Estimation of Limit Cycles and Signal Stabilization with Deadbeat approach in three Dimensional Nonlinear systems

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Abstract: - The present work narrates a clear logical approach to estimating Limit Cycles (LC) in 3×3 Nonlinear systems. The estimation of LC is done employing graphical method assuming harmonic balance approximation and are validated by computer simulations. The graphical method is developed using both computer graphics and geometric tools. The computer simulations are done by developing a suitable program with MATLAB code and also using the SIMULINK Toolbox of MATLAB software. Considering the structure of 3×3 nonlinear systems is a bit complex, the estimation of LC is done considering the frequency of LC remains the same at every point of the loop. Once the LC is predicted/detected in an autonomous state, the investigation explores the quenching of the oscillation at high frequency, ten times higher than the frequency of LC applied at the input node, which is normally termed a Signal Stabilization. The process of Signal Stabilization is a type of response which exhibits both transients and steady states. Of course, with the proper amplitude of the dither signal, the synchronizing frequency of the output should be the frequency of the dither signal at the steady state. However, the Signal Stabilization process is made faster and, in minimum time, the steady-state synchronizing value is realized without the transient and any ripples at steady state by a discrete signal which is termed as deadbeat approach to response. In this article, the Signal Stabilization with deadbeat approach has been explored analytically and is validated by computer simulations.

Key-Words: - Limit Cycles, 3×3 Nonlinear Systems, Signal Stabilizations, Dead Beat response, Describing functions

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1 Introduction

Exhibition of Limit Cycles in single-input and single-output (SISO) nonlinear (NL) systems has been considered to be the basic characteristics of instability [1-5]. For the last six decades, researchers have been focusing on the investigation of LC in 2×2 nonlinear systems [6-48]. In the estimation and analysis of LC in SISO/2×2 nonlinear systems, the use of describing functions (DFs) has gained much importance and is well established [4, 5, 10, 13, 16, 23, 29, 33, 49, 50, 51]. Some researchers [52, 53, 54, 55, 56, 57] have reported their observations of the exhibition of LC in flow and thermodynamics, in cell models, in the dynamic model where the NL system switches between a stable LC and the stable equilibrium point, in an autocatalytic system, in Biological Oscillators and in natural system respectively.

However, little literature is available where the prediction /investigation of LC for 3×3 nonlinear systems has been discussed [6, 38, 40, 41, 42, 58, 59].

In the case of occurrence/exhibition of LC, it is essential to quench the Limit Cycling oscillations. Signal stabilization is the most reliable and established method [5, 30, 31, 46, 49, 50] to quench the Limit Cycling oscillations. Signal stabilization has also been used in [5, 30, 31, 46, 49, 50] to quench such oscillations in the 2×2 nonlinear systems as well as in the 3×3 systems [58, 59].

It has been observed that during the process of signal stabilization, the synchronization causes the transients to have overshoots, undershoots and even some ripples [58]. In such a case, it is proposed to achieve a near-instantaneous response to a change in the reference signal, effectively eliminating any overshoot or settling time, resulting in a very fast and precise output with minimal error and ripples in the synchronization.

The work is presented in the following sequences:

Section 1, as an introduction, covers the literature survey, section 2 presents the graphical method of estimation of LC, which has been substantiated by computer simulation, section 3 discusses the signal stabilization with deterministic signals, section 4 proposes the Signal Stabilization with deadbeat approach, which aims at achieving deadbeat synchronization using discrete or digital controller [60]. This has been illustrated through examples. Section 5 ends with a conclusion.

2 Estimation of LC in 3x3 Nonlinear Systems

The analytical expression of prediction of LC is complex, involved and cumbersome [6,48,49]. Graphical method is preferred for determination LC which offers insight into the problem and such simpler for visualization.

2.1 Graphical Method:

The normalization phase diagram method is followed as has been used in [46] and [6] for 2x2 and 3x3 nonlinear systems respectively.

A class of 3x3 nonlinear system is considered as shown in Fig.1.

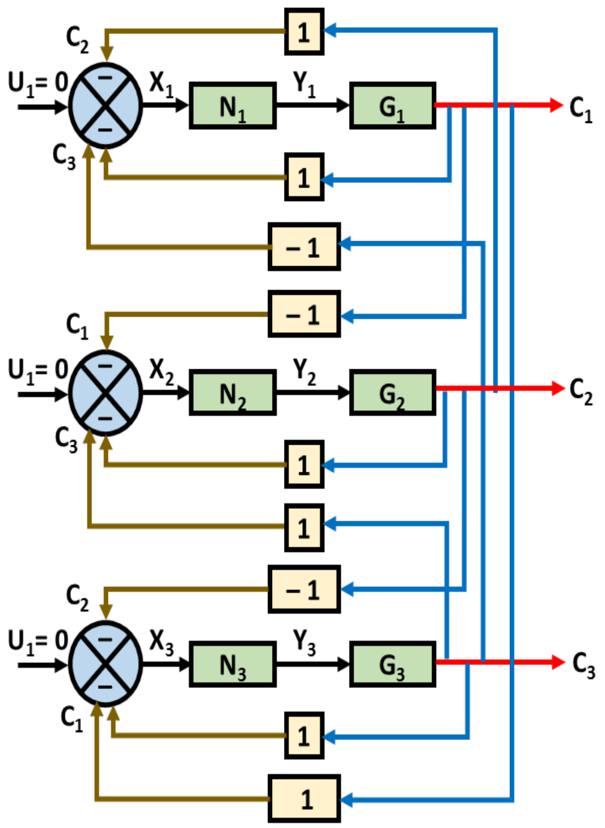


Fig. 1: A class of 3×3 multivariable nonlinear system

The three groups of normalized phase diagrams [6] are drawn as:

Group1: In subsystem S_1, S_2 & S_3: C_1 & C_2 are negative but C_3 is positive

Group2: In subsystem S_1, S_2 & S_3: C_2 & C_3 are negative but C_1 is positive

Group3: In subsystem S_1, S_2 & S_3: C_3 & C_2 are negative but C_2 is positive

Example 1 and 2: The linear elements in both the Examples are same i.e. $G_1(s) = \frac{2}{s(s+1)^2}$; $G_2(s) = \frac{1}{s(s+4)}$: $G_3(s) = \frac{1}{s(s+2)}$ and Nonlinear elements are shown Fig2(a) and Fig2(b) used in Ex. 1 and Ex. 2 respectively.

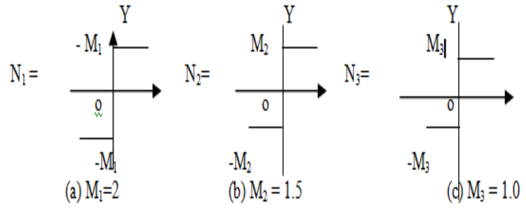


Fig. 2(a): Relay used in Ex. 1

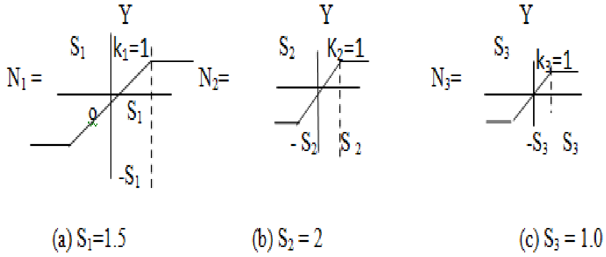


Fig. 2(b): Saturation NL elements used with slope k_1, k_2, k_3 in Ex. 2

In both the examples, non-memory NL elements are used. Considering the harmonic linearization concept, these NL elements are replaced by their respective DFs [28]. For non-memory elements, the DFs are real functions and it is not contributing to any phase angles. Only the transfer functions are complex functions of s (the Laplace operator).

It is to be noted that in frequency response, only sinusoidal input and steady state output are considered, so that in the analysis & is replaced by jω [6]. N_1, N_2 & N_3 are also absolute values of DFs representing the NL element subsystems S_1, S_2 & S_3 respectively. G_1, G_2 & G_3 are the absolute values of transfer functions representing the linear elements of subsystems S_1, S_2 & S_3 respectively. X_1, X_2 & X_3 are taken as the amplitude of the concerned sinusoidal inputs to the NL elements. C_1, C_2 & C_3 are taken also the amplitudes of sinusoidal outputs of subsystems S_1, S_2 & S_3 respectively.

For Ex. 1, the different quantities with different value of ω, are calculated and shown in Table 1.

$\theta L_1 = Arg.\ (G_1 cj\omega) = -90^0 - 2\tan^{-1}(\omega)$

$\theta L_2 = Arg.\ (G_2 cj\omega) = -90^0 - \tan^{-1}(\frac{\omega}{4})$

$\theta L_3 = Arg.\ (G_3 cj\omega) = -90^0 - \tan^{-1}(\frac{\omega}{2})$

$$N_2 = (11\text{-}3\omega^2)\omega^2 \pm \sqrt{(11-3\omega^2)\omega^4 - 8(\omega^2+16)(1-\omega^2)\omega^2} \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$N_1 = \frac{\omega^2-1}{8} N_2 + \frac{9\omega^2-\omega^4}{8} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{X_1}{X_2} = \frac{(1+\omega^2)\sqrt{\omega^2+16-2N_2)+N_2^2}}{2N_1\sqrt{\omega^2+16}} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\frac{X_1}{X_2} = \frac{BD\ i}{ADi} = \sqrt{\frac{(1-u_i)^2}{(1-u_i)^2+(u_i)^2}} \quad \ldots\ldots\ldots\ldots\ldots(4)$$

With a particular frequency, ω three groups of subsystems 1,2 & 3. The Normalized Phase Diagrams (NPD) are presented in figure 3(a), 3(b) & 3(c) respectively. Any group out of three is used for determination of LC and any other values as desired.

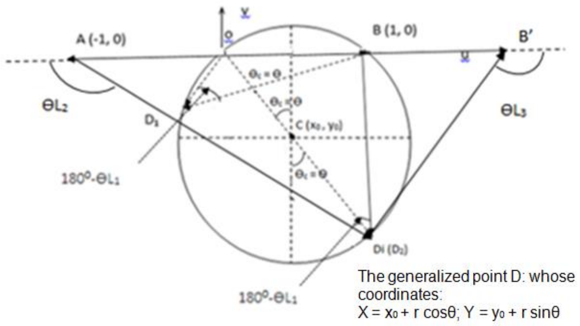


Fig. 3(a): NPD with C_1, C_2 & C_3 in the Group 1, where C_1 & C_2 are negative but C_3 is positive

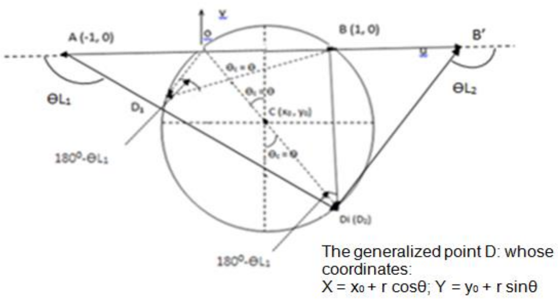


Fig.3(b): NPD with C_1, C_2 & C_3 in the Group 2, where C_2 & C_3 are negative but C_1 is positive

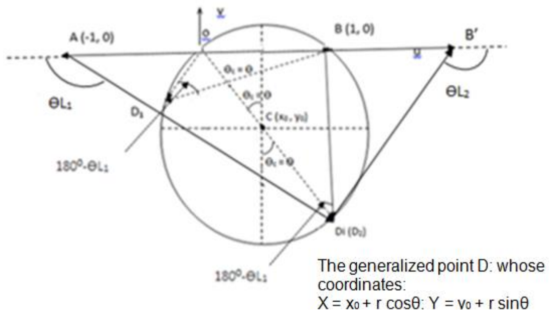


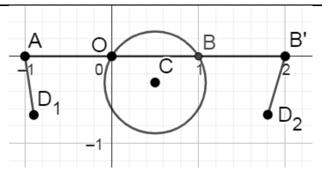
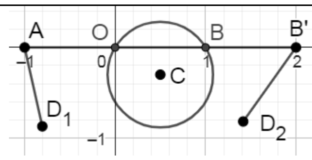
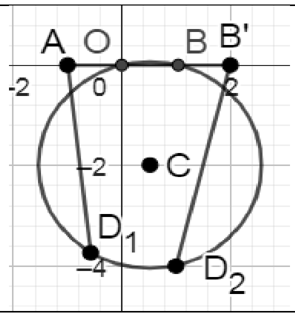
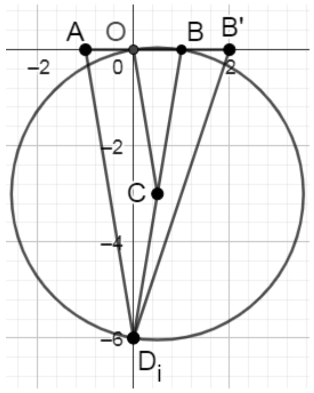
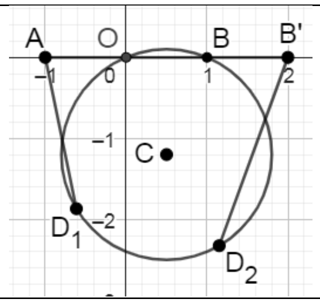
Fig. 3(c): NPD with C_1, C_2 & C_3 in the Group 3, C_3 & C_2 are negative but C_2 is positive

In the light of Fig. 3(a),(b) & (c) quenching in Group 1 depicts: X_2 traces a line drawn at an angle θ_{L2} with the C_2 (C_2 = - R_1) and X_3 traces a line drawn at an angle θ_{L3}. The intersections of two lines and the circle drawn having radius, r = ½ sin θ_{L1} would represent possible self-sustained oscillations (LC). Similar case would happen with Groups 2 & 3.

In order to estimate the LC if Group 1 is considered for which Table 1 shows: θ_{L1}, θ_{L2}, θ_{L3}, r =½ sin θ_{L1}, centre of the circle, C (1/2, -1/2tanθ_{L1}), and the intersecting point of the two lines and the circle conforming to a particular value of frequency ω for Ex. 1. For ω = 0.701,$\frac{X_1}{X_2}$, calculated from Equations 3 and 4 are matched which confirms the frequency of LC is 0.70 and other quantities of interest be read from the NPD.

Table 1: Shows the θ_{L1}, θ_{L2}, θ_{L3}, r=½ sin θ_{L1}, and the intersection points of the two lines with the circles for Group 1 related to Ex. 1.

Table 1: Phase diagrams for different ω and its resulting values of r for example 1 (Rectangular Hysteresis) using graphical methods

ω (rad/sec)	**θ_{L1} (degree)**	**θ_{L2} (degree)**	**θ_{L3} (degree)**	**$r=\frac{1}{2}\sin\theta_{L1}$ units**	**X_1/X_2 from eqn. 3**	**X_1/X_2 from eqn. 4**	**NPD**	**Remarks**
0.60	-151.9	-98.53	-106.70	- 0.55	-	-		The concerned lines and circle are not intersecting.
0.650	-156.05	-99.23	-108	0.58	-	-		The concerned lines and circle are not intersecting.
0.700	-159.98	-99.926	-109.29	-2.12	-	-		The concerned lines and circle are not intersecting.
0.701	**-160.06**	**-99.94**	**-109.32**	**-3.13**	**1.0**	**1.02 (matched)**	 D (1,2)	The concerned lines and circle are intersecting at D(1,2) confirming the occurrence of LC, ω=0.701 rad/sec, X1=BD2=6.0 units X2=AD2=6.0 units X3=B'D2= 6.3 units C1 = OD2 = 6 units C2= C3=1unit
0.750	-163.74	-100.62	-110.56	-1.35	-	-		The concerned lines and circle are not intersecting.

2.2 Digital Simulation

For Ex. 1 and Ex. 2, a MATLAB code is developed to validate the results obtained with the graphical method [6].

Fig 4(a) shows the canonical from of Fig. 1 for the Ex. 1 & 2.

Fig 4(b) shows the digital equivalent for Ex. 1 & 2.

Ex. 1 & Ex. 2: A 3×3 system shown in Fig.1 has three NL elements as represented in Fig.2(a) and Fig.2(b) for the respective Ex.1 & Ex.2. Also the three linear transfer functions are given by $G_1(s) = \frac{2}{s(s+1)^2}$; $G_2(s) = \frac{2}{s(s+4)}$ *and* $G_3(s) = \frac{1}{s(s+2)}$.

Partial Fraction Expansion of $G_1(s)$, $G_2(s)$ and $G_3(s)$:

$$G_1(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2} = \frac{A\,(s+1)^2 + Bs(s+1) + Cs}{s(s+1)^2}$$

$$Or\ \frac{s^2(A+B) + s\,(2A+B+C) + A}{s\,(s+1)^2} = \frac{2}{s(s+1)^2}$$

Or A=2; B=C= -2;

$$\text{Hence } G_1(s) = \frac{2}{s} - \frac{2}{s+1} - \frac{2}{(s+1)^2}$$
$$: \frac{2}{s}, \frac{-2}{s+1}, \frac{-2}{s+1}\left(\frac{1}{s+1}\right)$$

$$G_2(s) = \frac{1}{s(s+4)} = \frac{A}{s} + \frac{B}{(s+4)} = \frac{4A + s(A+B)}{s(s+4)}$$

$$Or\ 4A = 1: A = \frac{1}{4}, A + B = 0: B = -A = -\frac{1}{4}$$

$$\text{Hence } G_2(s) = \frac{0.25}{s} - \frac{0.25}{s+4}$$

$$G_3(s) = \frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} = \frac{A(s+2) + Bs}{s(s+2)} = \frac{2A + (B+A)s}{s(s+2)}$$

$$\text{Or } 2A = 1: A = \frac{1}{2}, A + B = 0: B = -A = -\frac{1}{2}.$$

$$\text{Hence } G_3(s) = \frac{0.5}{s} - \frac{0.5}{s+2}$$

If, sampling period, T is infinitesimally small, TG(z) ≈ G(s).

Z[G)s]:

$$G_1(s): \frac{2}{s} \Rightarrow \frac{2z}{z-1}; \frac{-2}{s+1} \Rightarrow \frac{-2z}{z-e^{-T}};$$

$$\frac{-2}{(s+1)^2} \Rightarrow \frac{-2Tz\,e^{-T}}{(z-e^{-T})^2}$$

$$G_2(s): \frac{0.25}{s} \Rightarrow \frac{0.25z}{(z-1)}; \frac{-0.25}{s+4} \Rightarrow \frac{-0.25z}{z-e^{-4T}};$$

$$G_3(s): \frac{0.5}{s} \Rightarrow \frac{0.5z}{(z-1)}; \frac{0.5}{s+2} \Rightarrow \frac{-0.5z}{z-e^{-2T}}$$

From the Fig. 4(a) and 4(b) following algorithm has been derived:

$$(1)\ \frac{OW1(z)}{Y_1(z)} = \frac{2z}{z-1} \Rightarrow 2Y_1(z) = \frac{z-1}{z} OW1(z) = OW1(z) - z^{-1}OW1(z)$$

Z^{-1} [OW1(nT)] = 2Y1 (nT) + OW1($\overline{n-1}$T)

$$(2)\ \frac{OW2(z)}{Y_1(z)} = \frac{-2z}{z-e^{-T}} \Rightarrow -2Y_1(z) = \frac{z-e^{-T}}{z} OW2(z) = OW2(z) - z^{-1}e^{-T}OW2(z)$$

Z^{-1} [OW2(nT)] = -2Y1 (nT) +e^{-T} OW2($\overline{n-1}$T)

$$(3)\ \frac{OW3(z)}{Y_1(z)} = \frac{-2Tze^{-T}}{(z-e^{-T})^2} \Rightarrow -2Tze^{-T}Y_1(z) = \frac{(z-e^{-T})^2}{z} OW3(z) = z^*OW3(z) - 2e^{-T}\,OW3(z) + e^{-2T}z^{-1}OW3(z)$$

Or -2T$e^{-T}z^{-1}Y_1$(z)=OW3 (z) -2$e^{-T}z^{-1}$OW3 (z) + $e^{-2T}z^{-2}$ OW3 (z)

Z^{-1} [OW3(nT)] = - 2T$e^{-T}Y_1(\overline{n-1}$T)+2e^{-T} OW3($\overline{n-1}$T) - e^{-2T}OW3 ($\overline{n-2}$T)

(4) $\frac{TU1(z)}{Y_2(z)} = \frac{0.25z}{(z-1)} \Rightarrow 0.25\, Y_2(z) = \frac{z-1}{z} TU1(z) - z^{-1} TU1(z)$

Z^{-1}[TU1(nT)] = 0.25Y_2 (nT) +TU1($\overline{n-1}$T)

(5) $\frac{TU2(z)}{Y_2(z)} = \frac{-0.25z}{(z-\mathrm{e}^{-4\mathrm{T}})} \Rightarrow -0.25\, Y_2(z) = \frac{z-\mathrm{e}^{-4\mathrm{T}}}{z} TU2(z) = TU2(z) - z^{-1}\mathrm{e}^{-4\mathrm{T}} TU2(z)$

Z^{-1}[TU2(nT)] = -0.25Y_2 (nT) + $\mathrm{e}^{-4\mathrm{T}}$TU2($\overline{n-1}$T)

(6) $\frac{TV1(z)}{Y_3(z)} = \frac{0.5z}{(z-1)} \Rightarrow 0.5\, Y_3(z) = \frac{z-1}{z} TV1(z) = TV1(z) - z^{-1} TV1(z)$

Z^{-1} [TV1 (nT)] =0.5Y_3 (n T) +TV1($\overline{n-1}$T)

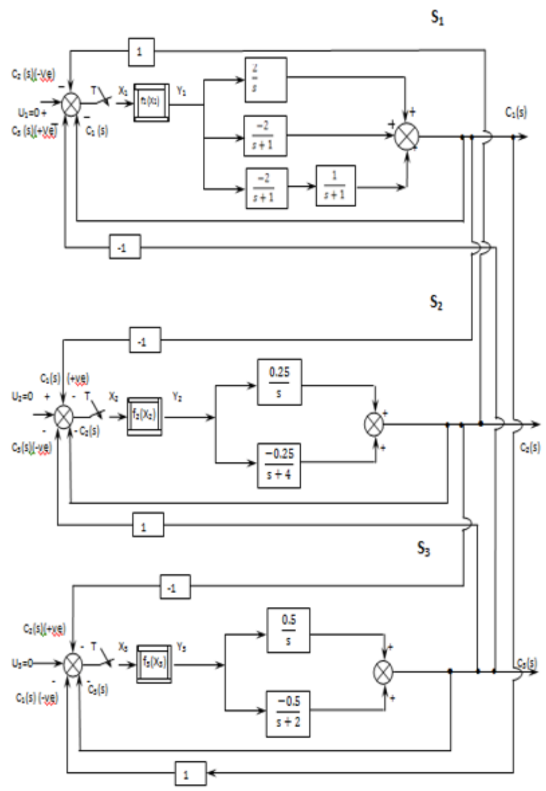


Fig. 4(a): Canonical form of Fig. 1 for Ex.1 & 2

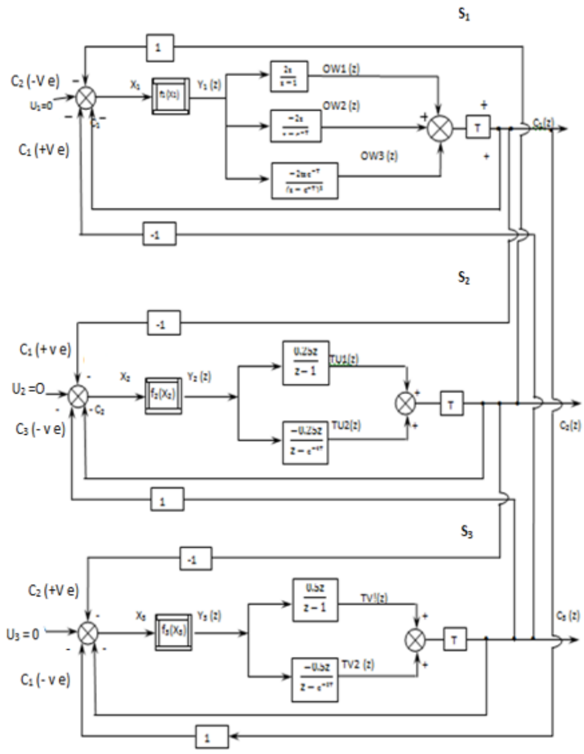


Fig. 4(b): The Digital equivalent of Fig. 1 for Ex. 1 & 2

(7) $\frac{TV2(z)}{Y_3(z)} = \frac{-0.5z}{(z-\mathrm{e}^{-2\mathrm{T}})} \Rightarrow -0.5\, Y_3(z) = \frac{z-\mathrm{e}^{-2\mathrm{T}}}{z} TV2(z) = TV2(z) - z^{-1} * AK2 * TV2(z)$

Z^{-1}[TV2(nT)] = -0.5Y_3 (n T) + AK2* TV2($\overline{n-1}$T)

Let us take($\overline{n-1}$T) is the zeroth instant; nT is the first instant, so we can write:

OW1($\overline{n-1}$T) = OW1Nϕ⇒OW1N; OW1 (n T) = OW1N1; OW2($\overline{n-1}$T) = OW2Nϕ⇒OW2N; OW2 (n T) = OW2N1

OW3($\overline{n-2}$T) = OW3N (-1)⇒OW3NN; OW3 ($\overline{n-1}$T) = OW3Nϕ⇒ OW3N ; OW3 (nT)=OW3N1

Now C_1 (nT) = OWN1 = T∗ [OW1N1 + OW2N1 + OW3N1] = T∗ [OW1 (nT) +OW2 (nT) + OW3 (nT)]

=T∗[2Y_1 (nT) + OW1N-2Y_1 (nT)+ AK∗OW2N– 2∗T∗ AK1 ∗ OY1N + 2∗AK1∗OW3N - AK2∗OW3 NN] =OWN1=C_1

Similarly,

TU1($\overline{n-1}$T) = TU1Nϕ $\Rightarrow$ TU1N; TU1 (nT) = TU1N1, TU2($\overline{n-1}$T) = TU2Nϕ= TU2N; TU2 (nT) = TU2N1

Now C_2 (nT) = TUN1 = T*[TU1N1+TU2N1] =T*[TU1 (nT) + TU2 (nT)] = T* [0.25 Y_2 (n T) + TU1N – 0.25Y_2 (nT) + AK3$*$TU2N] = TUN1 = C_2

Similarly,

TV1($\overline{n-1}$T) = TV1Nϕ $\Rightarrow$ TV1N; TV1 (n T) = TV1N1

TV2($\overline{n-1}$T) = TV2Nϕ $\Rightarrow$ TV2N; TV2 (n T) = TV2N1

Now C_3 (n T) = TVN1 = T*[TV1N1 + TV2N1] = T*[TV1 (n T) + TV2 (n T)]

= T*[0.5Y3 (n T) + TV1N-0.5Y3 (n T) +AK2$*$ TV2N] = TVN1 = C_3

Next Run:

R_1=ORN1=C_3 – C_2 =TVN1 – TUN1

R_2 = TRN1 = C_1 – C_3 = OWN1 – TVN1

R_3 = THRN1= C_2 – C_1 = TUN1 – OWN1

X_1 = OXN1 = ORN1 – OWN1, OYN1 = OF (OXN1); X_2 = TXN1 = TRN1 – TUN1, TYN1 = TF (TXN1)

X_3 = THXN1 = THRN1 – TVN1, THYN1 = THF (THXN1)

Using the above algorithm, a suitable program has been developed with MATLAB code. The results of which in image form shown in Fig.6 (a) and 6(b) for Ex. 1 and 2 respectively. The numerical results are shown in Table 2a and 2b for Ex. 1 and 2 respectively.

The numerical results are presented in Table 2(a) and Table 2(b) for Ex. 1 and 2 respectively.

2.3 Application of SIMULINK Toolbox of MATLAB:

Ex. 1 and 2 are revisited again.

Fig.5 (a) and (b) represent the simulation diagram for use of SIMULINK to investigate corresponding to the Ex. 1 & 2 respectively.

The SIMULINK Toolbox is used to determine X_1, X_2, X_3, C_1, C_2 & C_3 for both the Ex. 1 and 2 and the results are compared with the graphical method and digital simulation.

Fig. 5(a) and 5(b) represents the simulation diagram using SIMULINK Tool Box for prediction of LC in case of Ex. 1 and 2 respectively.

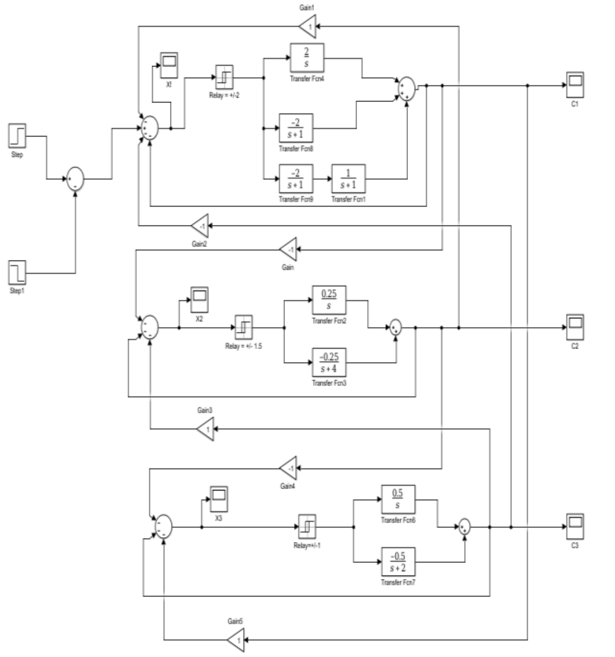


Fig. 5(a): Simulation block using SIMULINK for getting the solution of the Ex. 1

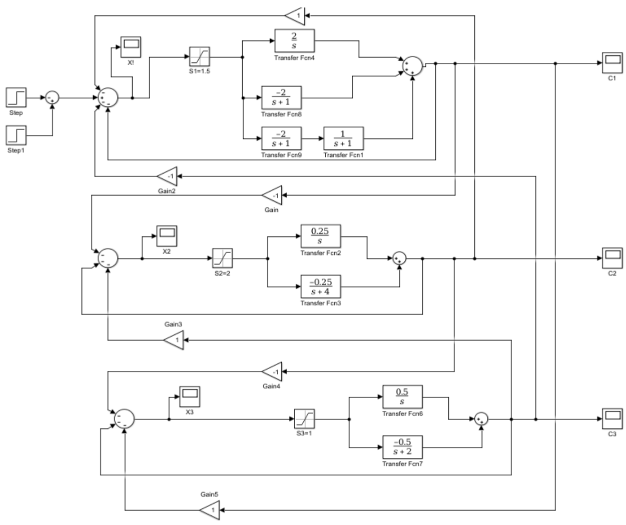


Fig. 5(b): Simulation block using SIMULINK for getting the solution of the Ex. 2.

The results thus obtained are entered in Tab. 2 (a) & Tab. 2 (b) for Ex. 1 and 2 respectively.

2.4 Comparison of Results

Figure 6(a) and 6(b) show the Images (Results) obtained from digital simulation (developed programme) and that of using SIMULINK tool box for Ex. 1 and 2 respectively.

Table 2(a): Results obtained using different methods for Ex.1 (Ideal Relay).

Sl. No.	Method	C_1	C_2	C_3	X_1	X_2	X_3	ω
1	Graphical	6.0	1.0	1.0	6.0	6.0	6.3	0.70
2	Digital Simulation	4.83	0.74	0.95	4.72	4.91	5.23	0.70
3	Using SIMULINK TOOL BOX OF MATLAB	5.95	1.01	0.96	4.84	5.12	5.62	0.70

Table.2(b): Results obtained using different methods for Ex.2 (Saturation)

Sl. No	Method	C_1	C_2	C_3	X_1	X_2	X_3	ω
1	Digital Simulation	4.345	1.06	1.06	4.464	4.581	4.762	0.628
2	Use of SIMULINK TOOL BOX OF MATLAB	4.30	1.05	1.05	4.425	4.534	4.74	0.6283

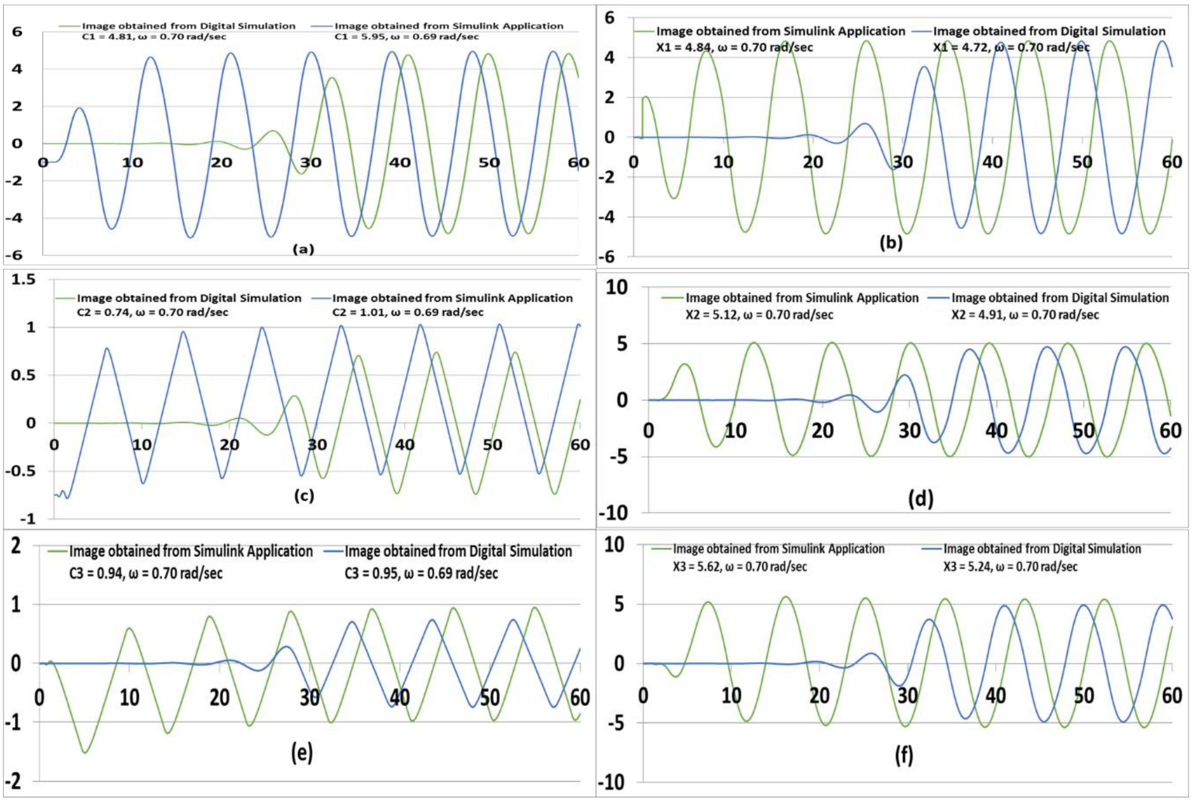


Fig. 6 (a): Images obtained using Digital Simulation and SIMULINK for C_1, C_2, C_3, X_1, X_2, X_3 of Ex. 1.

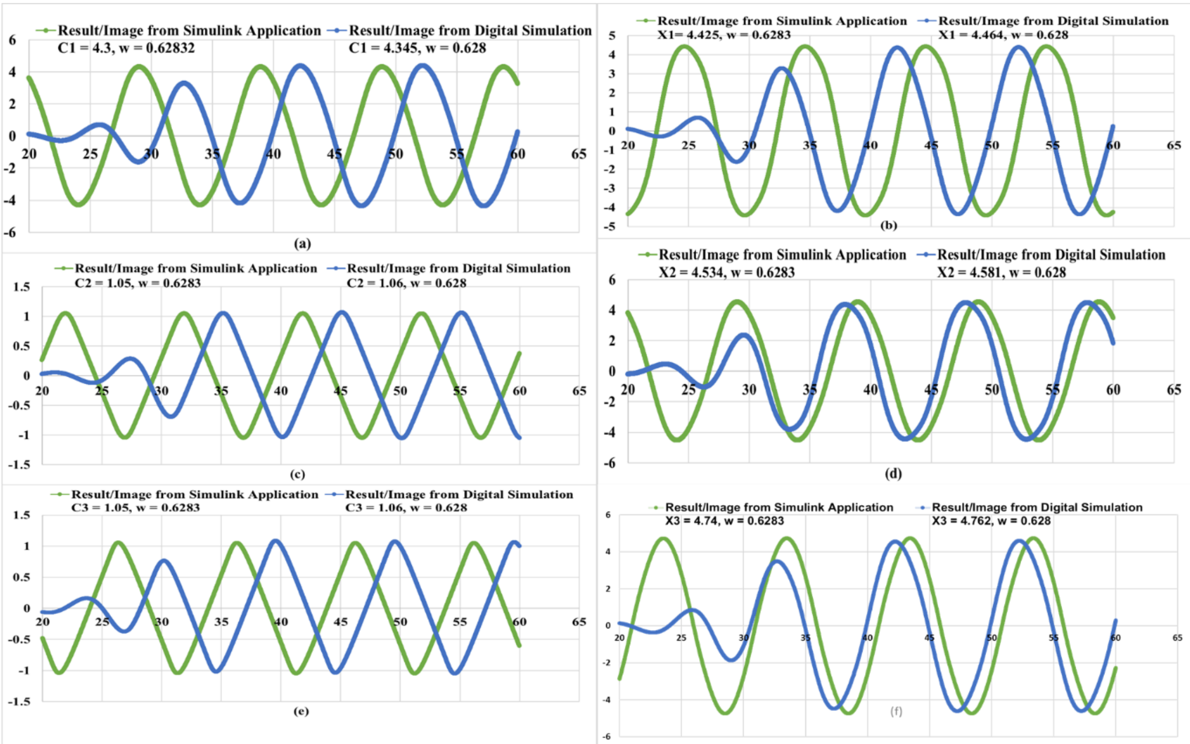


Fig. 6 (b): Images obtained using Digital Simulation and SIMULINK for C_1, C_2, C_3, X_1, X_2, X_3 of Ex. 2.

3. Quenching of LC with the method of Signal Stabilization in 3x3 Non-linear Systems:

3.1. Use of Deterministic Dither Signal

The concept of Signal Stabilization is narrated in 2x2 systems, as in [5,30,46,49,50]: In the event of exhibition of LC in 2x2 non-linear systems under autonomous condition, the quenching/extinction of LC can be done through injection of suitable high frequency dither signal, preferably 10 times more than the limit cycling frequency ωs. The process of quenching such oscillations is termed signal stabilization or forced oscillation, which has also been described appropriately in [30,33,46] for 2x2 systems and in [6] for 3x3 systems.

The quenching of limit cycling oscillations are achieved by the method of signal stabilization. This has been illustrated by injecting a high frequency signal 5 sin ωft where, ω = 7.5 rad/sec and 6.5 rad/sec shown in Fig. 7(a) and 7(b) for Ex. 1 & 2 respectively. The corresponding results/images are shown in Fig. 8(a) and 8(b) for Ex. 1 & 2 respectively.

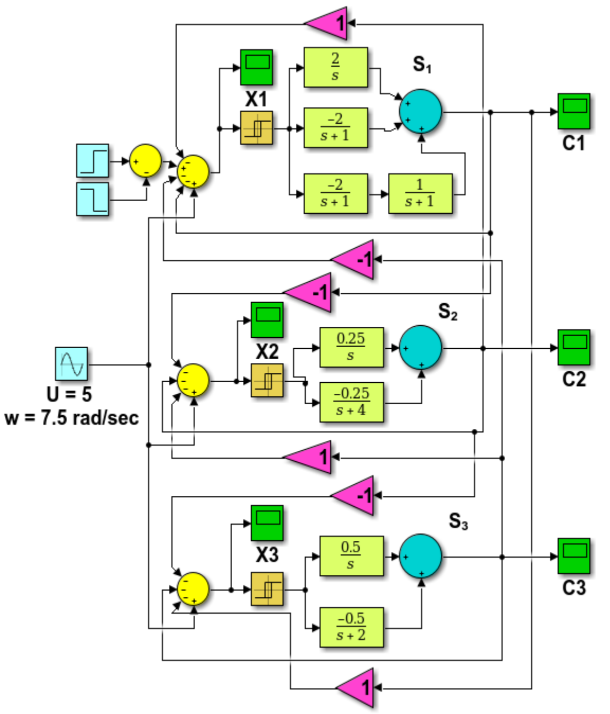


Fig. 7 (a): Equivalent block diagram of Fig1 for stabilizing the signal using SIMULINK Tool Box for Ex. 1.

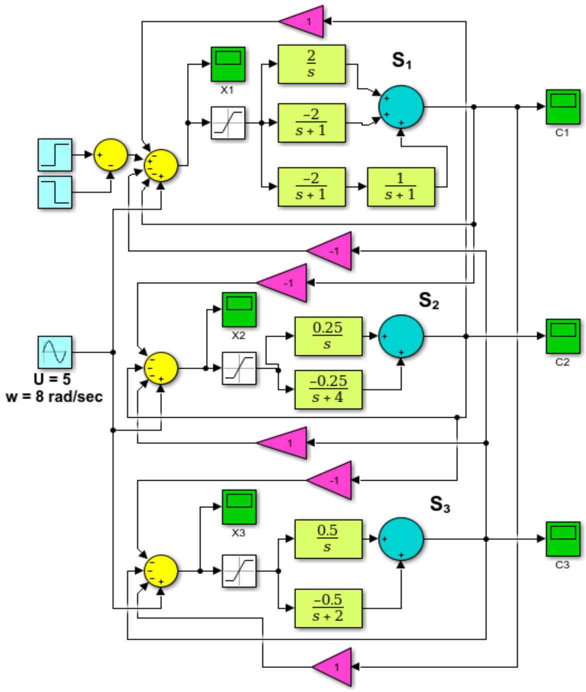


Fig. 7 (b): Equivalent block diagram of Fig1 for stabilizing the signal using SIMULINK Tool Box for Ex. 2.

3.1.1. Signal Stabilization with deadbeat approach

The process of signal stabilization is a type of response which exhibits both transients and steady states. Of course, with the proper amplitude of the dither signal, the synchronizing frequency should be the frequency of the dither signal at the steady state. However, the signal stabilization process is made faster and, in the minimum time, the steady-state synchronized value is determined without the transients and any steady-state error by a discrete signal which is termed as edged deadbeat approach of response

A digital system with all poles at origin and in Z-domain at z = 0 is termed as deadbeat [60].

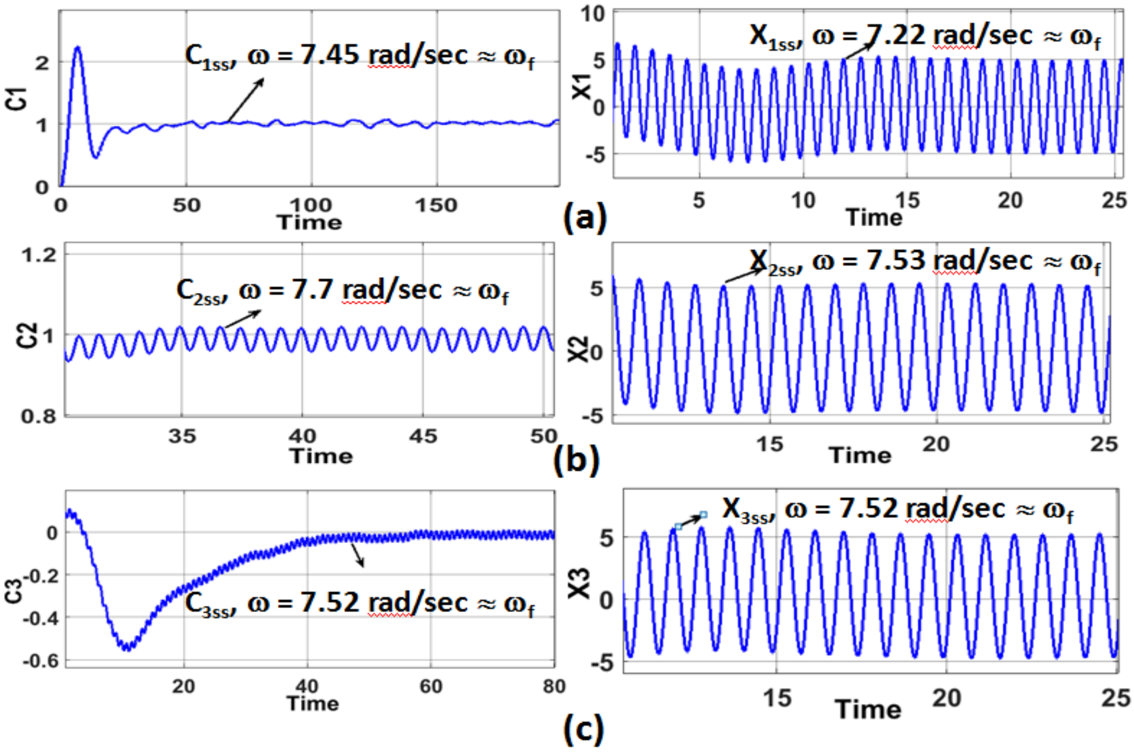


Fig. 8(a): Stabilized (Synchronized) results with Deterministic signal for Ex. 1. The forcing signal (Stabilizing signal) is U = 5 sinω_f t (ω_f = 7.5 rad / sec).

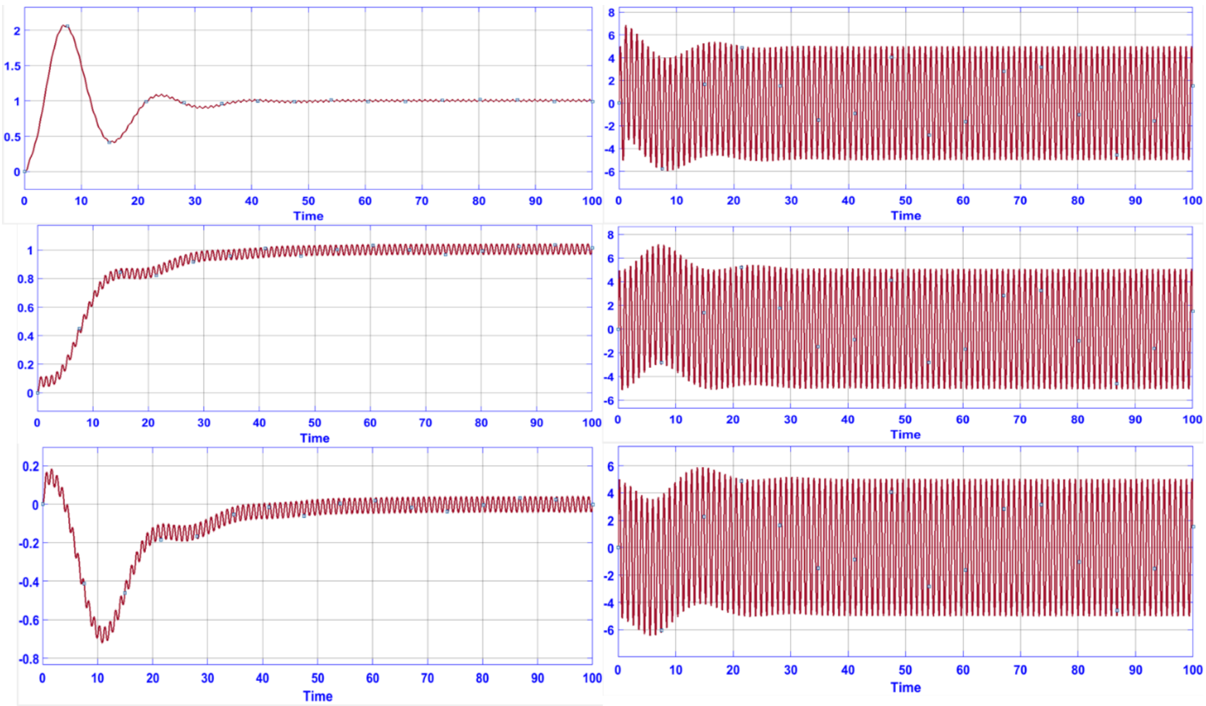


Fig. 8(b): Stabilized (Synchronized) results with Deterministic signal for Ex. 2. The forcing signal (Stabilizing signal) is U = 5 sinω_f t (ω_f = 6.5 rad / sec).

In design of a digital controller taking a simple model as shown in Fig. 9:

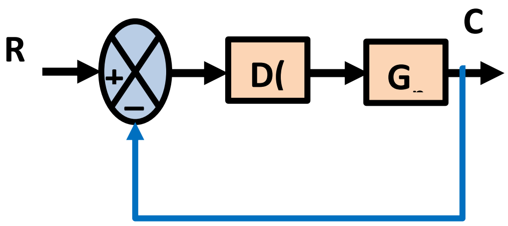


Fig. 9: A simple model of a Digital Controller with Unity Feedback

M(z) = Closed loop transfer function = $= \frac{C(z)}{R(z)} = \frac{1}{z^n}$ in order to have deadbeat response [61].

where n=No. of excess poles than zeroes.

In the signal stabilization the input is B sin $\omega_f t$.

Z[B sin ω_ft] = $\frac{\text{B z sin } \omega_f \text{T}}{z^2 - 2z \cos \omega_f \text{T} + 1}$, here n=1.

Hence $\frac{1}{z^n} = \frac{1}{z^1} = z^{-1}$. In the used of SIMULINK Tool Box a z^{-1} signal to be multiplied with the stabilized / synchronized output as shown in Fig. 10(a) and 10(b) for Ex. 1 & 2 respectively and corresponding results with deadbeat response has been shown in Fig. 11(a) & 11(b) for Ex. 1 & 2 respectively.

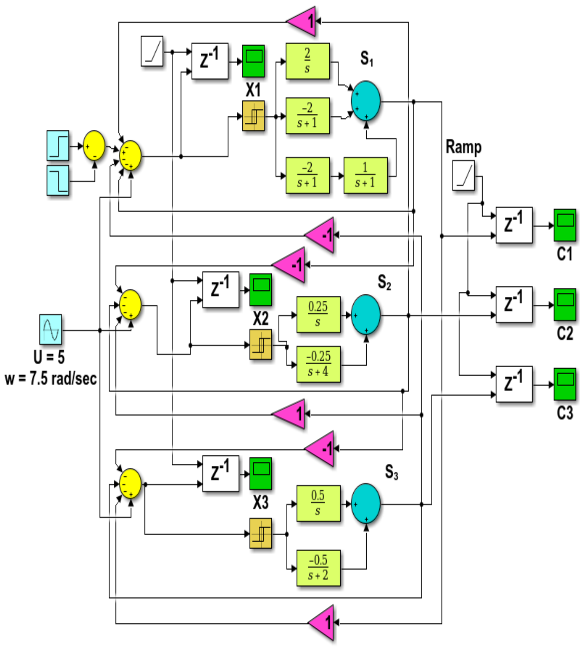


Fig. 10(a): Relevant Block Diagram of Fig. 1 for signal stabilization with deadbeat response using SIMULINK Tool Box for Ex. 1.

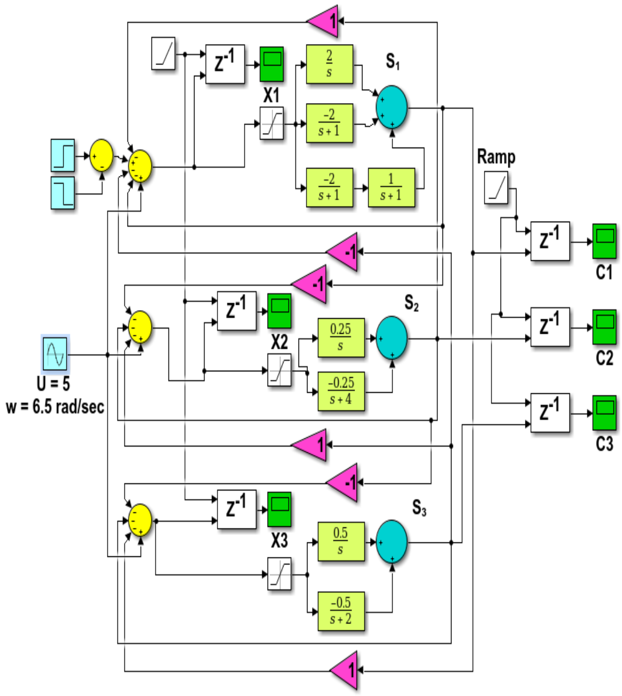


Fig. 10(b): Relevant Block Diagram of Fig1 for signal stabilization with deadbeat response using SIMULINK Tool Box for Ex. 2

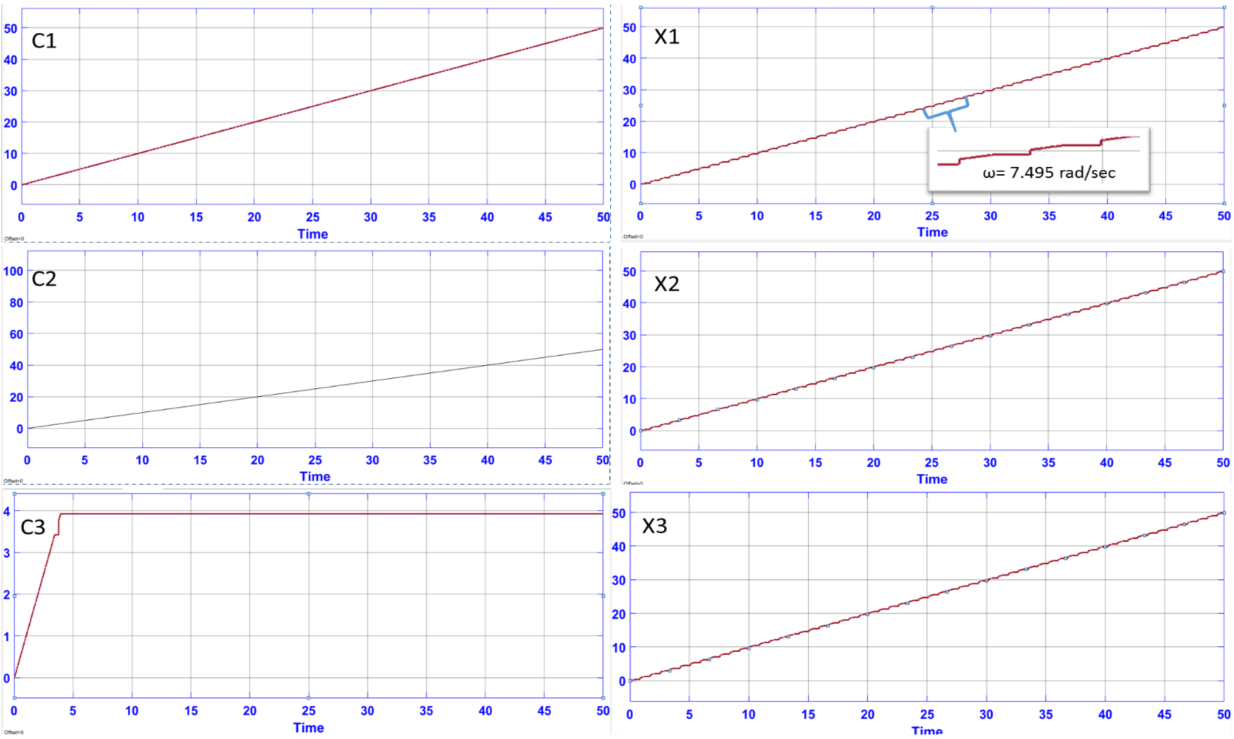


Fig. 11 (a): Stabilized with deadbeat signals using SIMULINK Tool Box for Example 1

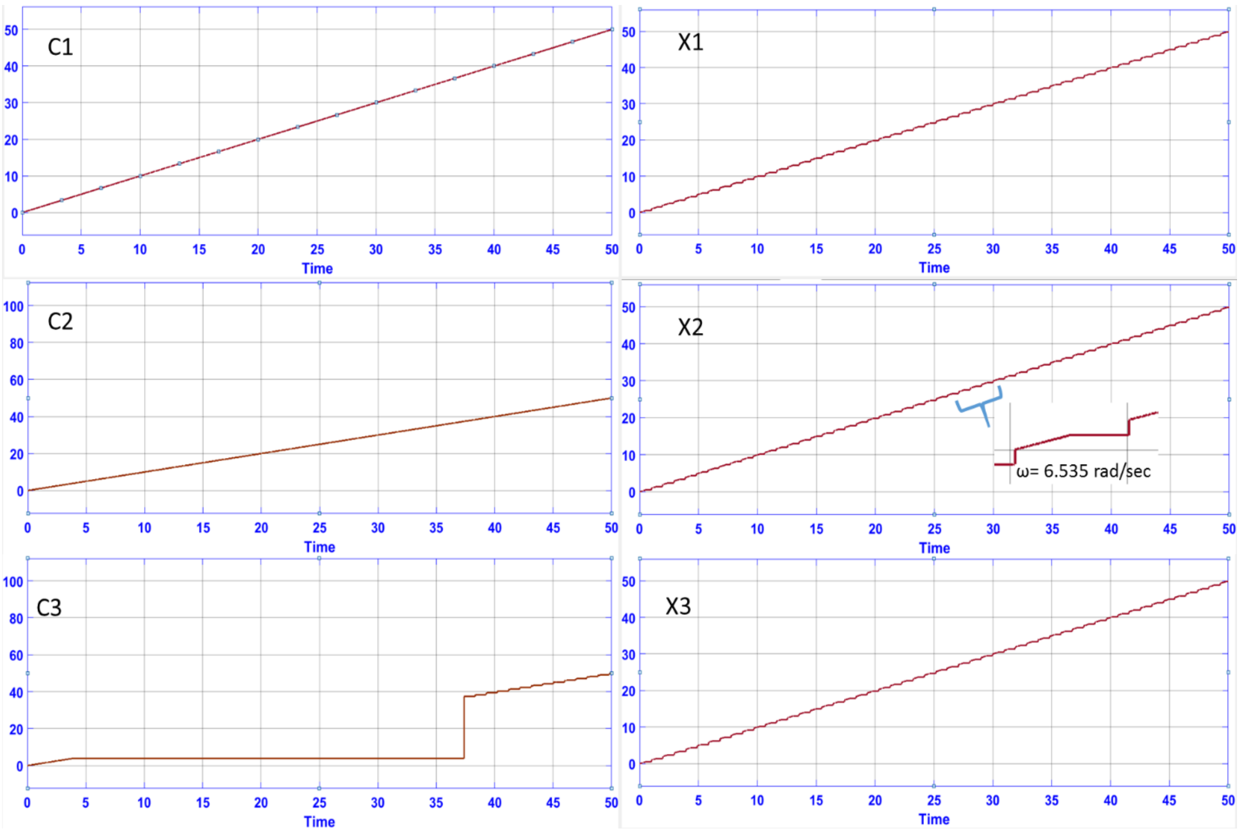


Fig. 11 (b): Stabilized with deadbeat signals using SIMULINK Tool Box for Example 2

4. Conclusion

It is always an approximation in Digital Control due to the sampling process and also an apprehension of an alias if there is any deviation in the sampling theorem in its execution / implementation. Still, the application of Digital Control is preferred because it provides a deadbeat response which is only possible in discrete systems that offer transient free, with no steady state error and a much faster response. This enables real-time applications due to their faster and error-free response.

Signal Stabilization with deadbeat response offered an exciting result in accuracy and fastness. The steady state was observed almost immediately after starting all the runs. Under steady state, the synchronized frequency was matched with the forcing signal frequency. In the present work, the novelty has been shown through digital simulation, which has also been predicted analytically. Signal Stabilization supported by dead beat response has never been attempted elsewhere, even in SISO systems. Because of its fastness in response, it can also be tried with real-time application in the future for any multivariable nonlinear systems where LC exhibits.

In comparison, of Fig. 8 (both (a) & (b)) with Fig. 11 (both (a) & (b)): Transient free response seen in Fig. 11 saves about 40 seconds (present in Fig. 8), which is a remarkable saving of time in response. This facilitates the adaption of such digital controllers in real-time applications.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article

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