}_p - m_p\mathbf{F}_e\right) + \mathbf{F}_{pe}. \end{aligned} \tag{3.60}$$

Introducing the quantity μ, called the reduced mass, and defined by

$$\mu = \frac{m_em_p}{m_e + m_p} \tag{3.61}$$

reduces the preceding equation into

$$\mu\frac{d^2\mathbf{r}}{dt^2} = \frac{m_e\mathbf{F}_p - m_p\mathbf{F}_e}{m_e + m_p} + \mathbf{F}_{pe}. \tag{3.62}$$

C. Our magnetism based gravity

Our hypothesis is that *an atom has a velocity relative to the centre of the universe; each individual charge in the atom then has a magnetic field; the nett sum of these fields at any point is not zero*. This non-zero sum is what we believe is the source of gravity. We will now build this up for two Hydrogen atoms, a distance r apart.

Let $\mathbf{r}_p$ be the position vector relative to the centre of the universe for a proton in a Hydrogen atom. The velocity is then $\dot{\mathbf{r}}_p$. Its magnetic field $\mathbf{B}_{pb}$ at position $\mathbf{a}_1$, occupied by a proton of another Hydrogen, is then

$$\mathbf{B}_{p,p} \;=\; \frac{\mu_0}{4\pi}\;e\;\dot{\mathbf{r}}_p \times \frac{\mathbf{d}_{p,p}}{d^3_{p,p}}. \tag{3.63}$$

Magnetic field of the electron at the same point is

$$\mathbf{B}_{e,p} \;=\; -\frac{\mu_0}{4\pi}\; e\; \dot{\mathbf{r}}_e \times \frac{\mathbf{d}_{e,p}}{d_{e,p}^3}. \tag{3.64}$$

The sum of the two gives

$$\mathbf{B} \;=\; \frac{\mu_0}{4\pi}\; e\; \left(\dot{\mathbf{r}}_p \times \frac{\mathbf{d}_{p,p}}{d_{p,p}^3} - \dot{\mathbf{r}}_e \times \frac{\mathbf{d}_{e,p}}{d_{e,p}^3} \right), \tag{3.65}$$

or

$$\mathbf{B} \;=\; \frac{\mu_0}{4\pi}\; e\; \left([\dot{\mathbf{r}}_e + \mathbf{u}] \times \frac{\mathbf{d}_{p,p}}{d_{p,p}^3} - \dot{\mathbf{r}}_e \times \frac{\mathbf{d}_{e,p}}{d_{e,p}^3} \right). \tag{3.66}$$

That is,

$$\mathbf{B} \;=\; \frac{\mu_0}{4\pi}\; e\; \left(\mathbf{u} \times \frac{\mathbf{d}_{p,p}}{d_{p,p}^3} + \dot{\mathbf{r}}_e \times \left(\frac{\mathbf{d}_{p,p}}{d_{p,p}^3} - \frac{\mathbf{d}_{e,p}}{d_{e,p}^3} \right) \right), \tag{3.67}$$

where $\mathbf{u} = \dot{\mathbf{r}}_p - \dot{\mathbf{r}}_e$.

From the Biot-Savart law:

$$\mathbf{F} = q\; \mathbf{v} \times \mathbf{B},$$

the force on the second proton moving with velocity $\mathbf{v}$ is

$$\begin{aligned} F_p \;&=\; \frac{\mu_0}{4\pi}\; e\; q\; \left(\mathbf{v} \times \mathbf{u} \times \frac{\mathbf{d}_{p,p}}{d_{p,p}^3} \right) \\ &+\; \frac{\mu_0}{4\pi}\; e\; q\; \left(\mathbf{v} \times \dot{\mathbf{r}}_e \times \left(\frac{\mathbf{d}_{p,p}}{d_{p,p}^3} - \frac{\mathbf{d}_{e,p}}{d_{e,p}^3} \right) \right). \end{aligned} \tag{3.68}$$

The identity $\mathbf{a} \times \mathbf{b} \times \mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ reduces it to

$$\begin{aligned} F_p \;&=\; \frac{\mu_0}{4\pi}\; e\; q\; \left(\left(\frac{\mathbf{d}_{p,p}}{d_{p,p}^3} \cdot \mathbf{v} \right) \mathbf{u} - (\mathbf{v} \cdot \mathbf{u}) \frac{\mathbf{d}_{p,p}}{d_{p,p}^3} \right) \\ &+\; \frac{\mu_0}{4\pi}\; e\; q\; \left(\mathbf{v} \times \dot{\mathbf{r}}_e \times \left(\frac{\mathbf{d}_{p,p}}{d_{p,p}^3} - \frac{\mathbf{d}_{e,p}}{d_{e,p}^3} \right) \right). \end{aligned} \tag{3.69}$$

The force on the electron is then

$$\begin{aligned} F_e \;&=\; \frac{\mu_0}{4\pi}\; e\; q\; \left(\left(\frac{\mathbf{d}_{e,e}}{d_{e,e}^3} \cdot \mathbf{v} \right) \mathbf{u} - (\mathbf{v} \cdot \mathbf{u}) \frac{\mathbf{d}_{e,e}}{d_{e,e}^3} \right) \\ &+\; \frac{\mu_0}{4\pi}\; e\; q\; \left(\mathbf{v} \times \dot{\mathbf{r}}_e \times \left(\frac{\mathbf{d}_{e,e}}{d_{e,e}^3} - \frac{\mathbf{d}_{p,e}}{d_{p,e}^3} \right) \right). \end{aligned} \tag{3.70}$$

This is our impression of a force between two electrically neutral objects.

D. The fifth, sixth and seventh force

The are three other forces from (3.70):

$$\begin{aligned} \mathbf{F}_{1,2,3} \;&=\; \frac{\mu_0}{4\pi}\; \frac{e^2}{s^2}\; \left((\mathbf{v} \cdot \hat{\mathbf{d}}_1)\mathbf{u} \right) \\ &+\; \frac{\mu_0}{4\pi}\; \frac{e^2}{s^2}\; \left((\mathbf{v} \cdot \hat{\mathbf{d}})\dot{\mathbf{r}}_e \right) \\ &-\; \frac{\mu_0}{4\pi}\; \frac{e^2}{s^2}\; \left((\dot{\mathbf{r}}_e \cdot v)\hat{\mathbf{d}} \right), \end{aligned} \tag{3.71}$$

They point to three different new directions. These do not surprise. The possibility of a fifth force has long being suspected, see [15] and [16].

It is also assumed that the two Hydrogen atoms are moving at the same speed, and in the same direction, hence

$$u \cdot v = v^2. \tag{3.72}$$

From $\mathbf{F}_N = \mathbf{F}_{EM}$, we have

$$-G\; \frac{m_1\; m_2}{s^2}\; \hat{\mathbf{d}}_1 \;=\; -\frac{\mu_0}{4\pi}\; \left[\frac{ev}{s} \right]^2\; \hat{\mathbf{d}}_1. \tag{3.73}$$

We replaced G_N with G, just to able to compare because our G is not really a constant, as equation (4.80) below suggests.

IV. DETERMINING v

It is assumed that the electron and proton are held together to form the hydrogen atom by some force $\mathbf{F}_{ep}$, which could be described through Coulomb potential or the Yukawa potential.

A. Condition condusive for easy evaluation of G

The special case in which

$$\frac{m_e \mathbf{F}_p - m_p \mathbf{F}_e}{m_e + m_p} = 0, \tag{4.74}$$

reduces the equation to

$$\mu \frac{d^2 \mathbf{r}}{dt^2} = \mathbf{F}_{pe}. \tag{4.75}$$

That is the case when the two Hydrogen atoms are far apart, because the gravitation diminishes with distance.

B. The internal force $\mathbf{F}_{ep} = -\mathbf{F}_{pe}$

Here $\mathbf{F}_{ep}$ is given by Coulomb's law:

$$F_{ep} \;=\; -\frac{1}{4\pi\epsilon_0} \left[\frac{e}{s} \right]^2. \tag{4.76}$$

That is, to find u and hence G, we have to solve the system constituted by

$$\frac{m_p m_e}{m_p + m_e} \frac{d^2 s}{dt^2} \;=\; -\frac{1}{4\pi\epsilon_0} \left[\frac{e}{s} \right]^2, \tag{4.77}$$

$$v \;=\; \frac{ds}{dt}, \tag{4.78}$$

and

$$G \;=\; \frac{\mu_0}{4\pi}\; \left[\frac{ev}{m_H} \right]^2. \tag{4.79}$$

These can be put together to give

$$\begin{aligned} G_{tt} - \left(\frac{2\pi}{\mu_0} \right) \frac{G_t^2}{v} \\ +\; \frac{\mu_0}{2\pi} \left(\frac{e}{m_H} \right)^2 \left(\frac{e^2}{4\pi\epsilon} \frac{m_p + m_e}{m_p m_e} \right) \frac{1}{v} = 0, \end{aligned} \tag{4.80}$$

with $G = G(t, s, v)$.

One other condition that G has to satisfy is that gravity fields travel a the speed of light. That is,

$$v_{ss} = c^2 v_{tt}, \tag{4.81}$$

where c is the speed of light.

The equation (4.77) solves into

$$\frac{v^2}{2} = A + \frac{\mu}{s}. \tag{4.82}$$

It simplifies further into

$$s = B\cos(ft) - \frac{\mu}{A}, \tag{4.83}$$

or

$$v = -Bf\sin(ft), \tag{4.84}$$

giving

$$v_{tt} = -f^2 v, \tag{4.85}$$

with

$$f = -\frac{2A^3}{\mu^2}, \tag{4.86}$$

where

$$A = \frac{v^2}{2} - \frac{\mu}{s_0}, \tag{4.87}$$

and s_0 represents the ground state of the Hydrogen atom.

Substituting v_{ss} and v_{tt} into (4.81) leads to the quintic equation (5.91).

V. A Lie group symmetrical solution

After a lengthy analysis, one of the symmetries of the equation leads to a simple solution of the form

$$G = G_N \cos(ft), \tag{5.88}$$

with f given by

$$f \; = \; \sqrt{\frac{\left(\frac{v^2}{2} - \frac{\mu}{s_0}\right)^3}{\mu^2}}, \tag{5.89}$$

and the gravitational constant G_N given by

$$G_N = \frac{i}{u\ Z_H \left(-K_e\frac{(e^-)^2}{9}\right)^2}\left(2K_v\frac{(e^-)^2}{9}\left(\frac{v^2}{2} - \frac{\left(-K_e\frac{(e^-)^2}{9}\right)}{r_0}\right)^3\right)\times \left(\frac{v^2\left(-K_e\frac{(e^-)^2}{9}\right)^2}{2\left(\frac{v^2}{2} - \frac{\left(-K_e\frac{(e^-)^2}{9}\right)}{r_0}\right)^3} + \left(s_0 + \frac{e^{\frac{i\pi}{3}}\left(-K_e\frac{(e^-)^2}{9}\right)}{\frac{v^2}{2} - \frac{\left(-K_e\frac{(e^-)^2}{9}\right)}{s_0}}\right)^2\right). \tag{5.90}$$

The parameter v is given by the quintic equation in v^2:

$$\alpha_5 v^{10} + \alpha_4 v^8 + \alpha_3 v^6 + \alpha_2 v^4 + \alpha_1 v^2 + \alpha_0 = 0. \tag{5.91}$$

Its parameters are given by

$$\begin{aligned}
\alpha_0 &= -4\mu^4,\\
\alpha_1 &= 8s_0\mu^3,\\
\alpha_2 &= -8c^2 s_0\mu^3,\\
\alpha_3 &= 12c^2 s_0^2\mu^2,\\
\alpha_4 &= -6c^2 s_0^3\mu,\\
\alpha_5 &= c^2 s_0^4\mu,
\end{aligned}$$

and $\mu = -K_e(e^-)^2/9$. with the physical parameters

$$\begin{aligned}
G &= 6.67259 * 10^{-11}\text{Nm}^2/\text{kg}^2,\\
&\quad \text{gravitational constant from experiment},\\
s_0 &= \frac{1}{2}(0.52917720859) * \text{Å},\\
&\quad \text{Bohr's atom size},\\
s_0 &= \frac{1}{2}(0.528) * \text{Å},\\
&\quad \text{hypothetical atom size},\\
e^- &= 1.602176487 * 10^{-19}\text{C},\\
&\quad \text{charge of an electron},\\
K_v &= \frac{\mu_0}{4\pi} = 1 * 10^{-7}\text{Wb/(A m)},\\
&\quad \text{permeability of free space},\\
K_e &= 8.987551787 * 10^9\text{N/C}^2,\\
&\quad \text{permittivity of free space},\\
c &= 2.99792458 * 10^8\text{m/s},\\
&\quad \text{speed of light in vacuum},\\
Z_H &= 1.000794,\\
&\quad \text{mass number of an Hydrogen atom},\\
u &= 1.660538782 * 10^{-27}\text{kg},\\
&\quad \text{atomic mass unit.}
\end{aligned} \tag{5.92}$$

A. A note on the quintic equation (5.91)

The tenth-order equation (5.91) is essentially a quintic in $V = v^2$, and is said to be unsolvable, algebraicly, otherwise it should have five roots. Lagrange gave up trying to solve the quintic, because every time he did, it evolved into another of a much higher degree. A closer look at the analysis reveals some fascinating observations.

Equation (5.91) can be solved numerically. Setting $v^2 = V$ transforms it into a proper quintic equation, say with solutions/roots V_1, V_2, V_3, V_4, V_5. One then expects the relation $v^2 = V \times 10^p$ to lead to an equation with solutions $V_i \times 10^p,\ i = 1, 2, 3, 4, 5$. But there is a value of p_0 at which the solutions suddenly change. The realisation is that one set of five solutions is valid for $p \leq p_0$, another for $p > p_0$.

The paragraph above suggests that there could be ten roots for a single quintic equation. Exploring this line of thought using other mathematical tools suggests that they are more forty. This means Lagrange was on the right track. He gave up too soon.

B. A calculated G

Solution of the quintic through numerical techniques yields ten roots. It is only two of them that lead to the correct answer. Bohr's atom size gives

$$v = \pm 6.89696\ i * 10^{-10} m/s, \tag{5.93}$$

from which the gravitational constant is found to be

$$G_N \ = \ 6.656 * 10^{-11}\text{Nm}^2/\text{kg}^2, \tag{5.94}$$

a 99.8% accuracy.

Our calculations suggest a Hydrogen atom size slightly larger by about 0.2%(what we refer to as the hypothetical atom size, above). It led to

$$v = \pm 6.90451\ i * 10^{-10} m/s, \tag{5.95}$$

from which the gravitational constant is found to be

$$G_N \ = \ 6.671 * 10^{-11}\text{Nm}^2/\text{kg}^2, \tag{5.96}$$

a value that compares more favorably with the experimental result at 99.98%.

The same effect can be realized by rounding off the Bohr obtained v to $\pm 6.9\ i * 10^{-10} m/s$, thus leaving Bohr's model intact.

VI. Applications

The formula (5.89) leads to the frequency $17.2Hz$, which we believe is the gravitational frequency for the Hydrogen atom. Other frequencies could be possible through the forces suggested in (3.71), the $17.2Hz$ has appeared in practise.

While studying the electromagnetic radiation in and around the Snowfru pyramids, Khavroshkin and Tsyplakov [17] noticed a peak around the $17Hz$ frequency. They attributed it to an external influence, that being the local power plant, generating electricity at $50Hz$. Their proposition was that the $17Hz$ frequency is a subharmonic of this frequency, meaning $17Hz \approx 50Hz/3$. Subsequently, they concluded that the frequencies falling within the interval $(16.5 - 17.2)Hz$ can be traced to the frequency attained by the electricity generators at the Aswan hydroelectric power plant, as water passed through the turbines.

Another chance detection of the frequency $17.2Hz$ was by Rakhmanov. This happened while studying the dynamics of mirrors in laser interferometric gravitational wave detectors [18], an antenna that will/is be suspended above earth, and will pass over rivers, dams, lakes and oceans. He explained the origin of the frequency as being indirectly induced by the electronics. Apparently, this frequency has a dampening effect on the motion of the mirrors.

This frequency is also encountered regularly in motions of ships, and has been accepted as the natural frequency of the masts. The cause is attributed to the engines. This example, together with the other two, are an empirical confirmations of the association of gravitational waves with ordinary mass.

A. The gravitational frequency of the element Platinum

The frequencies of heavier elements like Platinum, Gold and Uranium, on the other hand, are easier to measure and validate, unfortunately they involve too many calculations because of the many subatomic particles. Each leads to a many body problem.

The plot provided in Figure 1 is that of Platinum and its isotopes. It is in abundance all around us, be it in jewelery and vehicle exhaust systems. One only has to stand next to a traffic light and see Platinum and Gold peak on a spectrum analyzer, as cars go by. The Uranium peaks when standing next to mine dumps from mines that used to dig it up.

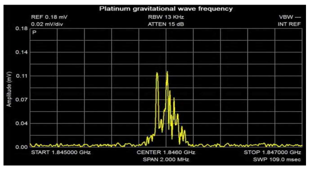

Fig. 1. A picture of Platinum frequency captured with a Signal Hound USB-SA44B spectrum analyzer, emanating from a catalytic converter. The calculated frequency in $1.88GHz$. One of the isotopes in the picture measured at $1.85GHz$.

VII. Discussion and conclusions

The objective of this work was to determine a formula for the gravitational constant G. This we did with 99.98% accuracy. The formula is in (5.90) and the calculated value in (5.96). The accuracy can however be improved through a quantum mechanical approach. We used Newton's mechanics.

There were serendipitous results. First, a frequency associated with Newton's gravitational force was observed. For the element Hydrogen, it turned out to be $17.2Hz$, thus answering the question for us on why this frequency is always observed in places where water is in close proximity. Unfortunately, those who observed it did not associate it with water or Hydrogen. They had their own explanations. This prompted us to seek an independent justification. We calculated the frequency for the element Platinum, and succeeded in observing it through a spectrum analyzer, presented in Figure 1(The calculations are not presented here, of course). We chose this element on the basis that it is so common, especially in the part of the world the authors of this contribution are.

Choosing $u = v$ in (3.72) was the right decision to take. Equation (3.70) then led to an attraction, thus agreeing with Newton's gravitational attraction law. The case $u = -v$ should lead to a repulsion, probably observable at astronomical scales. An astronomical object, say a galaxy, moving in a countering direction to others, should experience a repelling force from them. The three forces in (3.71) would then generate toques in the body, that would then proceed to turn it around. Thus raising the dark matter phenomena.

There was a third result. The use of Lie's symmetry group theoretical methods to solve differential equations analytically has not waned in more than a century, thanks to the practitioners in pure Mathematics, and analytical tools were absolutely necessary in this study. But as anyone in the physical sciences can attest, one has to be extra cautious when using this procedure. Some choose to stray away from it. We thus had to come up with a modification to the theory, given in (2.39), and it worked for us: We got a formula for G.

The final result regards Lagrange's work on the quintic equation, because our work led to an equation of this form. Pursuing his thoughts would have assisted us in removing the complex value i from the velocity v in (5.93) and (5.95). Unfortunately, this requires that a number of theorems in the Theory Of Equations be ignored; a very dangerous terrain to traverse.

Conflict of Interests

The author declares that there is no conflict of interest regarding the publication of this article.

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