

Visualization of graphs of functions of one and two variables

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Abstract: The paper deals with the problem of how to create graphs of different functions with the help of different types of software. Discussed are both graphs of functions of one variable and functions of two variables. As for the software, following four types are discussed: a) applets free of charge, b) special commercial software, c) widely used commercial software, d) software free of charge. For comparison, positives and negatives of each of them are mentioned in each section.

Key-Words: graph, function of one variable, function of two variables, Excel, Mathematica, teaching

1 Introduction

The use of computers in education is now very common.

Also in teaching mathematics different software is used for visualization and computing ([7]). Computers are used yet in basic courses of algebra ([8]), precalculus ([6]), and calculus ([1], [9]).

This article focuses on the use of computers in teaching mathematics, especially displaying graphs of functions. The term *function* is rather theoretic, on the contrary the graph of the function may be very concrete. Often speaking about a function, one thinks about its graph. Also describing the properties of the function can be done very well by displaying the graph of that function.

Visualization of graphs helps to understand functions and their properties. The teacher may use the computer from simple presentations of static slides to much deeper work with interactive programs. Students will certainly appreciate interactive and user-friendly applications.

Let us discuss four options of how to display graphs of the functions.

2 Free programs and applets

Probably the simplest way how to draw a graph of the function of one variable is to type the formula e.g. x^2 for $y = x^2$ into the search box of a common web browser like Microsoft Edge, Internet Explorer, Mozilla Firefox etc. The result is shown in Fig.1. There are also some interactive buttons (depending on the version of the browser), which serve for zooming or shifting the graph. To see the graph of another function, one must type again the

appropriate formula. This may be quite inconvenient, especially if the role of a parameter should be studied. E.g. how does the parameter c influences the graph of the given function $y = cx^2$?

There are dozens of similar on-line applets as well as programs, which are free of charge and may be run on-line or download and use. Many of them offer displaying of more than one graph and also have more interactive buttons (e.g.[2]).

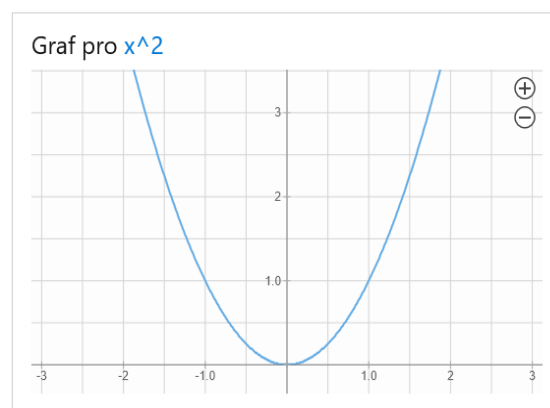


Fig.1 – Graph of the function $y = x^2$ created by the Google search engine in a common internet browser

The advantage of such applications is that the user sees the result immediately. On the other hand, there are usually no possibilities to adjust the output. However, some applications offer limited changes like coloring the curve or changing the gridlines etc.

Displaying the graph of the function of two variables by typing the formula into the browser's search box is not possible. To see a 3D graph it is necessary to use programs or applets, which allow it. There are many such applications and, again,

many of them are free of charge. Fig.2 shows a graph of the rotational paraboloid $z = x^2 + y^2$ as an output of Math3D program, which is offered free of charge [3].

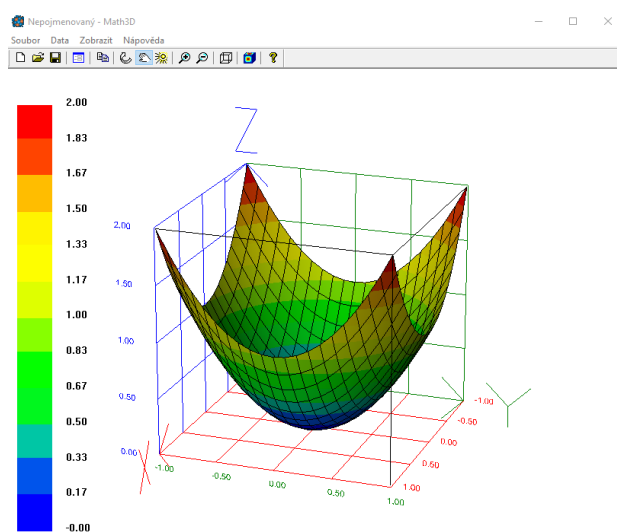


Fig.2 – Graph of the rotational paraboloid $z = x^2 + y^2$ created in Math3D

On the toolbar, there are different buttons for some settings and other options. In addition, by dragging the graph one may shift or rotate it. To change the scale, there are appropriate input fields and checkboxes on a panel where also the functional formula and other options could be entered or chosen (not shown in Fig.2).

3 Commercial software

For more accurate work with graphs, i.e. not only just display them, but also explore their properties, it is certainly better to use some commercial software. The user creates a program on his own, according to his demands. Hence, he is not limited by the possibilities of an applet. The main disadvantage could be the price. On the other hand, the license is usually paid by the academic institution, e.g. university. Or, one may use a software, which is free of charge. These possibilities mentioned above will be discussed in following paragraphs.

3.1 Special software

Software, like Mathematica, Matlab etc. are designed especially for mathematical computing and visualizing. As for the Mathematica, the license is quite expensive and time limited. On the other hand, the possibilities and utilization of the software are wide. Of course, the user has to know how to use

the software, which includes typing codes. However, the output is well-arranged and user-friendly.

In the Fig.3 there is an example of an interactive graph created in Mathematica. It not only shows the graph of the “basic” function $y = \sin x$, (solid line) but also, the influence of the parameters a, b, c, d , which can be changed by the user, on the graph of the function $y = a.\sin(b.x + c) + d$ can be explored. The dashed line shows the function $y = 3\sin(2x + \pi/2) - 1$.

Setting values of the parameters a, b, c, d , to 1, 1, 0, 0 respectively leads to the function $y = \sin x$. These values are set by default, so after launching the application, the user sees only one curve.

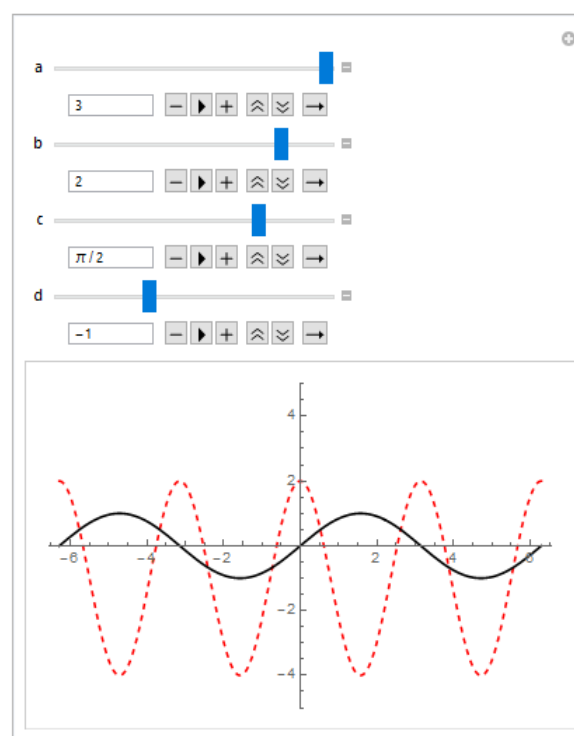


Fig.3 – Graphs of the functions $y = \sin x$, (solid) and $y = 3.\sin(2x + \pi/2) - 1$ (dashed) in Mathematica

Similar program can be easily created for the function of two variables. Fig.4 shows the graphs of the functions of $z = 0$ (plane) and $z = 3x^2 - 3y^2$ (hyperbolic paraboloid). Here the user may explore how parameters p and q change the graph of the function $z = px^2 + qy^2$. For both positive values the graph results to elliptic paraboloid, but for one parameter positive and the other negative, the graph turns out to hyperbolic paraboloid. As for the other function, it is a simple plane $z = r$, which is parallel to the xy plane. Changing the constant r results to shifting the plane vertically without changing its inclination. The intersections of the function of two

variables with such planes are so called horizontal cuts. Together with the intersections of the function with the planes parallel to the xz and yz planes these cuts are usually used when the graph of the function of two variables should be quickly sketched.

The program codes in Mathematica for displays of the graphs like in Fig.3 and Fig.4 are very short. Just few commands suffice to create them.

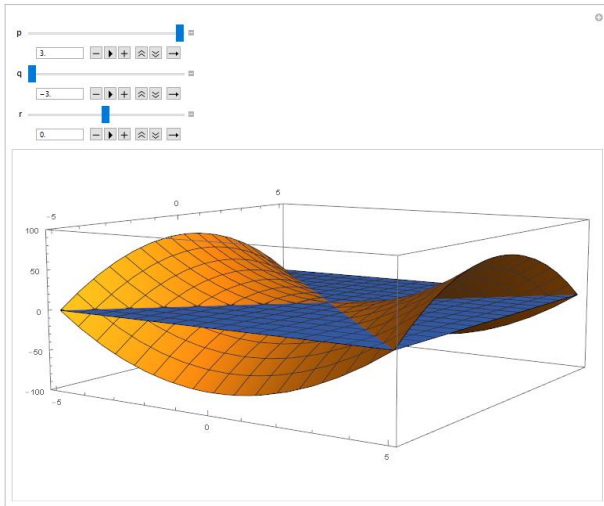


Fig.4 – Graphs of the functions $z = 0$ (plane) and $z = 3x^2 - 3y^2$ (hyperbolic paraboloid) in Mathematica

Another advantage of Mathematica is its elaborate help documentation with detailed description and many executable examples to each command. Of course, a lot of printed books and brochures regarding Mathematica are available. One may start e.g. with [4].

3.2 Widely used software

Another possibility of how to draw a graph using a software is to use a commercial commonly used spreadsheet calculators like Excel from the MS Office or LibreOffice Calc from the Open office packages.

To draw a graph of a function of one variable in such an environment, at first a table of x and y values must be prepared. Although it is nothing difficult – typing a formula of a desired function and dragging mouse through several cells to create x and y values, it is quite clumsy. In the Fig.6A a sample for the function $y = x^2$ is displayed. Secondly, the two columns have to be selected and finally an appropriate graph from the Insert→Graphs menu should be chosen.

Certainly, for the mere displaying of the graph one would prefer something more quick and convenient (e.g. a browser as described above in the

section 2). However, there is a possibility to explore the graph with Excel similarly as shown with Mathematica. The way, how to do it, is rather complicated in comparison with Mathematica; on the other hand, Excel is accessible for every common user.

In Mathematica, the interactive elements are created automatically as a part of the output of some commands. In Excel, the user has to insert them. On the Developer tab, in the Control group by clicking on Insert and under the ActiveX controls choosing Scroll Bar, also called a slider.

As the minimal value is always 0 and the increment must be integer with the minimal value of 1, the user must transform these values to the values which every parameter requires. The output of a slider is bounded with a cell, so one needs to transform the value of this cell. If an interval $[A, B]$ should be transformed to the interval $[a, b]$ with the same number of values given by the increment, then the value h from the interval $[a, b]$ equals to:

$$h = \frac{b-a}{B-A} \cdot H + \frac{aB-Ab}{B-A} \tag{1}$$

where H is the corresponding value from the interval $[A, B]$, i.e. the value from the cell corresponding to the value of the slider.

E.g. if the value of a parameter needed for the graph of a function should be from the interval $[-5, 5]$ with an increment of 0.1, then the values of the slider should be set from 0 to 100. Hence, every integer H from the interval $[0, 100]$ will be transformed to one value from the interval $[-5, 5]$ according to (1) in a following way:

$$h = \frac{5-(-5)}{100-0} \cdot H + \frac{-5 \cdot 100 - 0 \cdot 5}{100-0} = 0.1H - 5$$

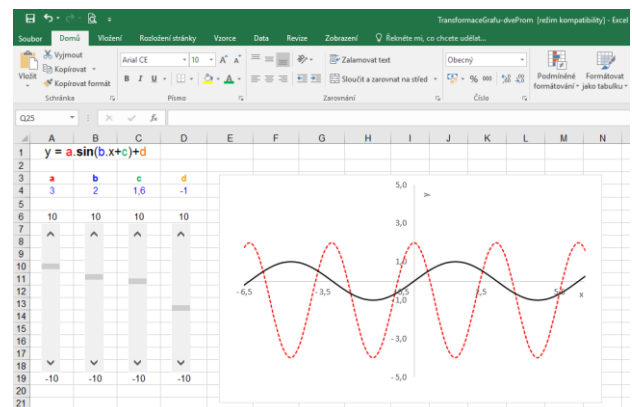


Fig.5 – Graphs of the functions $y = \sin x$, (solid) and $y = 3 \cdot \sin(2x + \pi/2) - 1$ (dashed) in Excel

In the Fig.5 there is an example of an interactive graph created in Excel.

Similarly to the same graph created in Mathematica and described in the section 3.1, it shows two graphs. One of them is the graph of the “basic” function $y = \sin x$, (solid line) and the other belongs to the function of $y = 3.\sin(2x + \pi/2) - 1$ (dashed line in Fig.5).

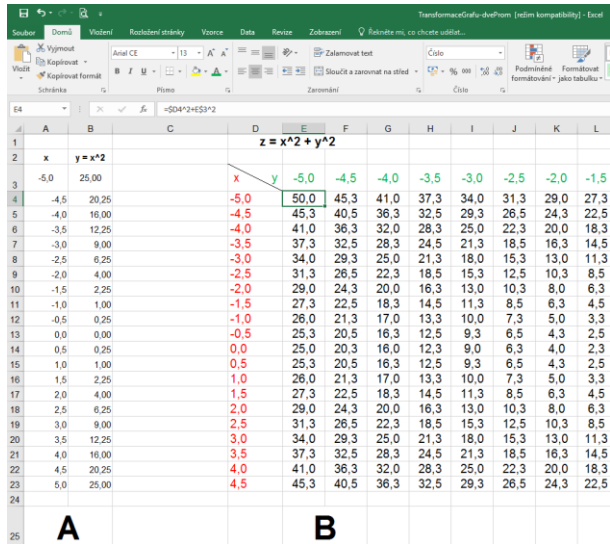


Fig.6 – Tables of values of functions of one variable (A) and of two variables (B) in Excel

As for the function of two variables, the use of the sliders is similar, but the preparation of the values is rather complicated. In comparison to the function of one variable, where only a simple table of two columns representing x and y values must be prepared (Fig.6A), for the function of two variables a matrix of values should be created (Fig.6B).

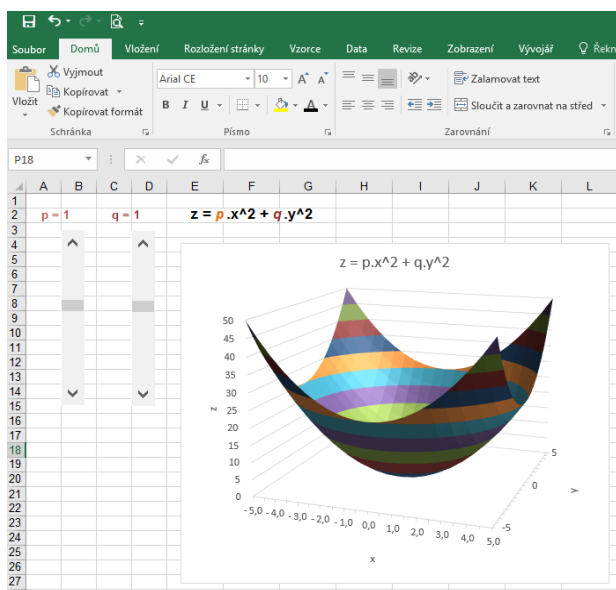


Fig.7 – Graph of the function $z = x^2 + y^2$ in Excel

In the Fig.6B a part of such a matrix is displayed. In order not to type a formula of a function into every single cell, a mixed cell reference is useful. It is seen in the formula bar in the Fig.6.

To explore the influence of some parameters on the graph of the function (in this case $z = px^2 + qy^2$), the user must prepare sliders for the values of the parameters, transform and assign their values to the desired parameters (now p and q) and redefine the formulae of the function in the matrix.

After that, to create a graph, the area of the matrix should be selected and then a 3D graph (surface chart) inserted. The result is seen in the Fig.7.

3.3 Software free of charge

Lastly, a software free of charge will be mentioned in this paragraph. The software GeoGebra [5] is shown as an example. This product may be downloaded on a computer, but the latest version is available also on-line.

The work with GeoGebra is very user-friendly and intuitive. The user may start with few icons, but there is a variety of commands. Their syntax as well as more detailed description can be found in an on-line help manual.

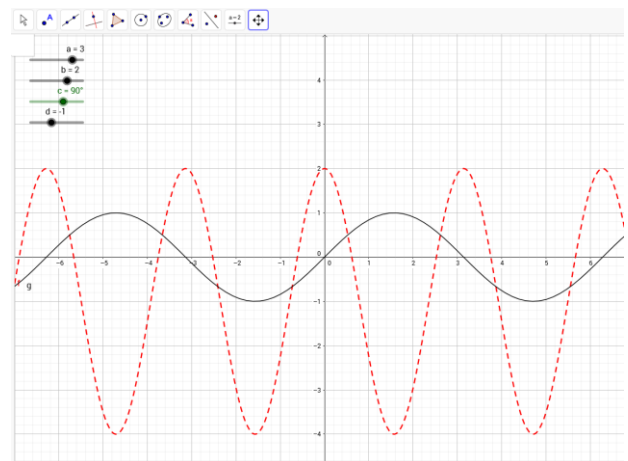


Fig.8 – Graphs of the functions $y = \sin x$, (solid) and $y = 3.\sin(2x + \pi/2) - 1$ (dashed) in GeoGebra

Depending on the choice, the environment for 2D graphs or 3D graphs should be open. The user can create sliders at first and then use their values in the formula for the desired function.

Fig.8 shows again the influence of several parameters on the graph of the function $y = a.\sin(b.x + c) + d$. While the graph of the function $y = \sin x$ remains unchanged (solid line), the graph of the function $y = a.\sin(b.x + c) + d$

changes (dashed line) as the user changes the values of the four parameters using the sliders in the top left corner as shown in the Fig.8.

In contrast to Excel, the preparation of sliders in GeoGebra is easier. It is because GeoGebra allows the minimal value of a slider to be also negative and the value of the increment can also be a decimal number. That is very convenient, because the user may directly set the desired values and no transformation of values is needed.

Fig.9 shows the graph of the function of two variables created in GeoGebra. The graph of the function $z = px^2 + qy^2$ depends on two parameters p and q , so, in order to change their values, two sliders are available.

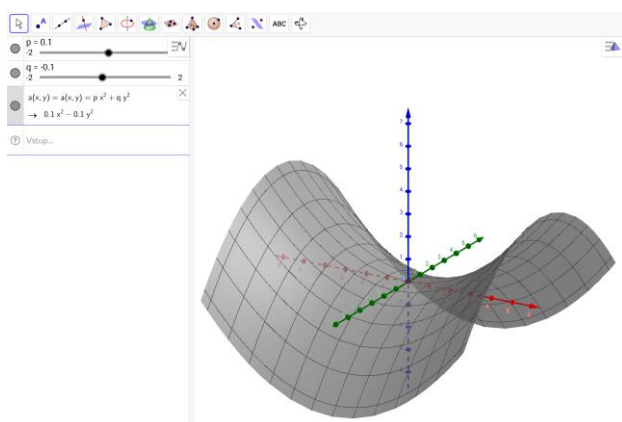


Fig.9 – Graph of the function $0.1x^2 - 0.1y^2$ (hyperbolic paraboloid) in GeoGebra

Similarly to Mathematica and Excel, in GeoGebra it is possible to shift and rotate the graph and it is also available to change some of its properties, like color, shading, opacity etc.

4 Conclusion

The article provides an overview of some possibilities to display the graph of a function of one variable or of two variables. Of course, it is impossible to count all possibilities, so four main types were discussed, namely: applets free of

charge, special commercial software, widely used commercial software and software free of charge.

The aim of the article was to point out the available software, certainly not to rate which one is better or worse. Some software enables to create the graph very quickly with no previous knowledge of programming. Some other requires programming skills. On the other hand, the output may also vary from a static curve to an interactive colorful picture.

The choice of which product to use depends on the user and circumstances. To see the graph just because to have an idea of the shape or trend, probably the simple applet will be used. Should the properties of the function be explored in more detail, then, certainly, the use of a more sophisticated software will be considered.

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