The Dual Aspects of Accounting Transactions and Asset Value Change in the Accounting Equation

FERNANDO JUÁREZ
Escuela de Administración
Universidad del Rosario
Autopista Norte, No. 200
COLOMBIA
fernando_juarez2@yahoo.com

Abstract: - The purpose of the study is to analyze the accounting equation and the relation between assets and claims on the assets (liabilities plus stockholders' equity), based on the dual aspects of accounting transactions. The methodology is rationalistic and analytical. The analysis consists of the application of the identity and characteristic functions, as well as a coordinate transformation. Results show that the accounting equation does not consist in a single addition, what would lead to inequality, but a series of addition functions when taking into account the dual aspects of accounting transactions. Due to the different number of summation dimensions on each side of the final equation, a coordinate transformation is applied resulting in a change in the value of assets; this value is not the same as that of the initial accounting equation.

Key-Words: Dual aspects, accounting transactions, accounting equation, balance sheet.

1 Introduction

This paper addresses the issue of the equality of assets to claim on the assets, in the balance sheet of financial statements. The idea that assets are equal to liabilities plus stockholders´ equity (claim on the assets) is crucial in financial statements. The bases for this idea are the dual aspect of the accounting transactions, the double-entry bookkeeping, and the accounting equation.

An accounting transactions must be recorded in two accounts with different signs in a double classification system [1], i.e. the recording of the same monetary units in two accounts with opposite signs. That is the recognition of its duality; every single transaction has two different properties, such as being a debit and a credit simultaneously. In accounting practice, the double-entry bookkeeping system is the way of registering the accounting transactions in financial statements, based on their dual aspects. Finally, the accounting equation is a checkpoint of the balance sheet correspondence with the accounting assumption and practice.

Some approaches to financial statements try to provide a different perspective, such as quantum accounting (see [2], [3]), and triple-entry bookkeeping ([4], [5]); however, they do not put into question either the dual aspect of accounting transactions or the accounting equation. Another way of depicting the financial reality of business is by fair value accounting; this method introduces a more comprehensive analysis of the accounting information [6]. It is, somehow, a critic of the dual aspects of accounting transactions, but without making it explicit. Due to the different assets and claims on the assets structures, another research proposed an inequality as the proper relationship between assets and claims on the assets [7].

The previous findings stir some issues that deserve attention.

2 Problem Formulation

The assets and claims on the assets sides of the accounting equation have different item structures; they do not comprise the same accounts. However, they have the same monetary units, by the dual aspects of the economic transactions.

It, therefore, seems comprehensible to review what the relationship between assets and claims on the assets is, taking into account the identity of the monetary units, which are located on both sides of the accounting equation.

2.1 Methodology

This research uses a rationalistic and analytical method. It is not based on empirical data but in the analysis of theoretical assumptions. It focuses on the accounting equation analysis using its basic mathematical operation. An identity function accounts for the dual aspect of accounting transactions. A characteristic function relates both sides of the equation identifying those monetary units that are equal. These functions, as well as the rest of the analysis, take claims on the assets as the range and assets as the domain. A reformulation of the accounting equation is made; finally, a coordinate transformation shows the change in value in assets between the two systems that come up in the reformulated accounting equation. These coordinate systems are the sides of the equation.

The type of accounting valuation method assumed in the analysis is book value method.

3 Problem Solution

2.1 The dual aspects of accounting transactions

This analysis will only take the lowest level accounts, i.e. those that have monetary units u_i with no aggregation into higher order accounts. Accordingly, the claims on the assets side of the equation includes accounts C_i (accounts with monetary units u_{ic}) and the assets side of the equation contains accounts A_i (assets accounts with monetary units u_{ia}).

It must be pointed out that claims on the assets do not comprise the typical aggregated liabilities and stockholder´s equity items or any other aggregation account, but only the lowest level items located beneath them. Doing it in this way does not change the result and provides a clearer explanation.

The accounts A_i and C_i are already sorted, because their sequence in balance sheet follows national or international standards. Thus, a correspondence exists between the natural numbers set $\mathbb{N}$ and the accounts A_i and C_i; we can assign serial numbers A_1, A_2, … A_n, and C_1, C_2, …. C_n to these accounts. To the purposes of this research, the monetary units located in the accounts do not need to follow any order.

To the analysis, the value of the monetary unit is irrelevant; it can be the legal tender or any other, as long as it remains the same for all of the assets and claims on the assets accounts.

Every monetary unit u_{ia} in an account A_i, is simultaneously located in an account C_i as u_{ic}, but still it is the same monetary unit, so $u_{ia} = u_{ic}$. Accordingly, there must be a function relating the monetary units u_{ia} of the accounts A_i with the monetary units u_{ic} of the accounts C_i, based on this characteristic. Due to the dual aspect of accounting transactions assumption and the unicity of the monetary units, an identity function f must exist for every monetary unit u_{ia} of all A_i and u_{ic} of all C_i. In fact, the accounting equation requires assets being equal to claims on the assets, and its mathematical expression is $A = C$, with C being liabilities (L) plus stockholders´equity (E), $C = L + E$.

The identity function can take two directions. The first one takes claims on the assets as the domain and assets as the range, F: $C \rightarrow A$. The second one takes assets as the domain and claims on the assets as the range, F: $A \rightarrow C$. None of this direction is the most relevant, and the duality assumption does not privilege any direction. To the purpose of this research, the analysis will assume the function F: $C \rightarrow A$.

2.2 The dual aspects of accounting transactions and the inequality between assets and claims on the assets

The function relating claims on the assets and assets takes each element u_{ci} of every C_i as the first component, and each element u_{ai} of every A_i as the second component of the pair (u_{ci}, u_{ai}). To every u_{ci} corresponds an identity image u_{ai} and $f(u_{ci}) = u_{ai}$, and $u_{ci} = u_{ai}$; accordingly, it exists a unique monetary unit u_i, named u_{ci} in C_i and u_{ai} in A_i, located on a claim on the asset and asset accounts simultaneously. That is in accordance with the dual aspects of accounting transactions, so the identity function denotes that a monetary unit is equal to itself despite its location at different and opposite places.

Additionally, the monetary units u_{ci} of a single C_i are distributed in several A_i, i.e. the range of the function f for a C_i is not in a single A_i. That is so because financial statements do not classify the monetary units by identity groups, but by accounts. The classification is different on both sides of the equation because assets and claims on the assets have different accounts. Therefore, to contain all the images of a C_i, f must be a family of functions for that C_i, each function having some (but not all) of the monetary units u_{ci} of C_i (domain) and some (but not all) of the monetary units u_{ai} in an A_i (range). To collect all the identity images of all u_{ci} for every C_i requires many functions.

To get some monetary units in each function, the analysis uses a characteristic function 1_{Ci}. For every C_i and A_i the characteristic function 1_{Ci} assigns ‘1’ to the u_{ai} of A_i, which are identity images of an u_{ci} located in C_i, and ‘0’ to that that is not. Then, they are multiplied by the value of u_{ai} of A_i, which is the value of the monetary unit in the financial statements. The full expression including the characteristic function and its multiplication by u_{ai} is

$u_{ai}(1_{C_i}(u_{ci})) = u_{ai}\ (1 \mid u_{ci} = u_{ai};\ 0 \mid u_{ci} \neq u_{ai})$ (1)

Since the images of all of the monetary units of each C_i are in several A_i, each C_i range, comprising u_{a1}, u_{a2}, u_{a3} … u_{an}, is located in several A_i. It is uncommon to find one A_i with all the images of a domain C_i. The domains and the ranges show no one-to-one correspondence.

Let us put an example; given a C_i with n u_{ci} monetary units, it might exist an A_1 with m_1 u_{ai} and another A_2 with m_2 u_{ai} monetary units. Part of the A_1 monetary units are an image of some monetary units of C_1 and part of the monetary units of A_2 are an image of some monetary units of C_1. Based on this different ranges, the C_i n u_{ci} monetary units are divided into n_1 and n_2, each one corresponding to each range in A_1 and A_2, with $n = n_1 + n_2$. However, A_1 has more monetary units than those that are an image of some of the monetary units of C_i and, then, the monetary units of A_1 are $m_1 = m_{1c} + m_{1x}$, with m_1 being images of some monetary units of C_i and m_{1x} being no images of C_i. Similarly, A_2 has more monetary units than those that are an image of some of the monetary units of C_i and, then, the monetary units of A_2 are $m_2 = m_{2c} + m_{2x}$, with m_{2c} being images of some monetary units of C_i and m_{2x} being no images C_i. In this way, the images of the monetary units of C_i spread over several partial ranges (m_{1c} of A_1 and m_{2c} of A_2).

Now, instead of adding all of the asset values and all of the claims on the asset values, what would lead to the standard accounting equation, one can add domains on one side of the equation and ranges on the other side; it means to add by the identity of monetary units.

The linear accumulation of a single domain C_i with n monetary units is:

$$SC_i = \sum_{i=1}^{n} u_{ci} \quad (2)$$

To every C_i there are several A_i with partial ranges of the function f so we can choose the first A_i, in the order they are arranged in the balance sheet; its linear accumulation is:

$$SA_i = \sum_{i=1}^{n} u_{ai} \quad (3)$$

It must be noted that all the monetary units of A_i that are not images of C_i were removed in this A_i by the characteristic function in (1), what results in $SA_i < SC_i$ for each A_i, C_i. That is the reason why SC_i is not equal to SA_i, the addition of the same monetary units is not possible when considering a C_i and a sole A_i for that C_i, because other ranges A_i for the domain C_i would require other functions f to have them added.

Extending the previous computation for a C_i to all the n C_i domains with m elements in each domain, their sum SC_T is:

$$SC_T = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{cij} \quad (4)$$

Applying again one single function f that takes every domain C_i and one single range A_i, and not all of them, for every domain C_i, the sum SA_p is:

$$SA_p = \sum_{h=1}^{k} \sum_{i=1}^{\exists! n} \sum_{j=1}^{\exists! m} u_{ahij} \quad (5)$$

with k = sequence number for C_i; n = sequence number for A_i; m = number of images in a A_i of some of the monetary units in a C_i; $\exists! m$ = a unique range in a unique A_i for a C_i exists and is found, and $\exists! n$ = the function f takes a single range A_i for every domain C_i. As it happened with a single C_i in (2) and (3), it results in the following inequality:

$$A_s \neq C_s \quad (6)$$

$$\sum_{h=1}^{k} \sum_{i=1}^{\exists! n} \sum_{j=1}^{\exists! m} u_{ahij} \neq \sum_{i=1}^{n} \sum_{j=1}^{m} u_{cij} \quad (7)$$

This formula is an inequality because the images of every C_i are distributed in partial ranges A_i and a run of the function f only takes one range in an A_i for each C_i. In doing so, the accounting equation in its standard formulation do not reflect the real relationship between assets and claims on the assets, which is, actually, an inequality. The standard equation is:

$A_s \neq C_s$ (8)

$C_s = L + E$ (9)

$A_s \neq L + E$ (10)

Typically, $A_s < C_s$ because ranges are restricted to those obtained in a unique f for each C_i. To be both sides equal, the range that was obtained should be multiply by a coefficient, to artificially increase its value.

2.2 The Assets Value Transformation in Coordinate Systems

First of all, it must be noted that it is not possible to obtain the total value of assets by adding all A_i in the usual form:

$$A_s = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ahij} \tag{11}$$

with n = sequence number for A_i; m = number of images of any C_i in a A_i. That is so because the addition based on the dual aspects of accounting transactions must follow the financial statements classification, and its corresponding partition in function domains and ranges, and not to add all the ranges of different functions in one run. The addition of the ranges (it is a procedure with several runs) must stop in every run once the function finds a partial range.

To get together all the m images of each one of the n ranges for a particular C_i, the procedure must be run n times, resulting in many partial functions. In this way, the procedure picks up all the images for a C_i. The recursive procedure to get all the A_i images for all the C_i is:

$$A_s = \sum_{h=1}^{k} \sum_{i=1}^{\exists n} \sum_{j=1}^{\exists m} u_{ahij} \tag{12}$$

with k = sequence number for C_i; n = sequence number for A_i; m = number of images in an A_i of the monetary units in a C_i; $\exists m$ = a range in a A_i for a C_i exists and is found, and $\exists n$ = the function f takes, if it exists, a range in every A_i for every domain C_i. It must be noted the difference of this equation with (5), where the equation states that a *unique* range exists in an A_i, while in (12) it states that a partial range exists in many A_i. The latter allows for running the procedure multiple times for every C_i until all its ranges are found.

Finally, the equation:

$$A_s = C_s \tag{13}$$

results in:

$$\sum_{h=1}^{k} \sum_{i=1}^{\exists n} \sum_{j=1}^{\exists m} u_{ahij} = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{cij} \tag{14}$$

One can take both sides of the equation as coordinate systems with three (asset side) and two (claims on the asset side) dimensions, and transform the three summation terms on the left side into two summation terms to make both coordinate systems equal. To this, the computation must introduce scaling parameters s_i and c_i, slope and constant respectively. The dimensional systems have a common axis for both terms, the axis C_i, which can be taken as the y-axis in the three-dimensional system. Then, it is needed to project the values in the axis x (a_x) and z (a_z) of the three-dimensional system onto the axis x (a_x) and y (a_y) in the new two-dimensional system. The equations must be linear, like the accounting equation simple calculation, and they are in the form:

$$\begin{aligned} a_x &= s_x a_x + c_x \\ a_y &= s_z a_z + c_z \end{aligned} \tag{15}$$

Let us take the axis x (a_x), in the three dimension system, as the partition of all the A_i by image groups of every C_i; the axis z (a_z) is the monetary units u_{ahij}, in that system too. Once the y axis (C_i accounts) is removed, and substituting in (15) the computations for each monetary unit are:

$$\begin{aligned} A_x &= s_x A_i + c_x \\ u_{ay} &= s_z\, u_{ahij} + c_z \end{aligned} \tag{16}$$

Each function f: $C_i \rightarrow A_i$ creates a set of parameters (s_x, c_x, s_z, c_z) for some monetary units of the domain C_i, and the range found in an A_i. All monetary units in each function have the same computation parameters, because they belong to the same partial function f: $C_i \rightarrow A_i$. The scaling parameters for the new axis A_x, in the two dimension system, are s_x and c_x; they are position transformations in an ordered sequence of ranges, starting with the range 1 in an A_i for the first domain C_i, and ending with the last range in an A_i for the last domain C_n. It is important to note that this transformation does not change the value of the monetary units but the position of the domains A_i; however, it will have an impact when aggregating accounts.

Also, the new axis u_{ay} is the value of the monetary units of each domain A_i in the new system, and it is a transformation of the original value u_{ahij} in the three-dimensional system. The parameters s_z and c_z are not the same as those for A_x; their value come from the fact that the monetary units in an A_i could be images of different C_i, and to collect them involves several functions for each C_i. Accordingly, the values of s_z and c_z must depend on each C_i function, and all of them could be different.

Accordingly, the accounting equation is as follows:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}u_{cij} = \sum_{i=1}^{n_1}\sum_{j=1}^{m_1}u_{aij} + \cdots + \sum_{i=1}^{n_n}\sum_{j=1}^{m_n}u_{aij} \tag{17}$$

with n = sequence number for C_i; m = monetary units u_c in each C_i,; n_1 = sequence number for A_i ranges in the new system; m_1 = monetary units u_a in each A_i range in the new system.

Substituting the monetary units in the new system for their coordinate transformation

$$\sum_{i=1}^{n}\sum_{j=1}^{m}u_{cij} = \sum_{i=1}^{n_1}\sum_{j=1}^{m_1}(s_z u_{ahij} + c_z)_{ij} + \cdots + \sum_{i=1}^{n_n}\sum_{j=1}^{m_n}(s_z u_{ahij} + c_z)_{ij} \tag{18}$$

with n = sequence number for C_i,; m = monetary units u_c in each C_i,; n_1 = sequence number of A_i in the new two coordinate system; m_1 = monetary units u_a in each A_i in the new two coordinate system; u_{ahij} the monetary unit value in the old three coordinate system, and s_z and c_z the slope and constant transformation parameter values.

The s_z and c_z parameter values need a definition for each group of monetary unit images A_i in the three coordinate system; they depends on the range of every C_i for which they are images. This computation results in a change of the asset values; as claims on the assets do not change their value, to keep the equality with the new terms, the asset value must change. Some of the assets can increase their value and others can decrease it; however, a transformation from a three coordinate system to a two coordinate system cannot keep the same scale.

4 Conclusion

By introducing the simply mathematical computations of the accounting equation combined with a coordinate transformation, the results confirm that the accounting equation, based on the dual aspects of accounting transactions, is more than just an addition, but a sequence of functions. That is of great relevance.

These mathematical equations, even they are just a successive layers of computations, are intended to be the basis for more complex mathematical analysis.

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